The 6th KMI International Symposium @ Nagoya Univ.

Stochastic GW background as a probe of the early universe

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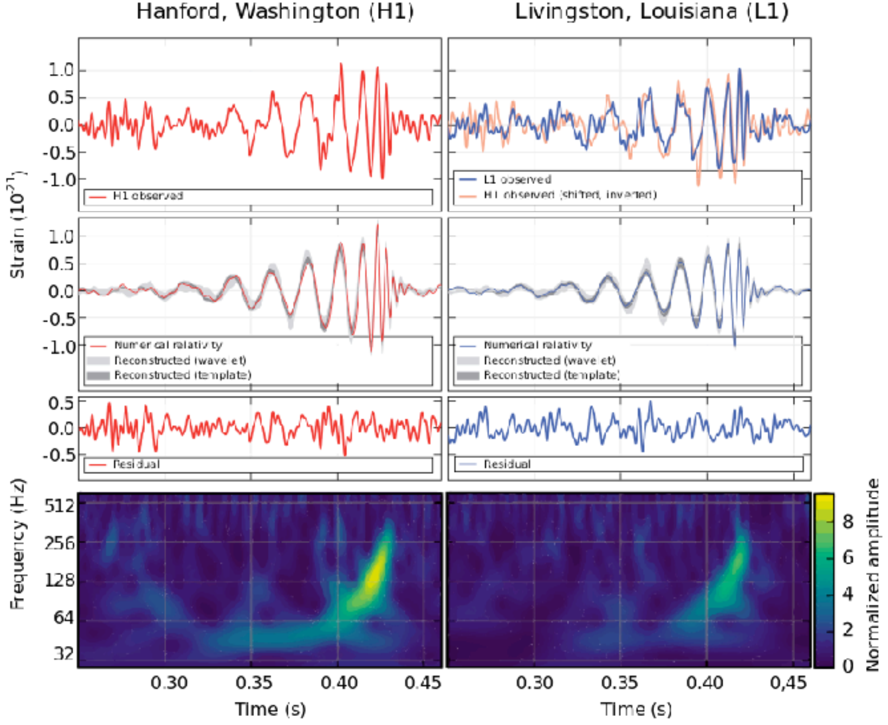
Gravitational wave (GW) observation

LIGO-India (2027-)

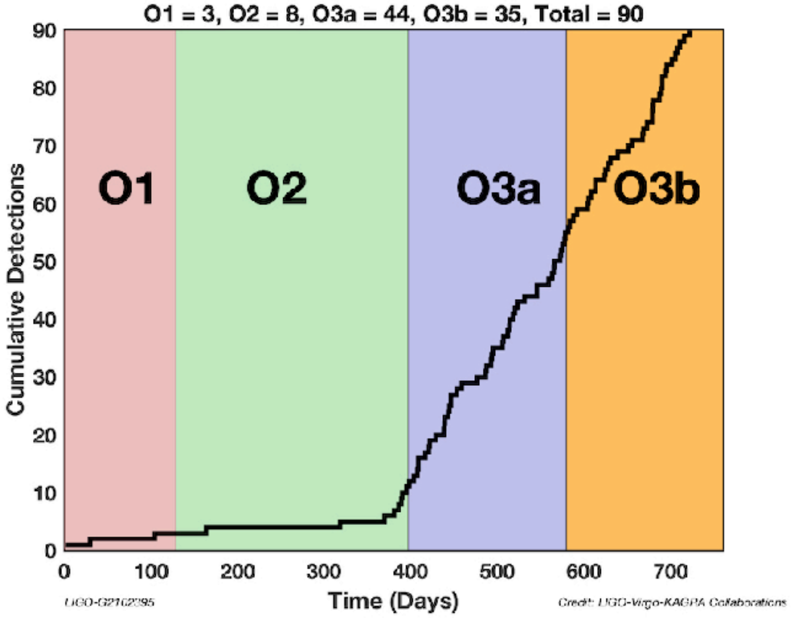
KAGRA (2020-)

Advanced-LIGO (2015-)

Advanced-VIRGO (2017-)



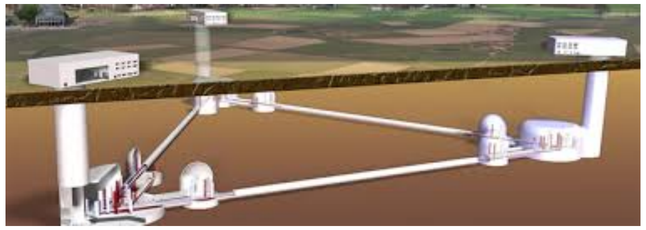
The first detection: GW150914



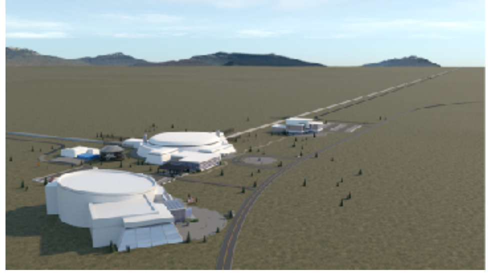
O4 started in May 2023
and still ongoing

Future projects

- Einstein Telescope



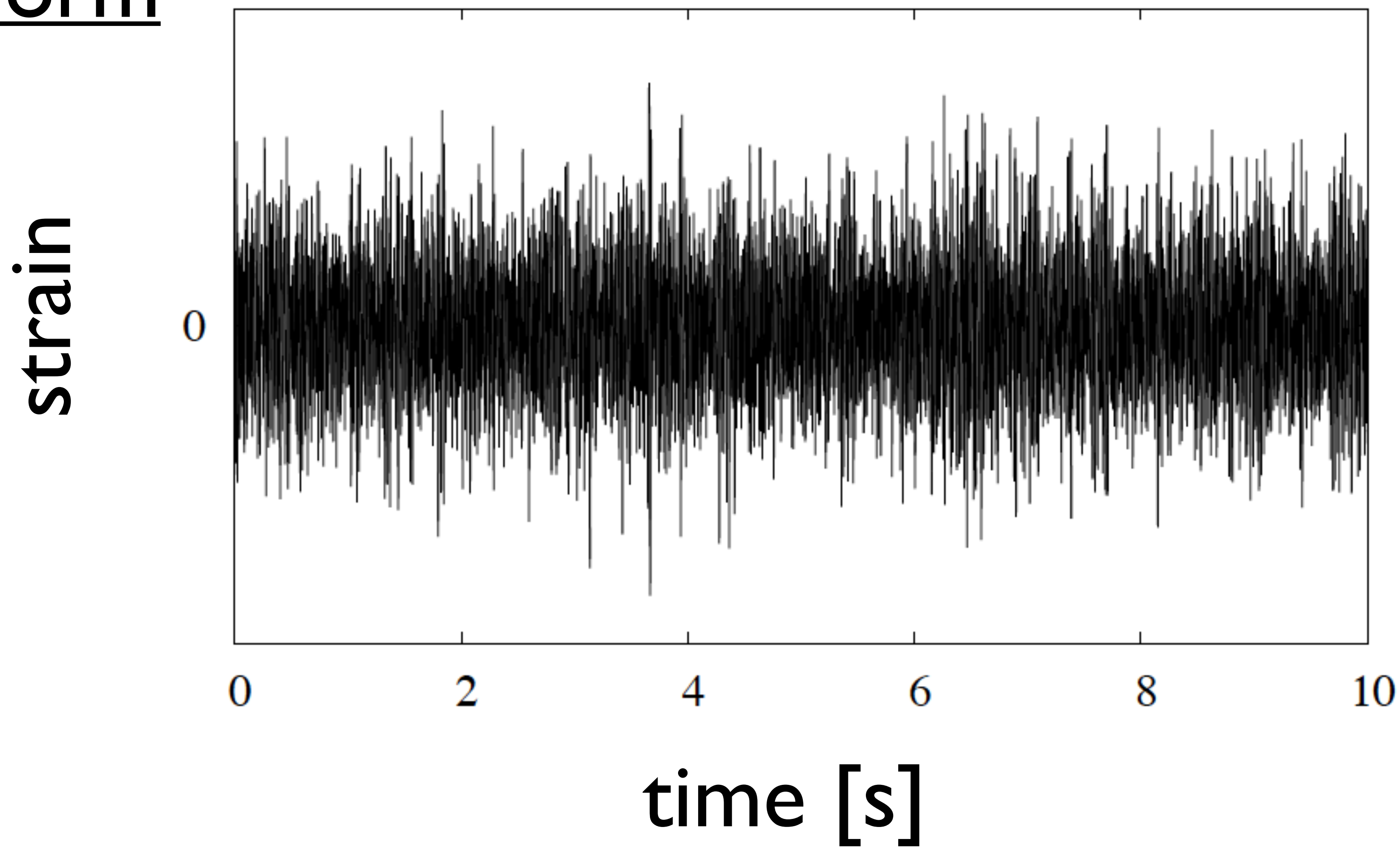
- Cosmic Explorer



etc.

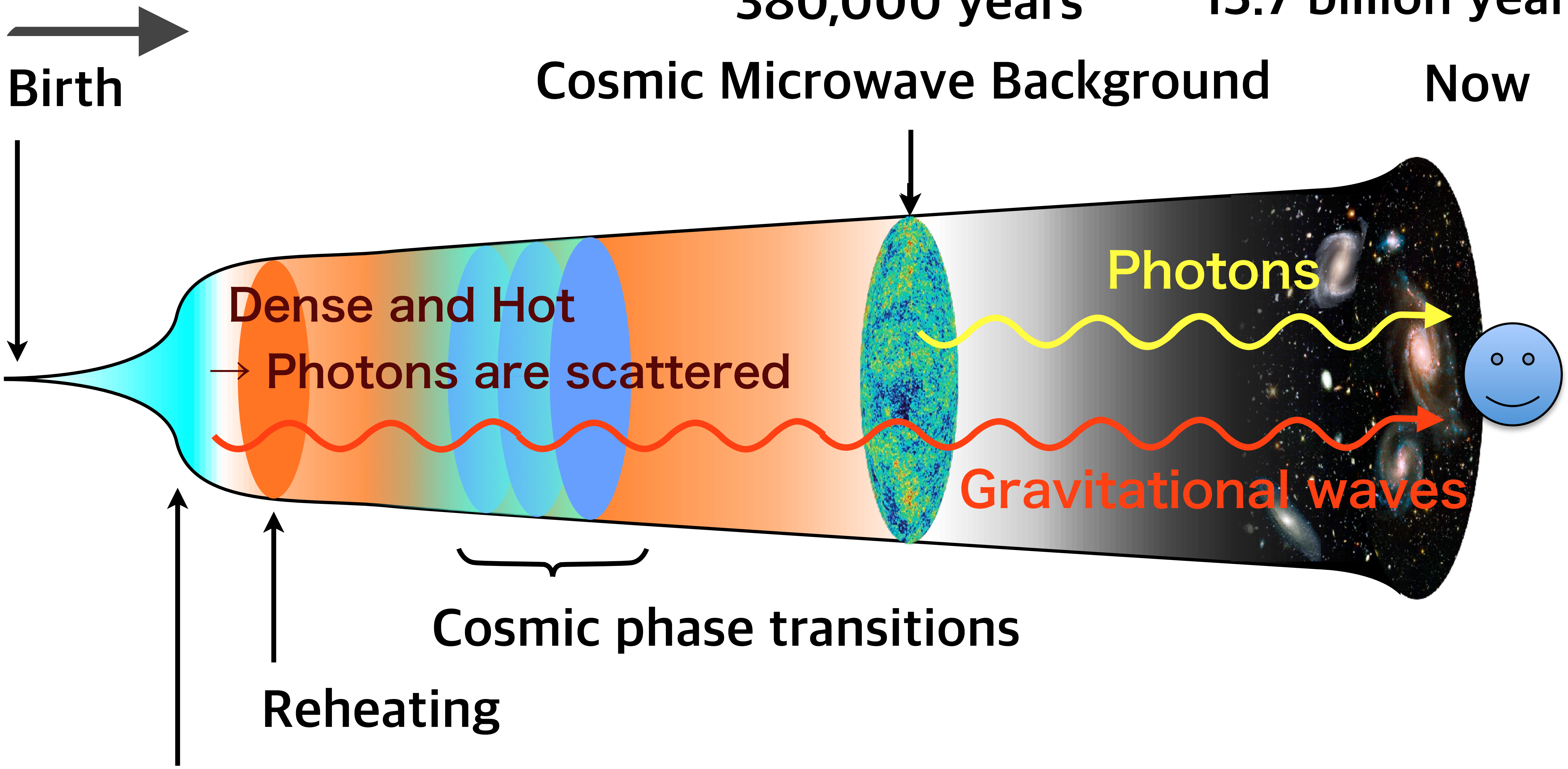
Stochastic GW background

Waveform



Continuous and random GW signal
coming from all directions → very similar to noise

Stochastic GW background as a probe of the early universe



Time

Birth

380,000 years

Cosmic Microwave Background

13.7 billion years

Now

Dense and Hot

→ Photons are scattered

Photons

Gravitational waves

Cosmic phase transitions

Reheating

Inflation

→ generate a stochastic background

→ **a unique test of high energy theories**

How to detect a stochastic background

Cross Correlation

detector1

$$s_1(t) = h(t) + n_1(t)$$

detector2

$$s_2(t) = h(t) + n_2(t)$$

s: observed signal
h: gravitational waves
n: noise

$$\langle S \rangle = \int_{-T/2}^{T/2} dt\ \langle s_1(t) s_2(t) \rangle$$

$$= \int_{-T/2}^{T/2} dt\ \langle h^2(t) + \underbrace{h(t)n_2(t) + n_1(t)h(t) + n_1(t)n_2(t)}_{\text{no correlations} \to 0} \rangle$$

$$= \int_{-T/2}^{T/2} dt\ \underline{\langle h^2(t) \rangle}$$

GW signal

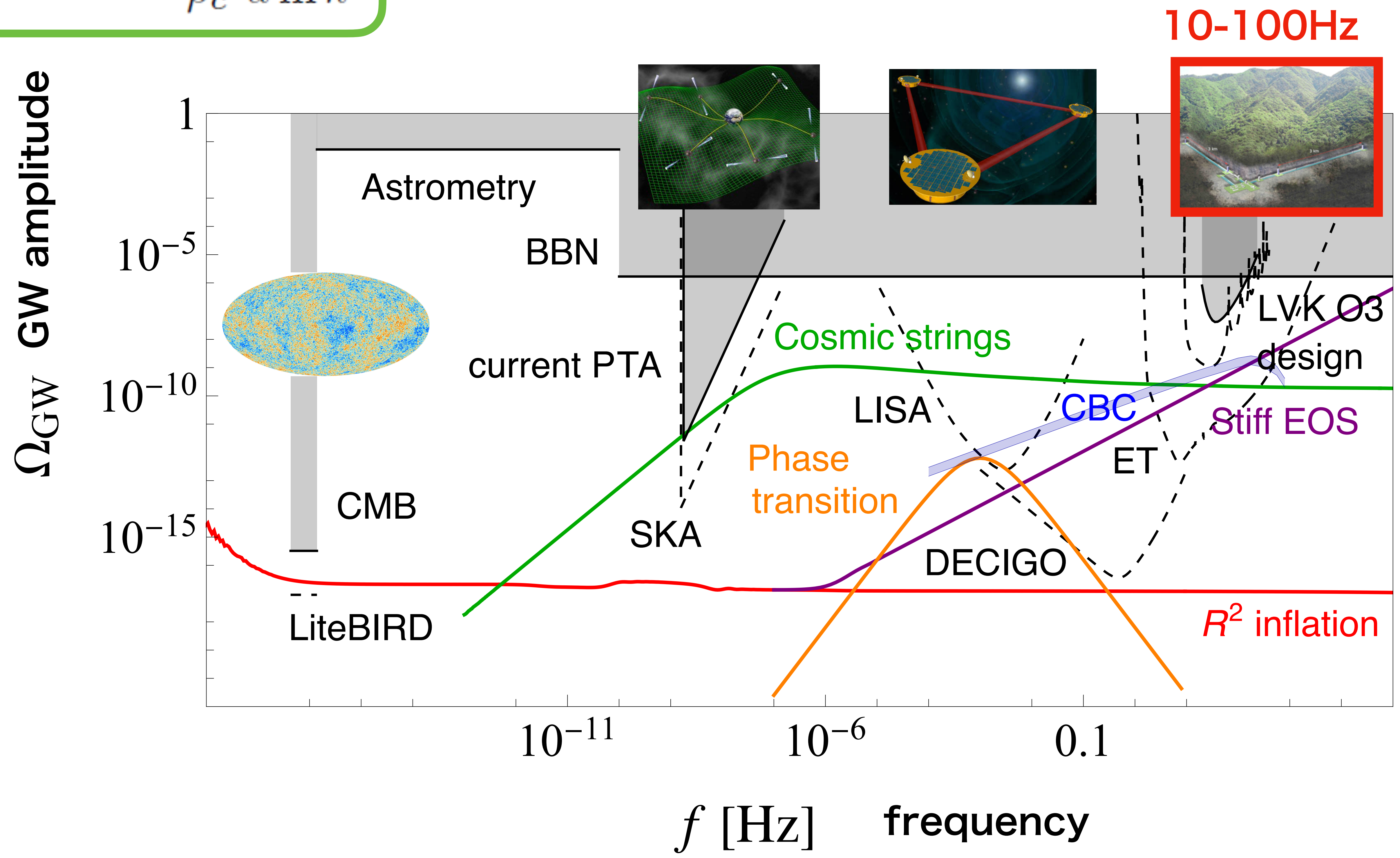
(for detectors at the same location)

Experiment’s sensitivities

$$\Omega_{\mathrm{GW}} \equiv \frac{1}{\rho_c}\frac{d\rho_{\mathrm{GW}}}{d\ln k}$$

ρ_{GW}: Energy density of GWs

ρ_c: Critical density of the Universe



Upper bounds on stochastic background

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_c} \frac{d\rho_{\rm GW}}{d\ln k}$$

$\rho_{\rm GW}$: Energy density of GWs

ρ_c : Critical density of the Universe

BBN + CMB bound:
$\Omega_{\rm GW} < 2.7 \times 10^{-6}$

Observation run	**Year**	**95% Upper bound** (for a flat spectrum)
LIGO S6	2009	$\Omega_{\rm GW} < 6.9 \times 10^{-6}$
Advanced LIGO O1	2016	$\Omega_{\rm GW} < 1.7 \times 10^{-7}$
Advanced LIGO O2	2019	$\Omega_{\rm GW} < 6.0 \times 10^{-8}$
Advanced LIGO O3 (+ Virgo)	2021	$\Omega_{\rm GW} < 5.8 \times 10^{-9}$
Advanced LIGO O4a	2025	$\Omega_{\rm GW} <$???

→ becoming a competitive tool to constrain early universe models

Upper bounds on stochastic background

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Let us provide detailed constraints!
(but using O3 data; O4a data release expected in August 2025)

Generation mechanisms

Evolution equation for GWs

anisotropic stress

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = 16\pi G\Pi_{ij}$$

1. Non-negligible initial condition

- **Inflation**

→ quantum fluctuations

2. Sourced by matter component of the Universe

- **Preheating**

→ rapid particle productions

- **Phase transition**

→ bubble collisions

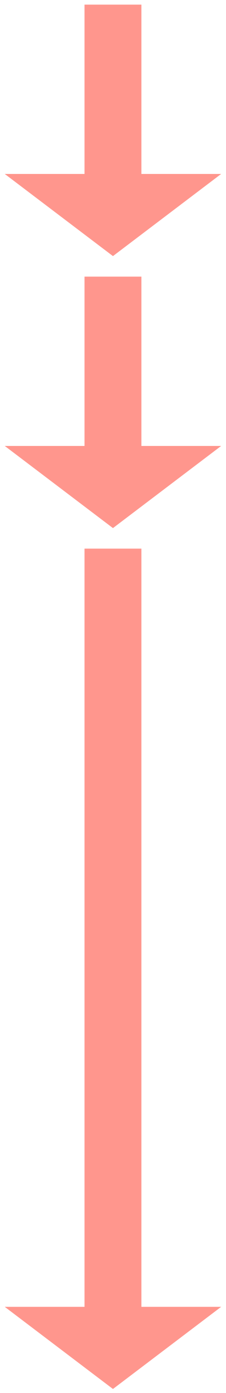
- **Cosmic strings**

→ heavy strings generated in phase transition

Models to investigate

Several early universe models predict a relatively large GW amplitude

Universe's history



Inflation

Reheating

Radiation dominated

Matter dominated

GWs from inflation: Typically very small $\Omega_{GW} < 10^{-16}$ Type 1

① SU(2) Gauge field during inflation Type 2

→ act as a source of GWs

② Kination phase after inflation Type 1

→ inflationary GWs decay slower than usual

③ Scalar induced GWs Type 2

→ large curvature perturbations (that eventually forms PBHs) act as a source of GWs

④ Primordial black hole (PBH) mergers Type 2

→ PBHs form binary system and emit GWs superpositions form a stochastic background

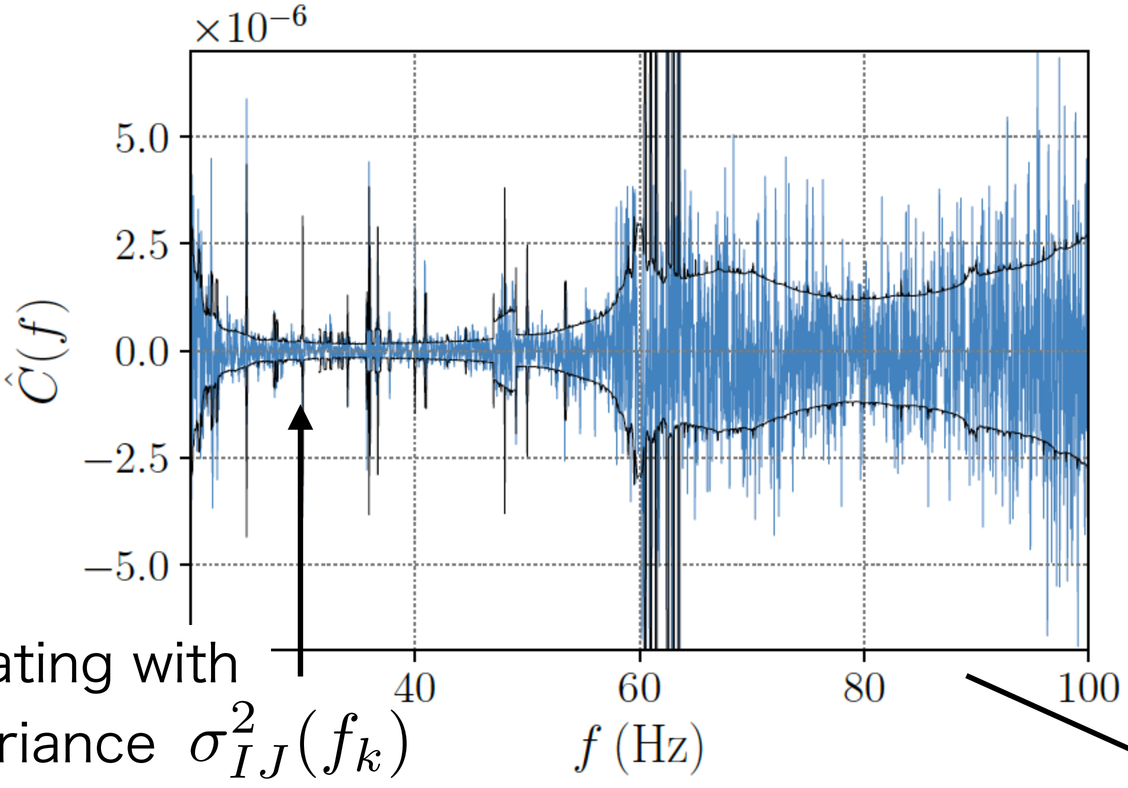
Likelihood analysis

Renzini et al., ApJ 952, 25 (2023)

pygwb 1.5.1

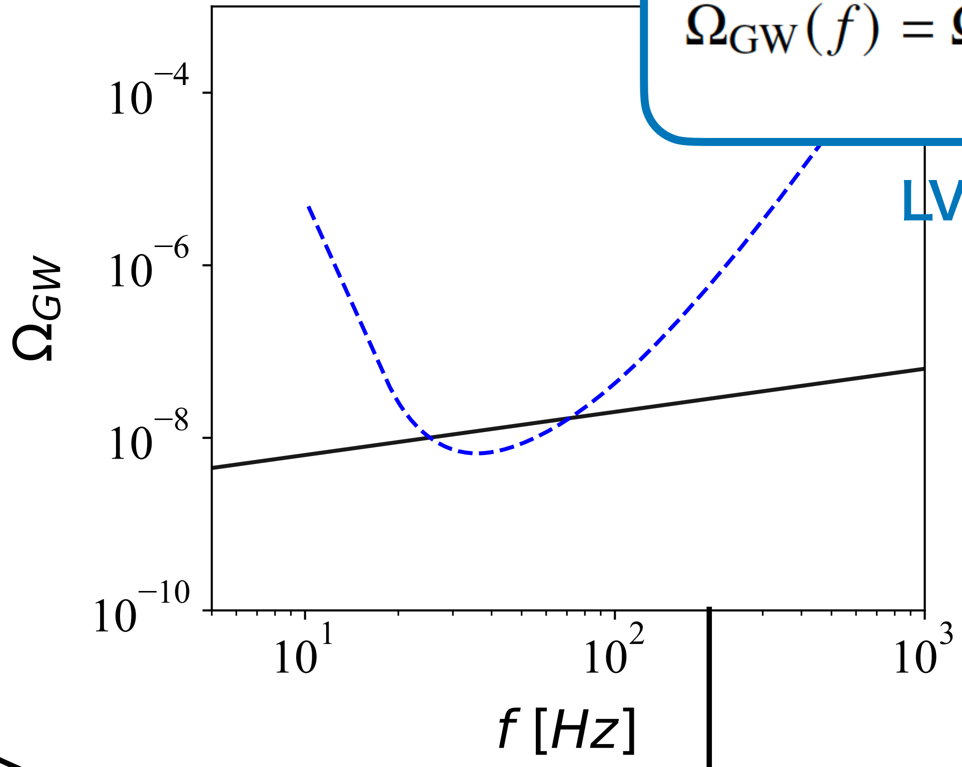
`pip install pygwb`

data (cross-correlated spectrum)



fluctuating with
the variance $\sigma^2_{IJ}(f_k)$

Your model



Model = power-law

$$\Omega_{\mathrm{GW}}(f) = \Omega_\alpha \left(\frac{f}{f_{\mathrm{ref}}}\right)^\alpha$$

LVK analysis

Likelihood

$$p\left(\hat{\Omega}^{IJ}_{\mathrm{GW},f}|\mathbf{\Theta}\right) \propto \exp\left[-\frac{1}{2}\sum_{IJ}^{B}\sum_{f}\left(\frac{\hat{\Omega}^{IJ}_{\mathrm{GW},f}-\Omega_{\mathrm{M}}(f|\mathbf{\Theta})}{\sigma^{IJ}_{\mathrm{GW},f}}\right)^2\right]$$

variance

IJ: detector combinations
k: frequencies

Prior

Posterior distribution

$$p(\mathbf{\Theta}|C_k^{IJ}) \propto p(C_k^{IJ}|\mathbf{\Theta})p(\mathbf{\Theta})$$

① SU(2) gauge field during inflation

Coupling between inflaton and SU(2) gauge field

$$S_{\rm CNI} = \int d^4x\sqrt{-\det(g_{\mu\nu})}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \underline{\frac{\alpha_f}{4}\phi F^a_{\mu\nu}\tilde{F}^{a\mu\nu}}\right]$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\varepsilon^{abc}A^b_\mu A^c_\nu$$

→ Anisotropic stress of the gauge field sources GWs during inflation

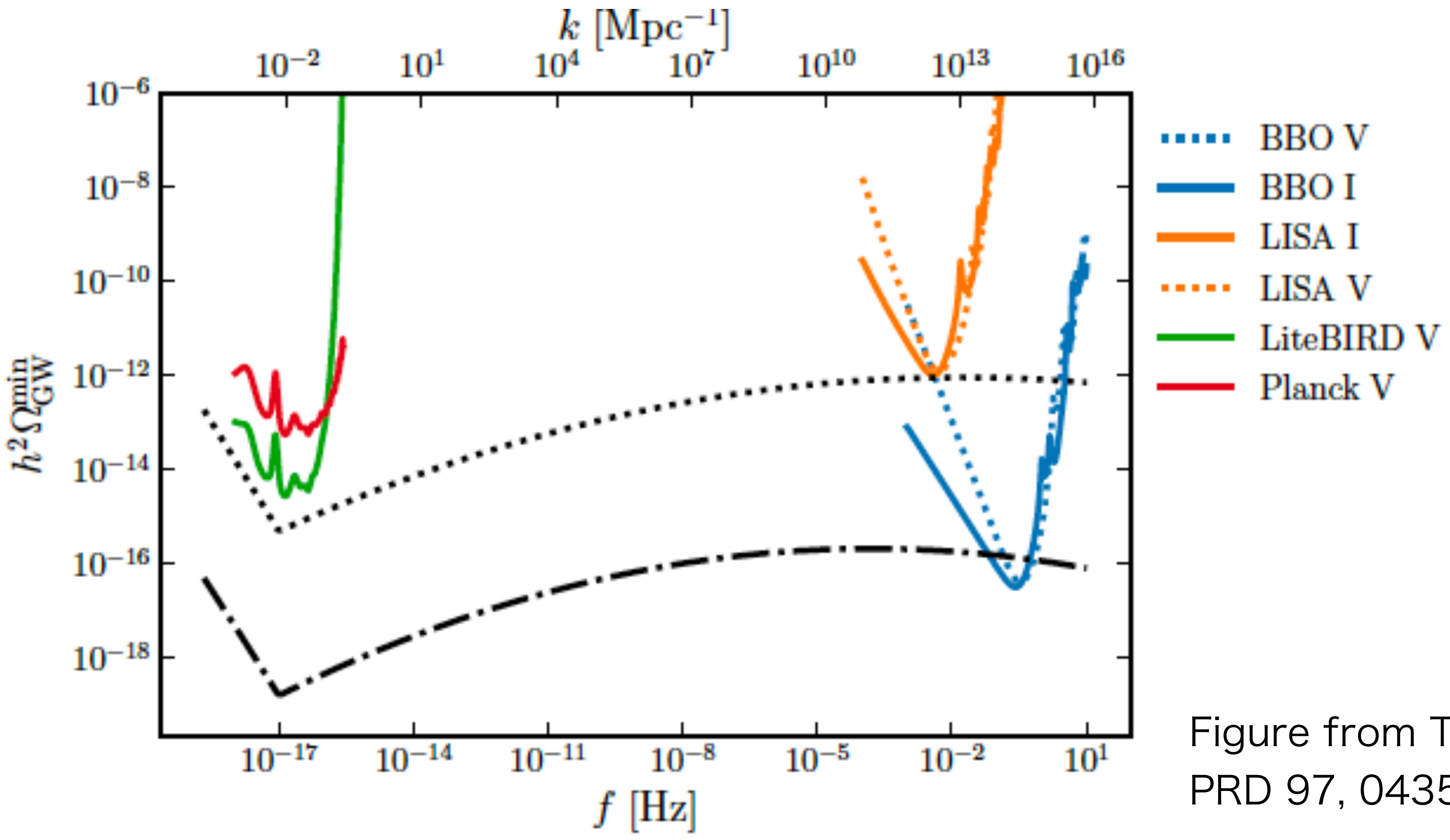
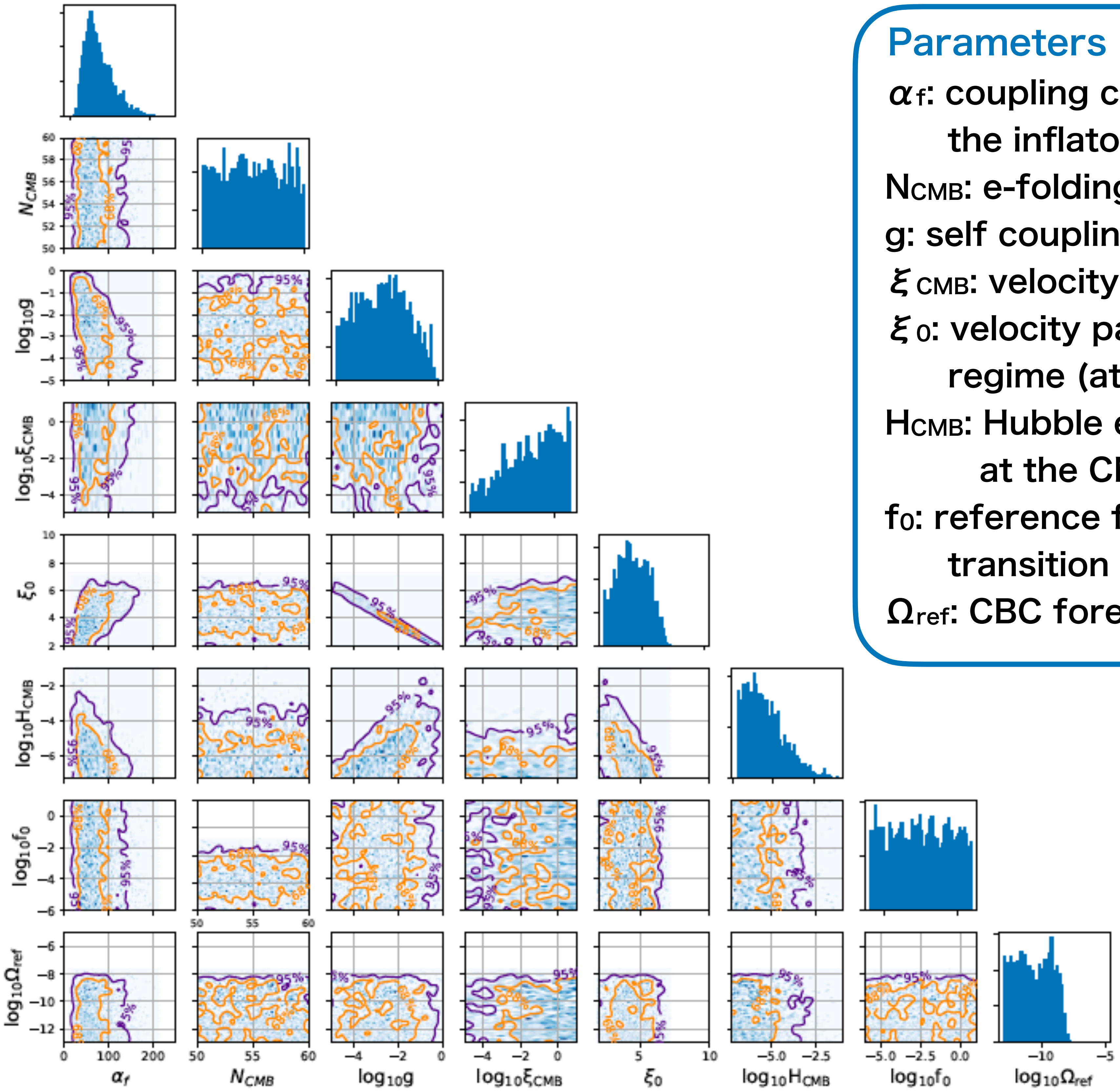


Figure from Thorne et al., PRD 97, 043506 (2018)

① SU(2) gauge field during inflation



Parameters

α_f: coupling constant between
the inflaton and the gauge field

N_{CMB}: e-folding number

g: self coupling constant of the gauge field

ξ_{CMB}: velocity parameter at the CMB

ξ_0: velocity parameter in the non-abelian
regime (at the LVK)

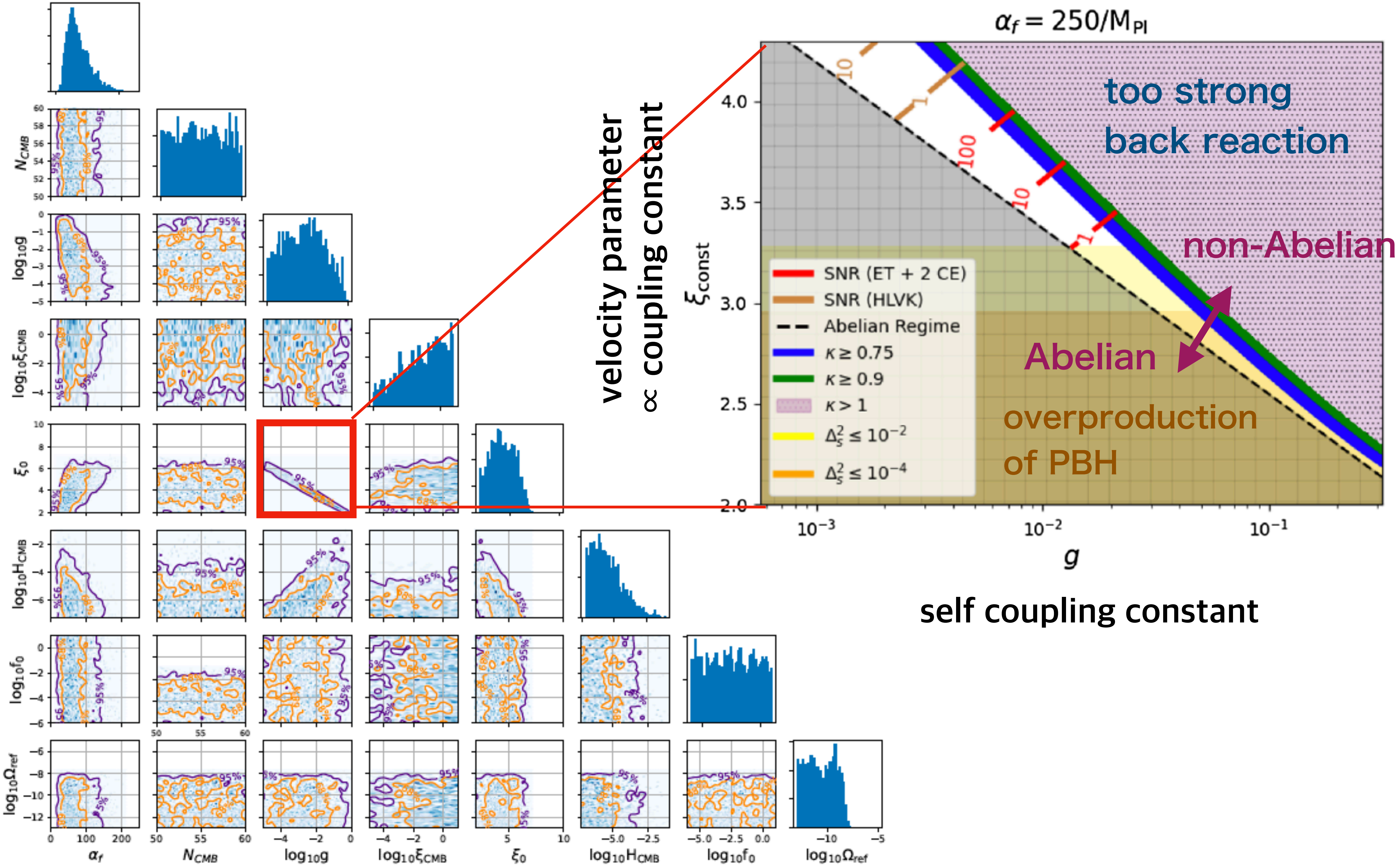
H_{CMB}: Hubble energy scale of inflation
at the CMB scale

f_0: reference frequency corresponding to the
transition from Abelian to non-Abelian regime

Ω_{ref}: CBC foreground

C. Burger et al. (+SK), PRD 110.084063 (2024)

① SU(2) gauge field during inflation



velocity parameter
∝ coupling constant

too strong
back reaction

non-Abelian

Abelian

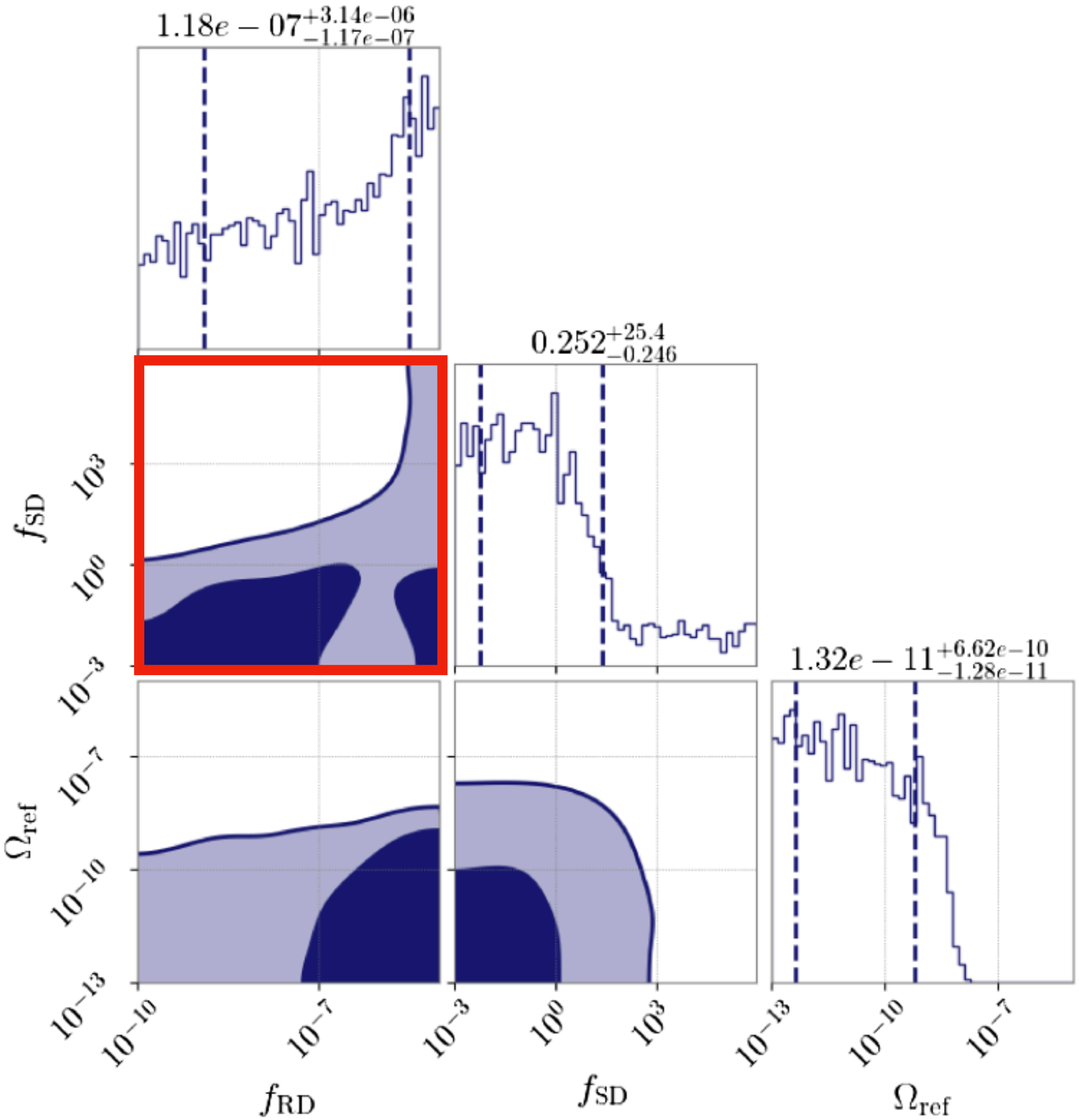
overproduction
of PBH

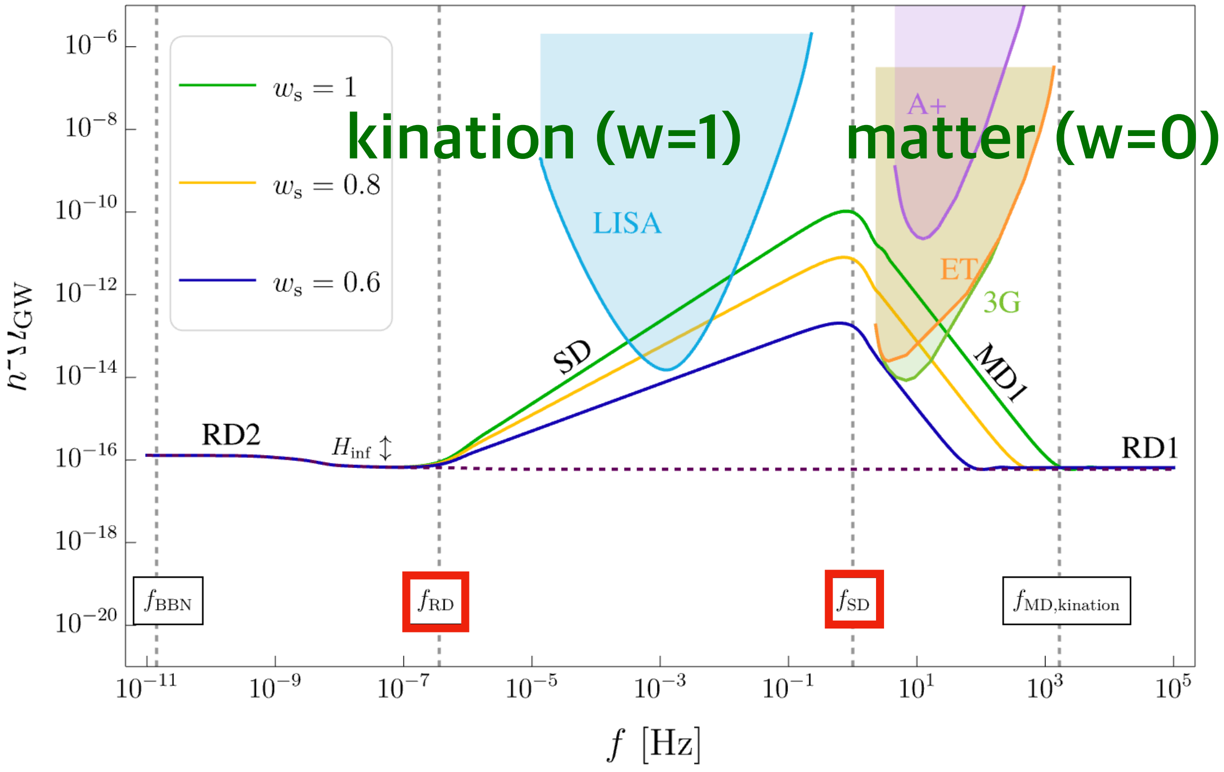
self coupling constant

C. Burger et al. (+SK), PRD 110.084063 (2024)

② Early kination phase (stiff EoS)

Equation of state (EoS, w) larger than 1/3 leads to an enhancement in inflationary GWs





kination (w=1)

matter (w=0)

Early matter & kination domination (realized in axion motivated model)

Ko & Harigaya, PRL 124, 111602 (2020)

Gouttenoire et al. arXiv:2111.01150

$f_{\rm RD}$, $f_{\rm SD}$: transition frequencies

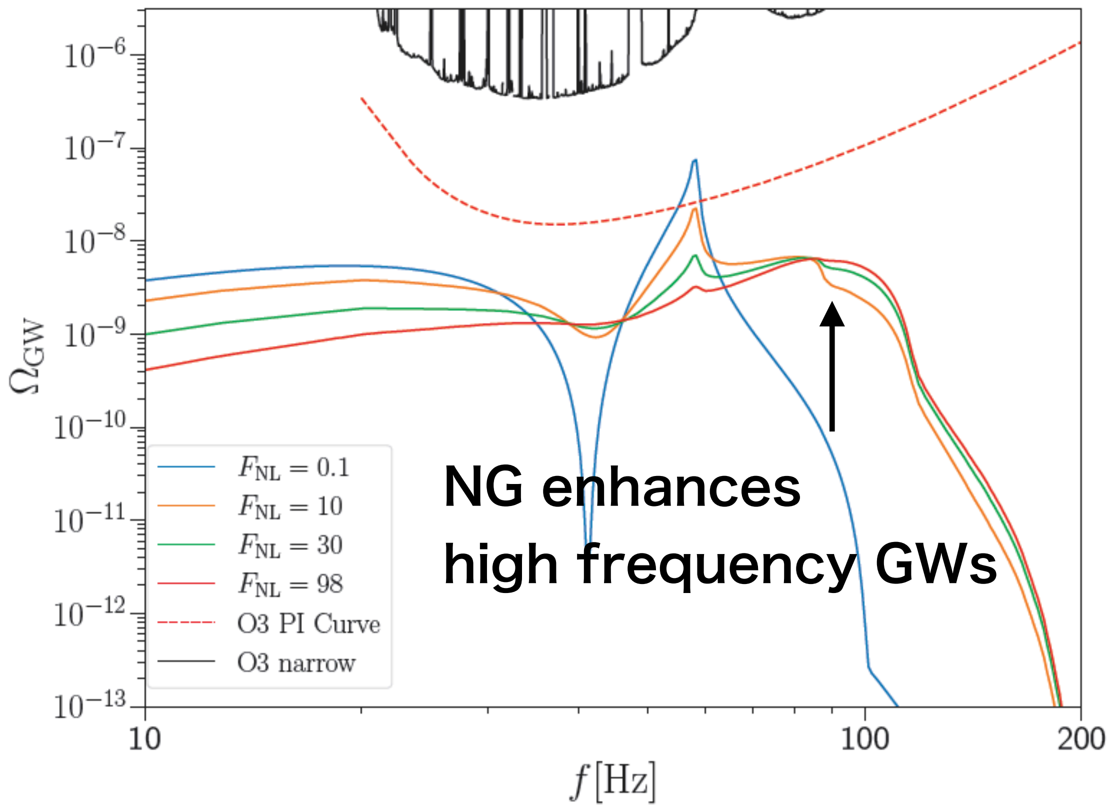
→ Connects to the energy scale of the Universe at the time of transitions (MD→KD→RD)

H. Duval et al. (+SK), PRD 110.103503 (2024)

③ Scalar induced GWs

Enhanced primordial curvature perturbations source both PBHs and GWs

GW spectrum



NG enhances
high frequency GWs

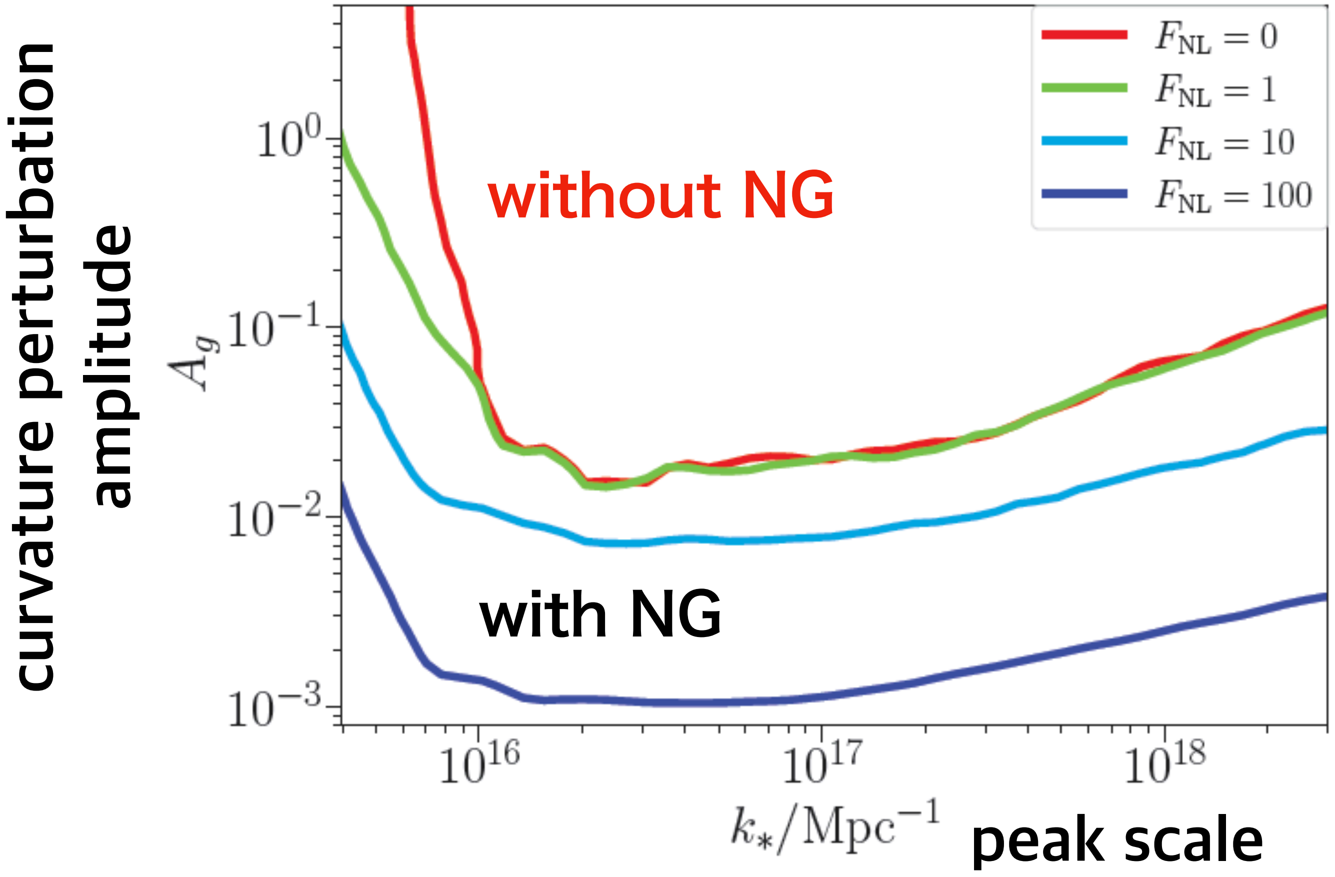
Assumption:
local type non-Gaussianity

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + F_{\rm NL}\zeta_g^2(\mathbf{x})$$

Note: Parametrization with $F_{\rm NL}$
covers limited cases

Many inflationary models predicting large curvature perturbations (and producing PBHs) exhibit Non-Gaussianity (NG)

- ultra slow roll inflation
- multi field inflation
- couplings leading to particle production, etc.



without NG

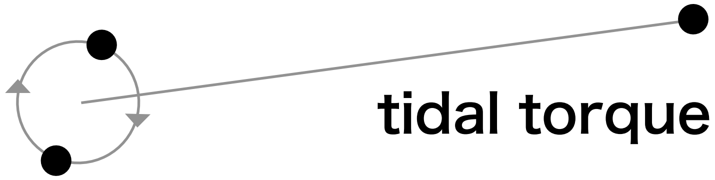
with NG

R. Inui, S. Jaraba, SK, S. Yokoyama, JCAP 05 082 (2024)

② PBH binaries

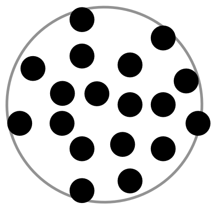
Superpositions of PBH binaries form a stochastic background

Early binary formation

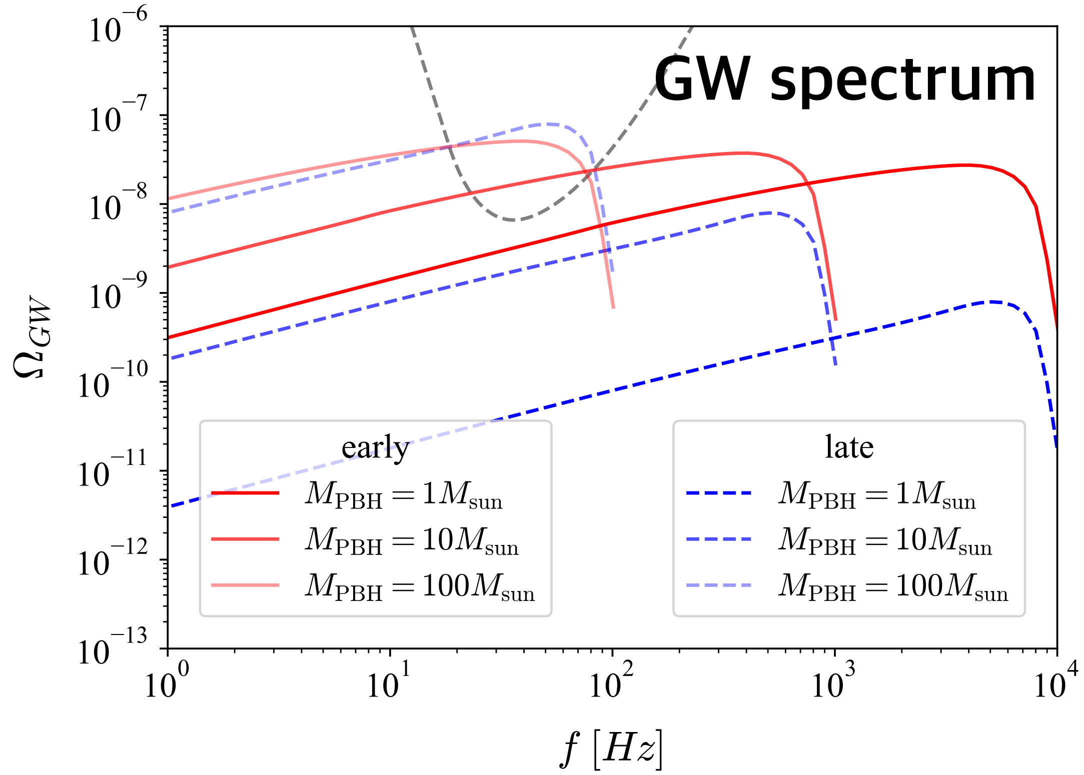


tidal torque

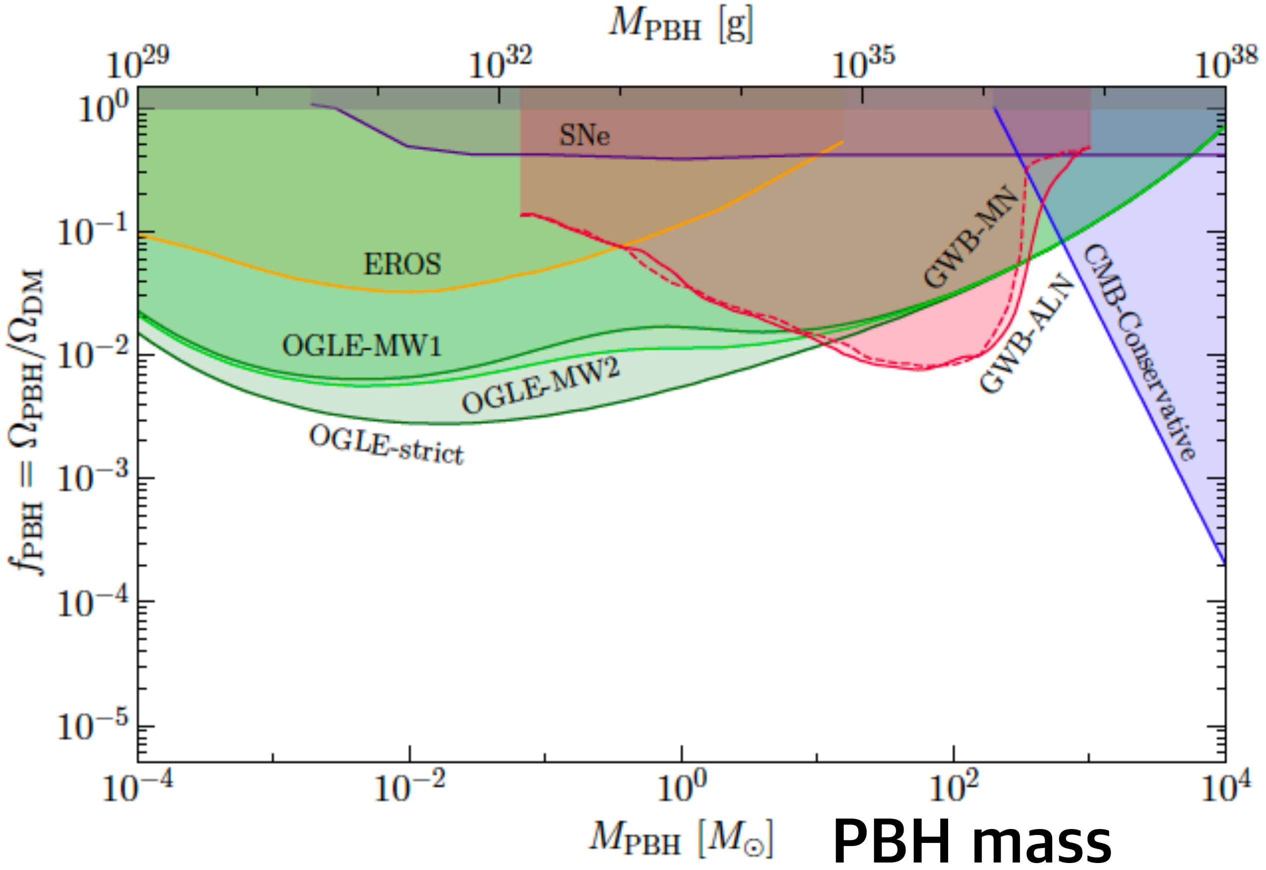
Late binary formation



dynamical capture
in a cluster



fraction of PBHs
with respect to DM

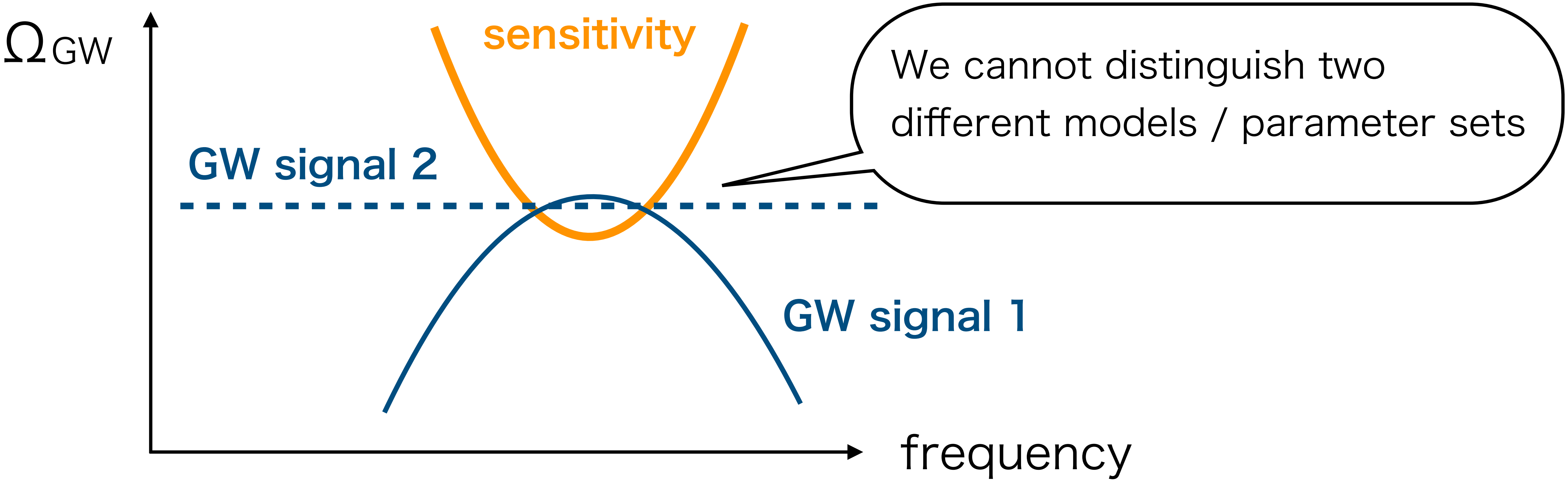


PBH mass

T. Boybeyi, S. Clesse, SK, M. Sakellariadou, arXiv:2412.18318

Note

GW observation currently provide limited information:
Only Ω_{GW} amplitude in the narrow frequency range



We cannot distinguish two different models / parameter sets

→ There are often degeneracies in parameter space when the model has many parameters.

Multi-band observations (CMB, PTA, space, ground, etc.) may play a key role in the future.

Summary

Stochastic GW background can be a unique probe of high-energy physics in the early universe

- Upper bounds on the stochastic background amplitude keeps improving with upgraded sensitivities and longer observation time.

- We can provide constraints on different types of model using the public code by the LVK collaboration (pygwb)

- We have provided constraints on model parameters for the early universe using the LVK O3 data

1. **SU(2) gauge field inflation** Burger et al. (+SK), PRD 110.084063 (2024)
2. **Early kination phase** Duval et al. (+SK), PRD 110.103503 (2024)
3. **Scalar induced GWs** Inui et al. (+SK) JCAP 05, 082 (2024)
4. **PBH binaries** Boybeyi et al. (+SK), arXiv:2412.18318