Vlasov-Poisson dynamics of cold dark matter around collapse

Shohei Saga (KMI, Nagoya)

KMI2025, Mar 5−7, 2025

1. Cold dark matter (CDM)

The evolution of the large-scale structure is dominated by the gravity of **CDM**

a self-gravitating collisionless fluid following **Vlasov-Poisson equations**

phase-space distribution function

Vlasov equation $$\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{v}}{a}\cdot\frac{\partial}{\partial \boldsymbol{x}} - \frac{1}{a}\boldsymbol{\nabla}\Phi\cdot\frac{\partial}{\partial \boldsymbol{v}}\right] f(t,\boldsymbol{x},\boldsymbol{v}) = 0$$

Poisson equation $$\frac{1}{a^2}\nabla^2\Phi = 4\pi G\left[\frac{1}{a^3}\int \mathrm{d}^3\boldsymbol{v}\, f(t,\boldsymbol{x},\boldsymbol{v}) - \bar{\rho}_{\mathrm{m}}\right]$$

Need to solve the 6D phase space evolution – computationally costly
(N-body simulation is an “effective” solver of Vlasov-Poisson equations)

2. *Cold* nature

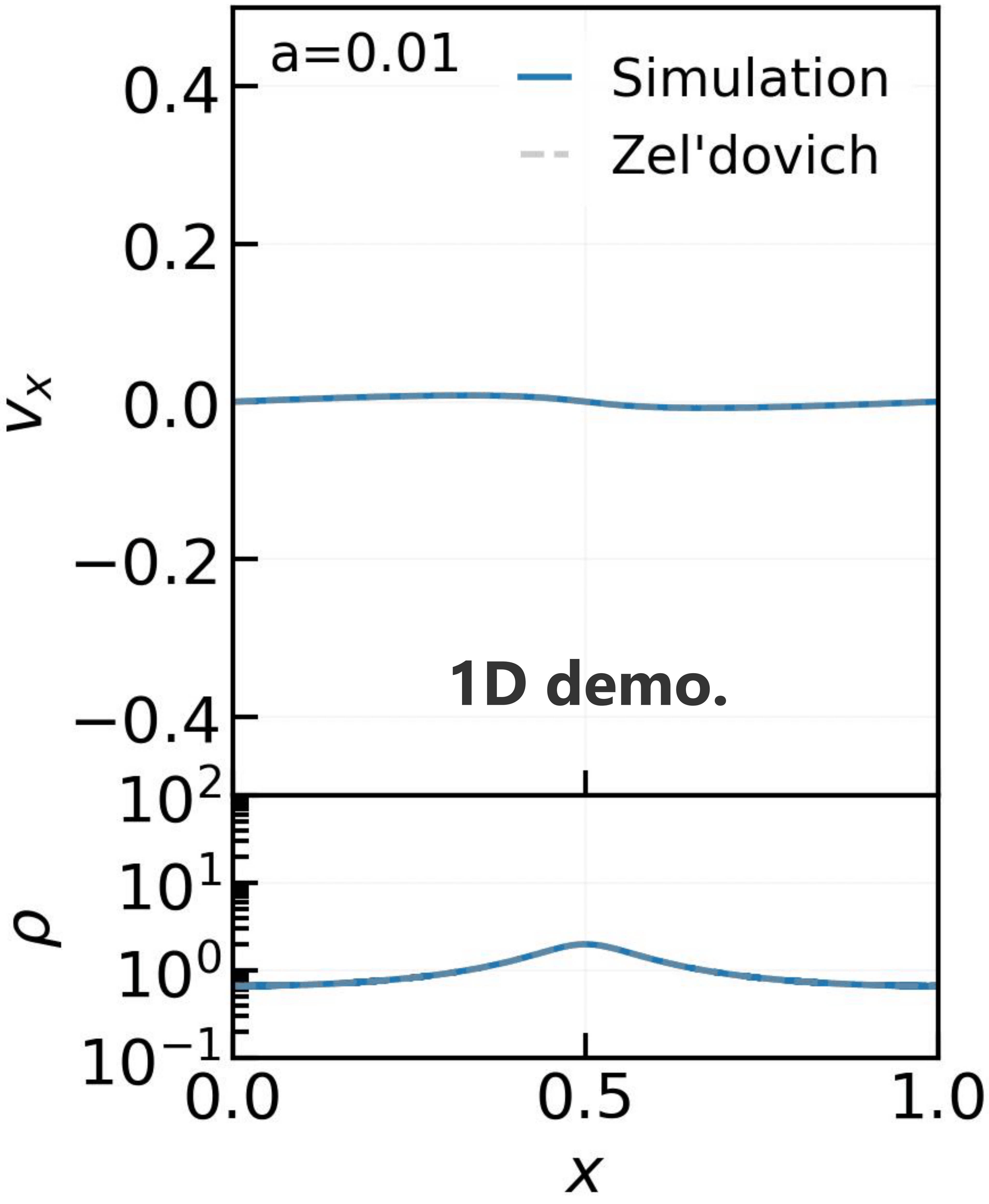
Initially negligible velocity dispersion

$$f(t_{\rm ini}, \boldsymbol{x}, \boldsymbol{v}) = \bar{\rho}_{\rm m}\, a^3 \left(1 + \delta_{\rm m}(t, \boldsymbol{x})\right) \boxed{\delta_{\rm D}(\boldsymbol{v} - \bar{\boldsymbol{v}}(t, \boldsymbol{x}))}$$

(single stream assumption)

➡ 3D phase space ***sheet*** moving in 6D phase space

vlafroid
(S.Colombi & A.Taruya)



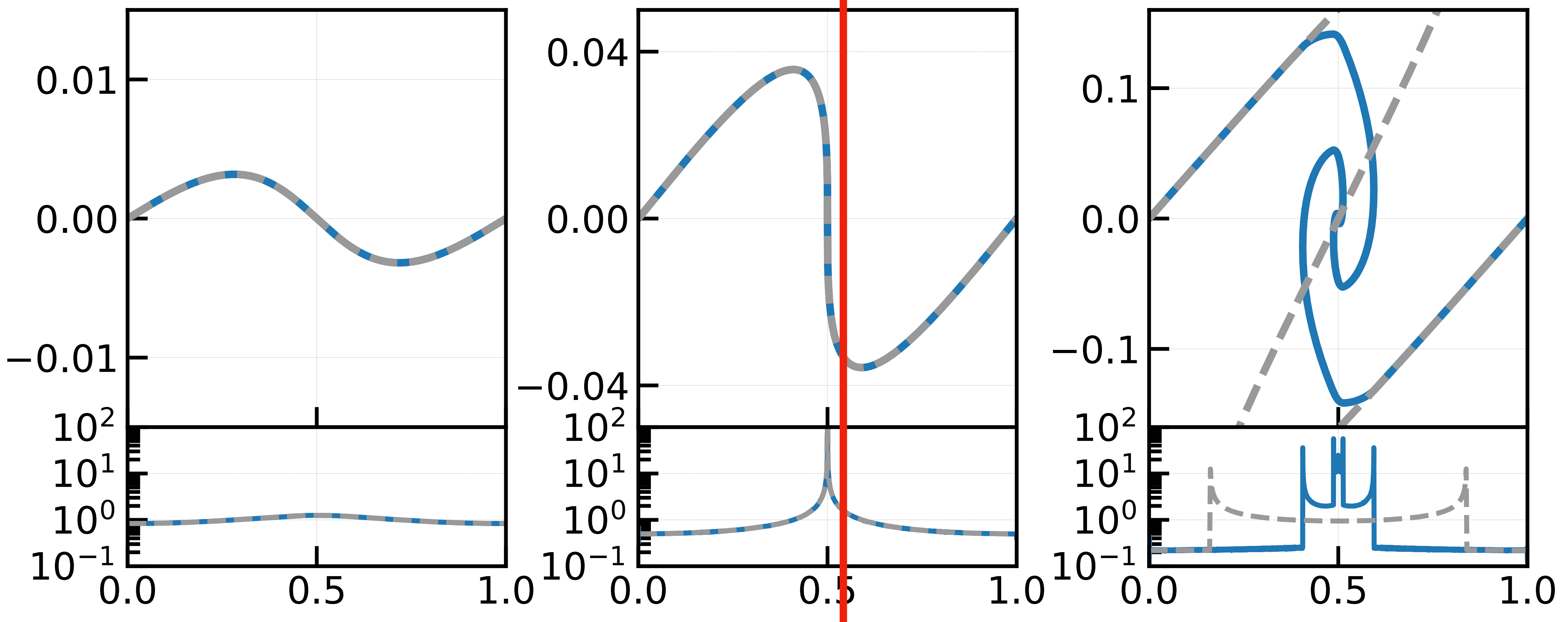
3. Overview of halo formation history

3. Overview of halo formation history

single stream

shell crossing

multi stream



exact in 1D

Zel'dovich solution

Perturbation theory

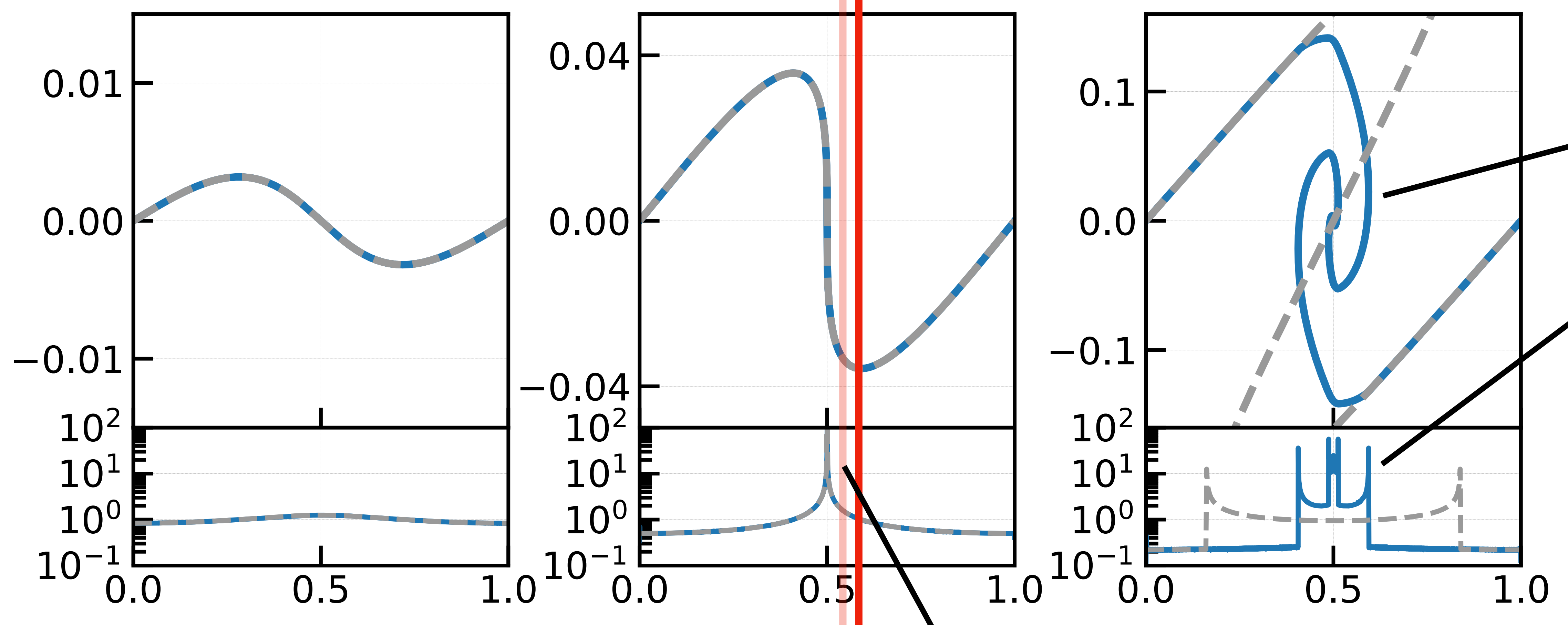
A. Pre-collapse phase

3. Overview of halo formation history

single stream

shell crossing

multi stream



multi-valued function

caustics - boundary separating single/multi stream regions

divergence

Zel'dovich (1970)
Shandarin & Zeldovich (1989)

Baumann et al. (2012)
e.g. Carrasco et al. (2012)
Effective theory

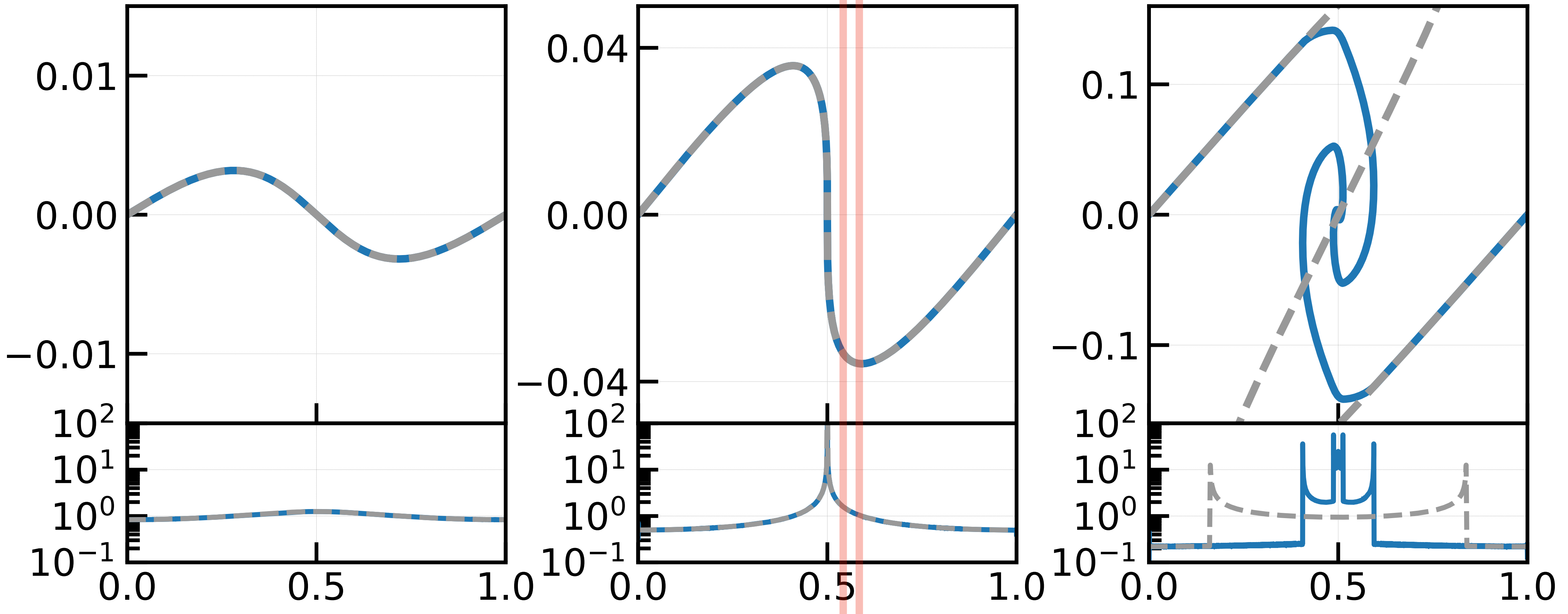
B. Post-collapse phase

3. Overview of halo formation history

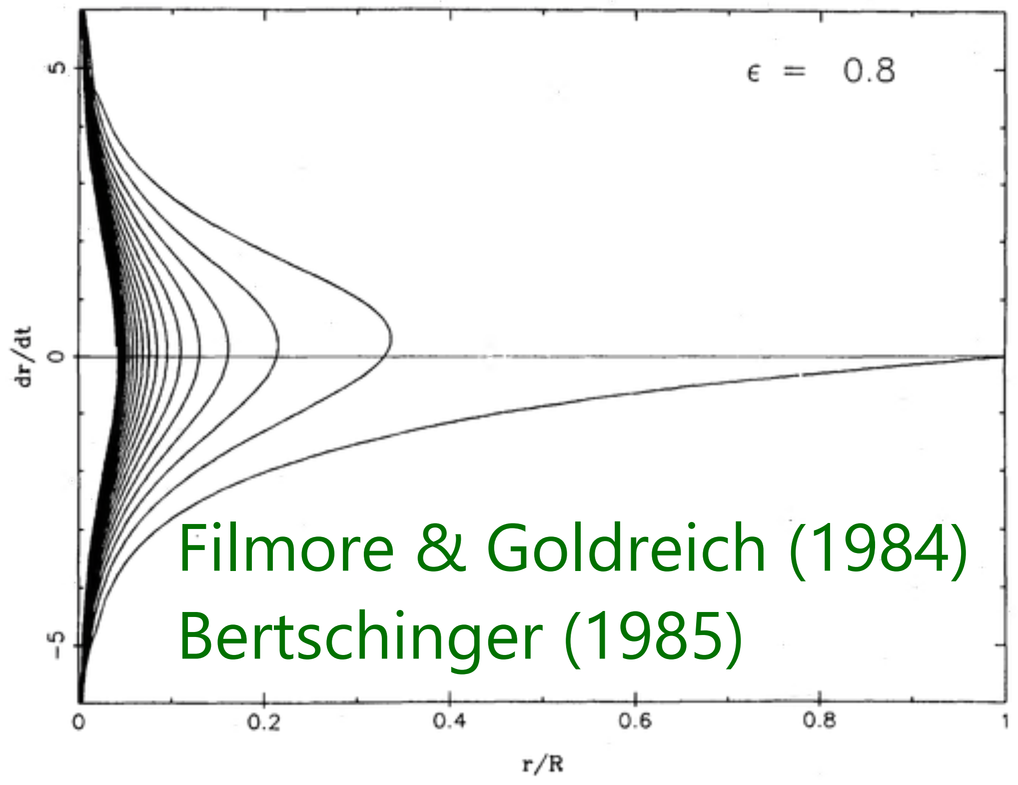
single stream

shell crossing

multi stream



(Violent) relaxation
Self-similar structure
Cuspy protohalos



Filmore & Goldreich (1984)
Bertschinger (1985)

e.g.
Self-similar model

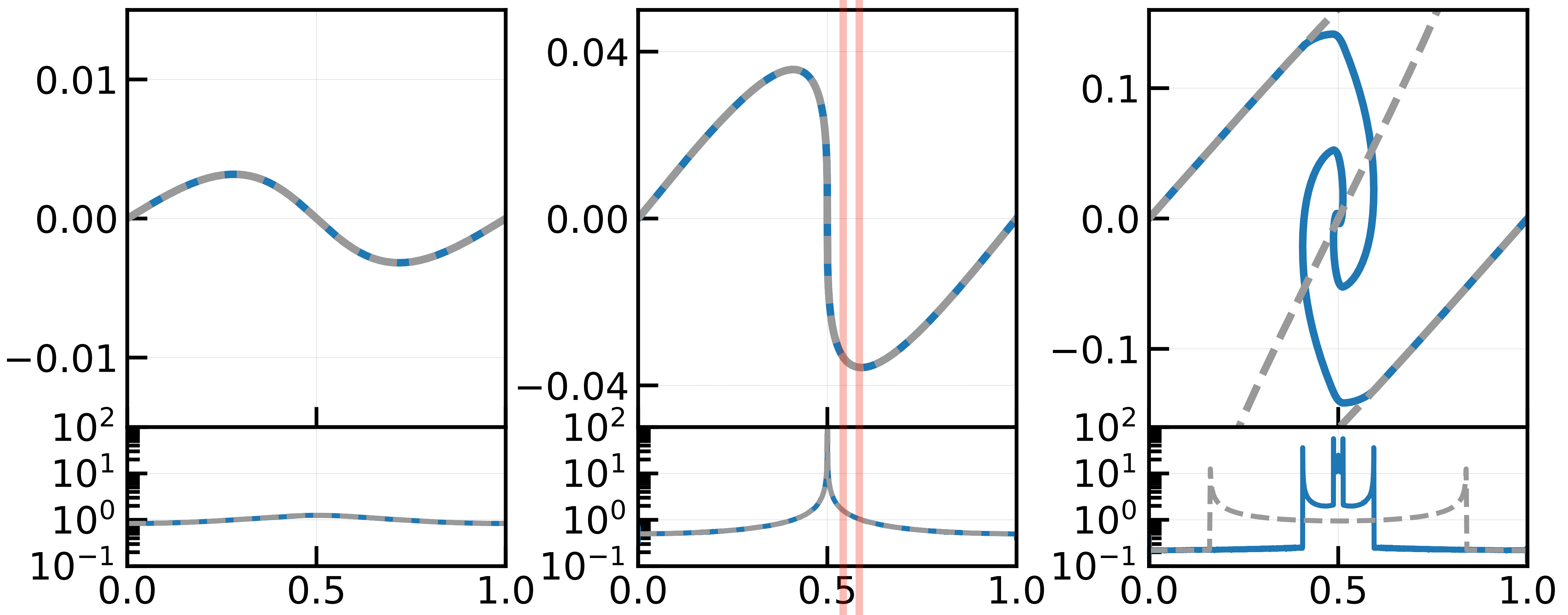
C. Self-similar phase

3. Overview of halo formation history

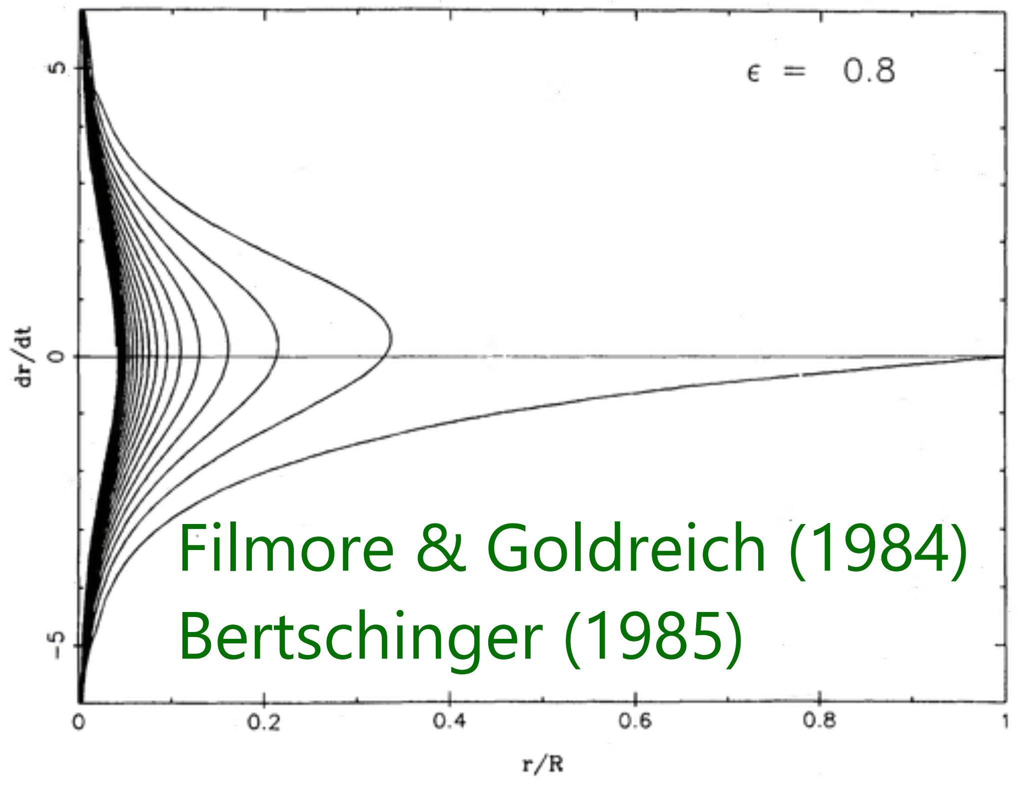
single stream

shell crossing

multi stream

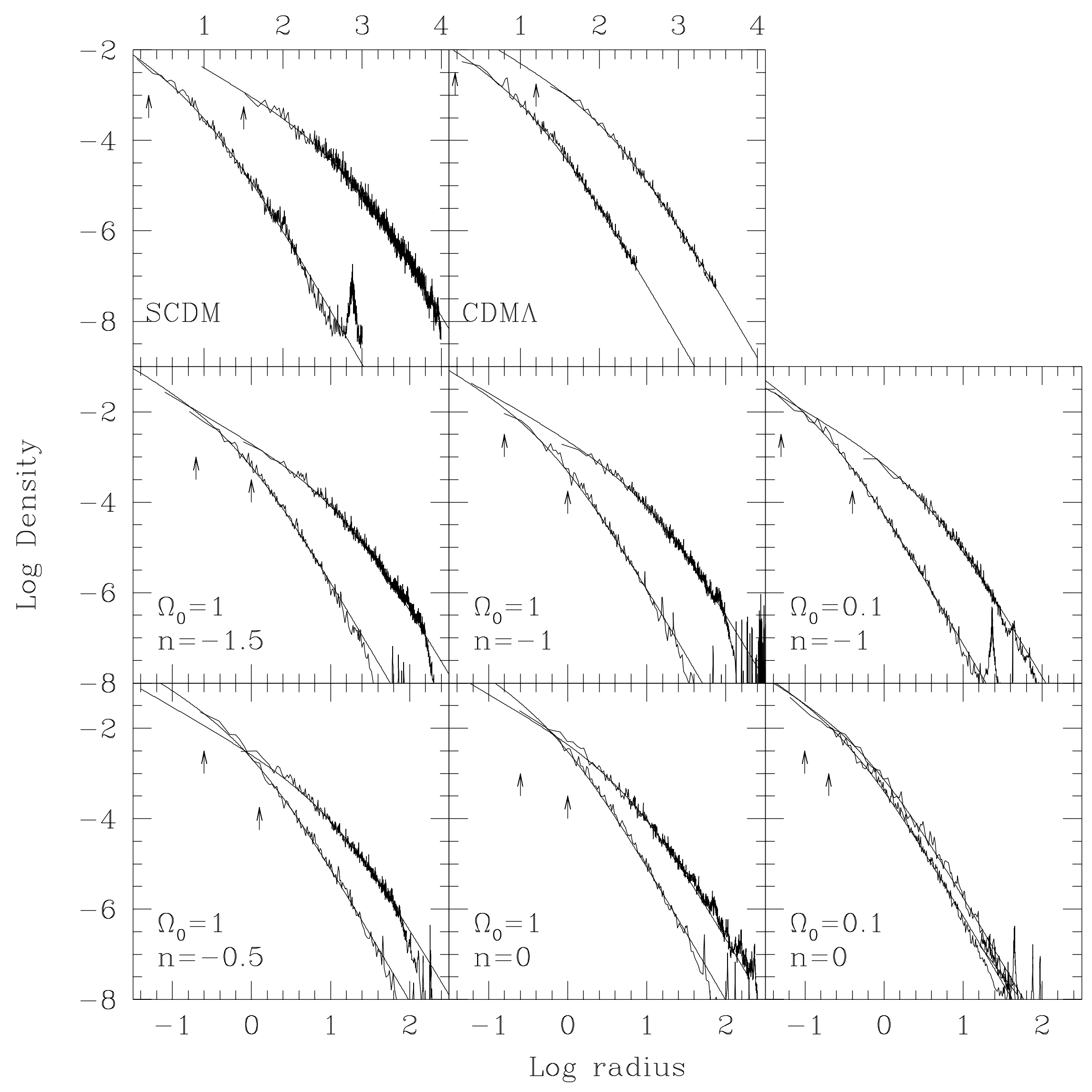


(Violent) relaxation
Self-similar structure
Cuspy protohalos



Accretion & Mergers

Universal NFW profile



Navarro, Frenk & White (1996, 1997)

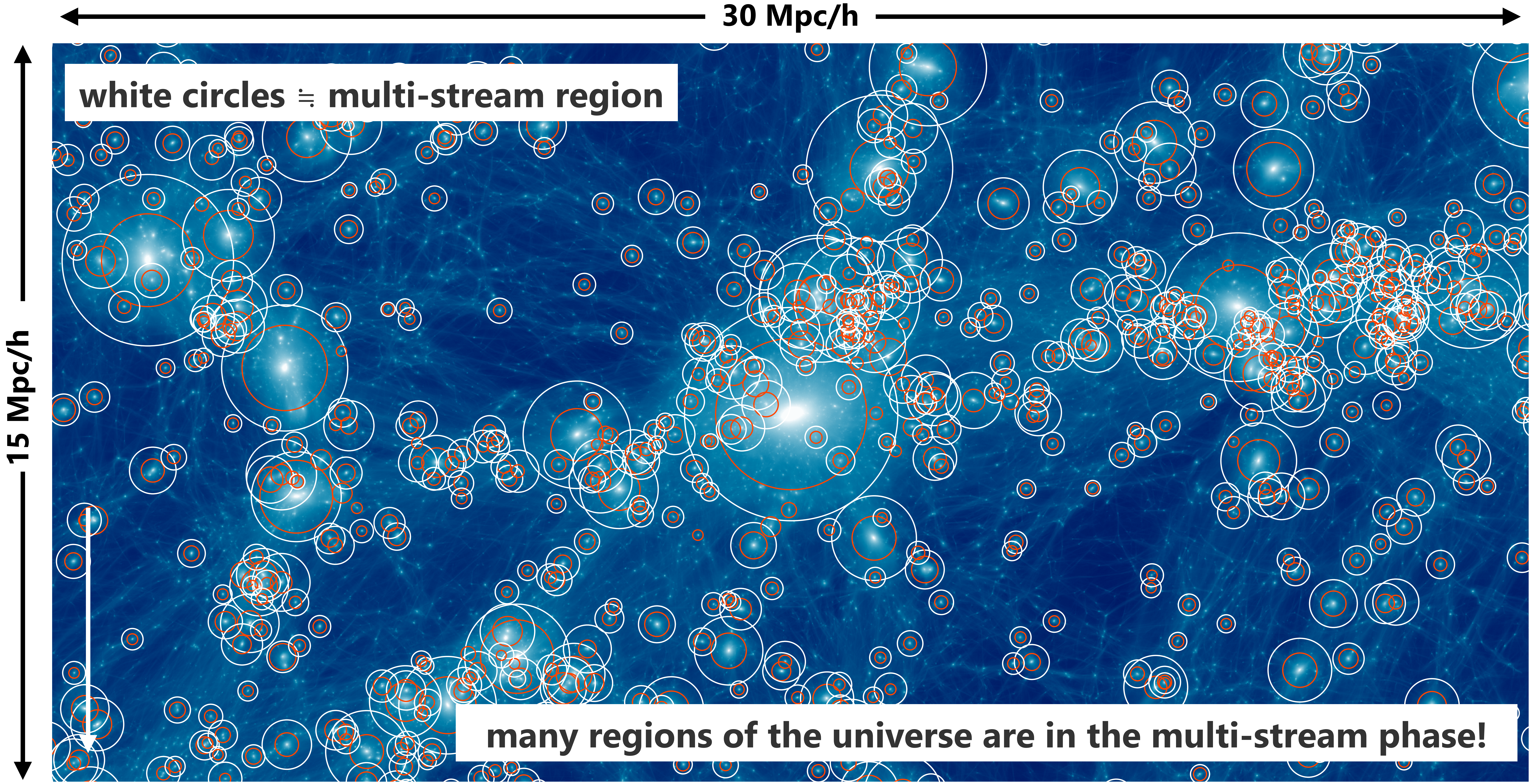
D. Evolution towards a dynamical atractor

3. Overview of halo formation history

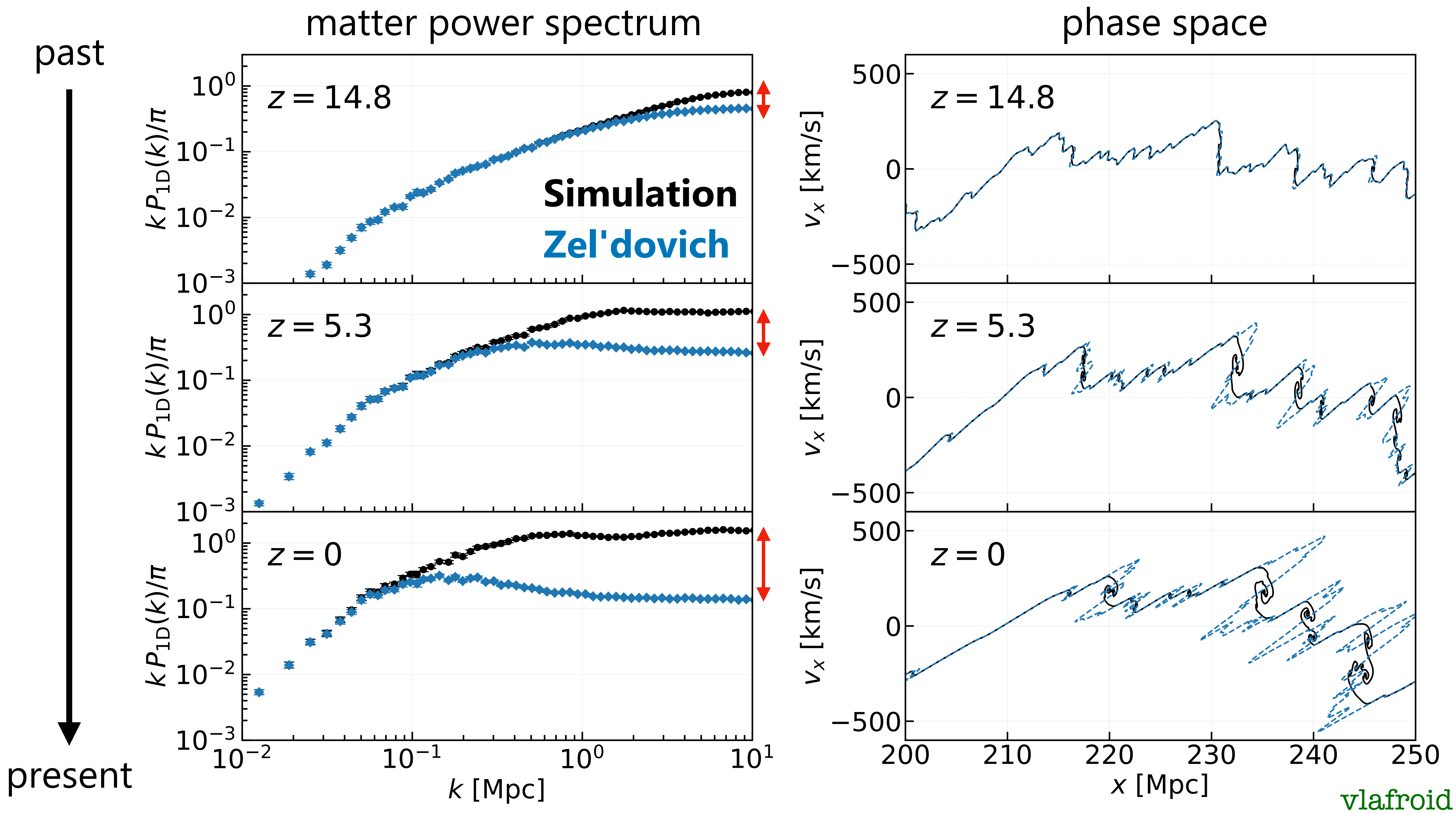
Most of the evolution of CDM is governed by the intricate dynamics in *multi-stream* phases

4. How important is multi-stream region?

Diemer et al. (2017)



5. How important is multi-stream region?



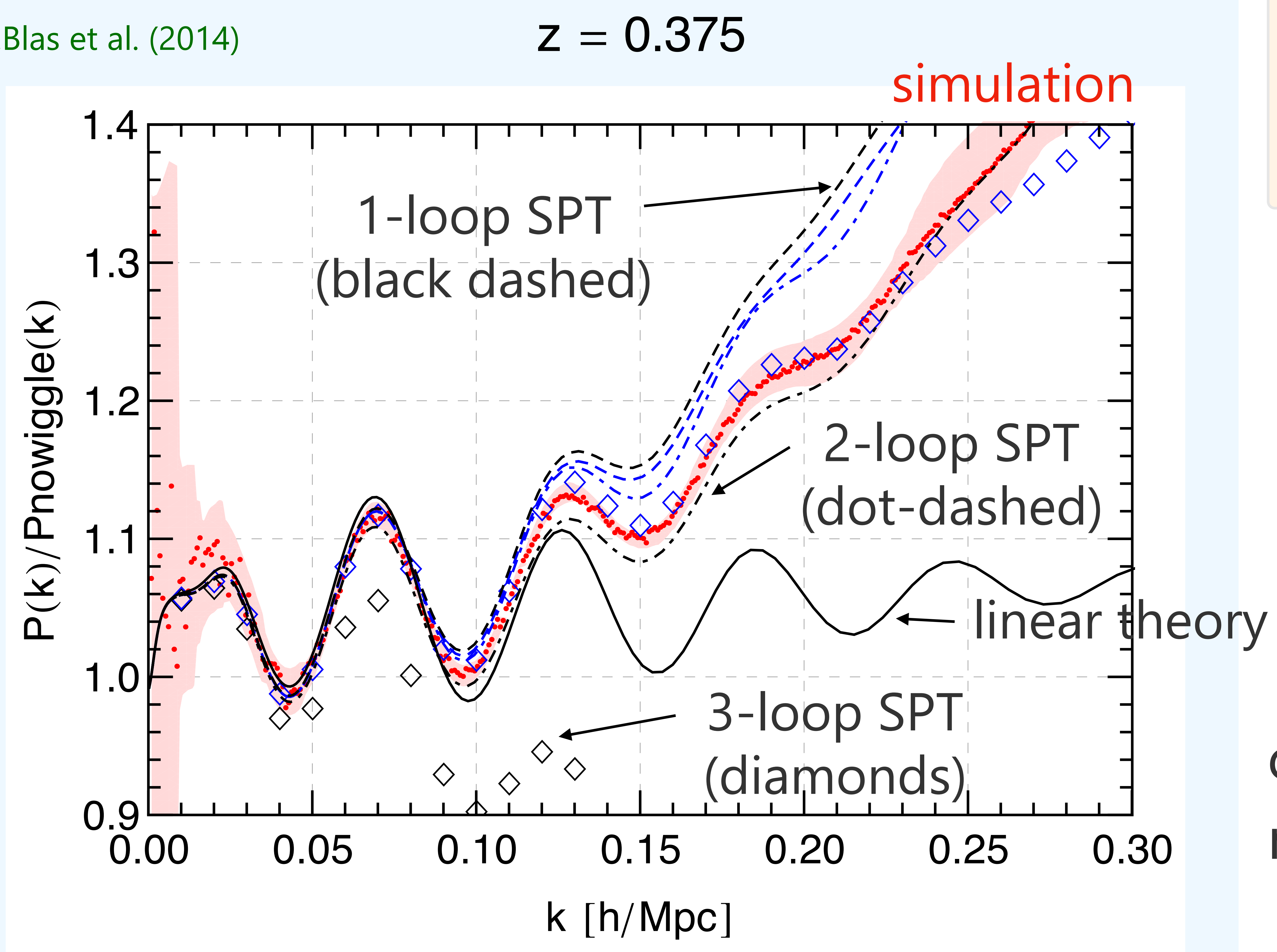
6. How important is multi-stream region?

The single-stream assumption is no longer valid at late time or at small scales.
→ the convergence of perturbative calculations is not ensured in the standard PT

Eulerian PT based on the single-stream approx.

$$\delta = \delta^{(1)} + \delta^{(2)} + \cdots$$

D.Blas et al. (2014)

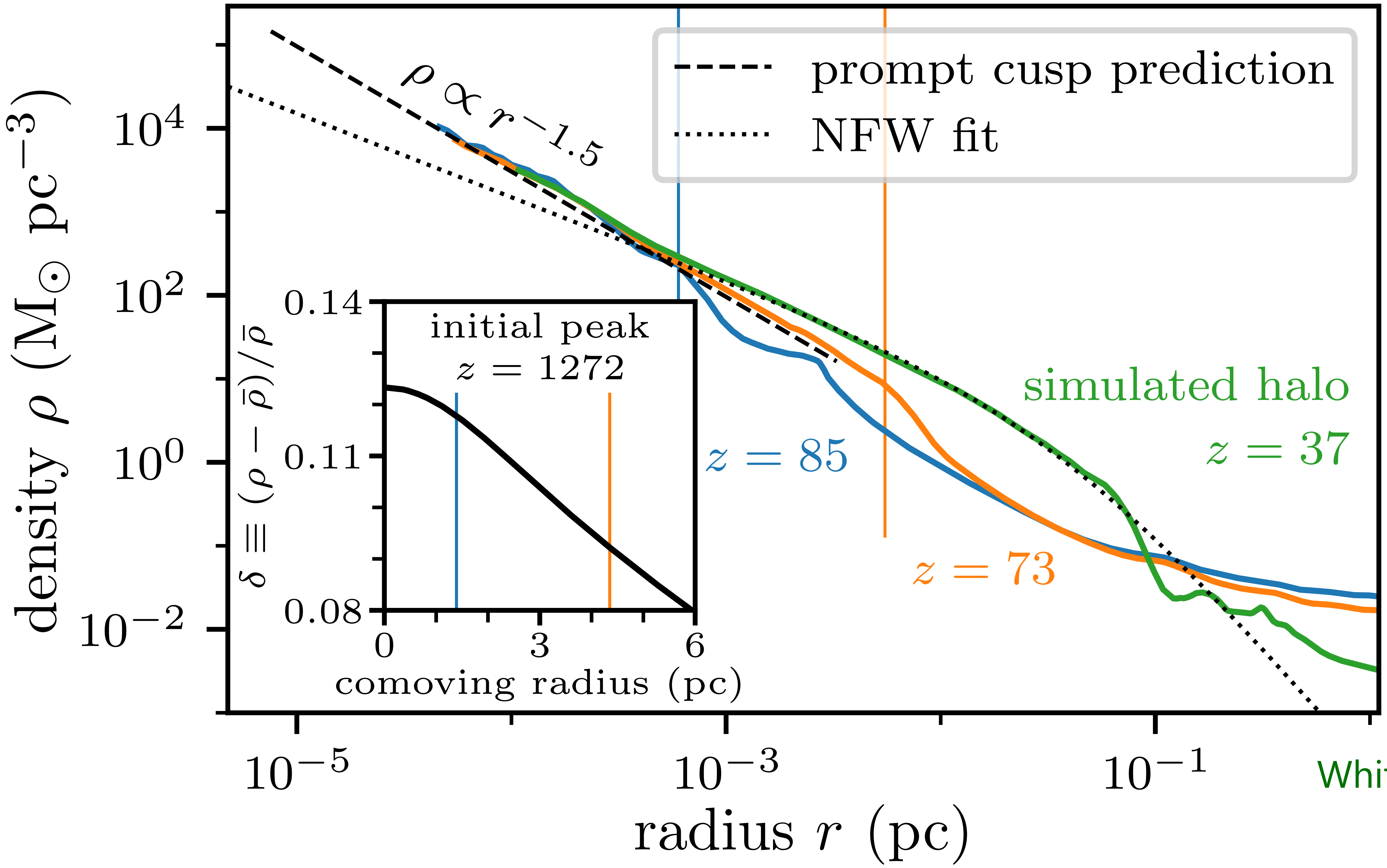


coupling between large- and small-scale modes becomes strong at 3-loop

Bernardeau et al. (2014)
Nishimichi et al. (2016)

7. How important is multi-stream region?

During **“C. Self-similar phase”**, cuspy structure presents and survives long enough to be observationally relevant (e.g. enhancement of the dark matter annihilation signal)



White (2022), Sten Delos & White (2023)

Still little theoretical work on prompt cusps;

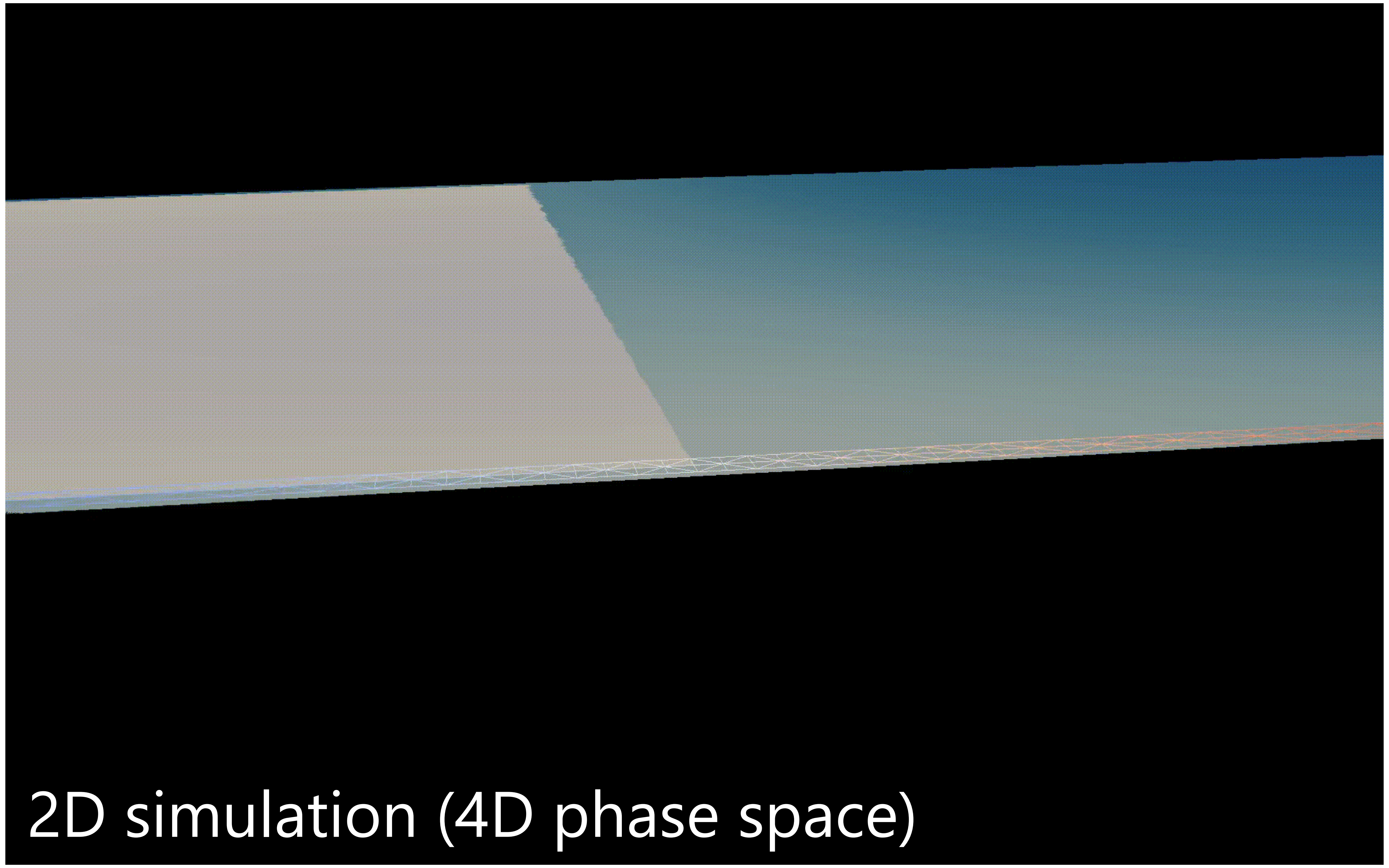
how they emerge dynamically?

how stable they are against halo mergers?

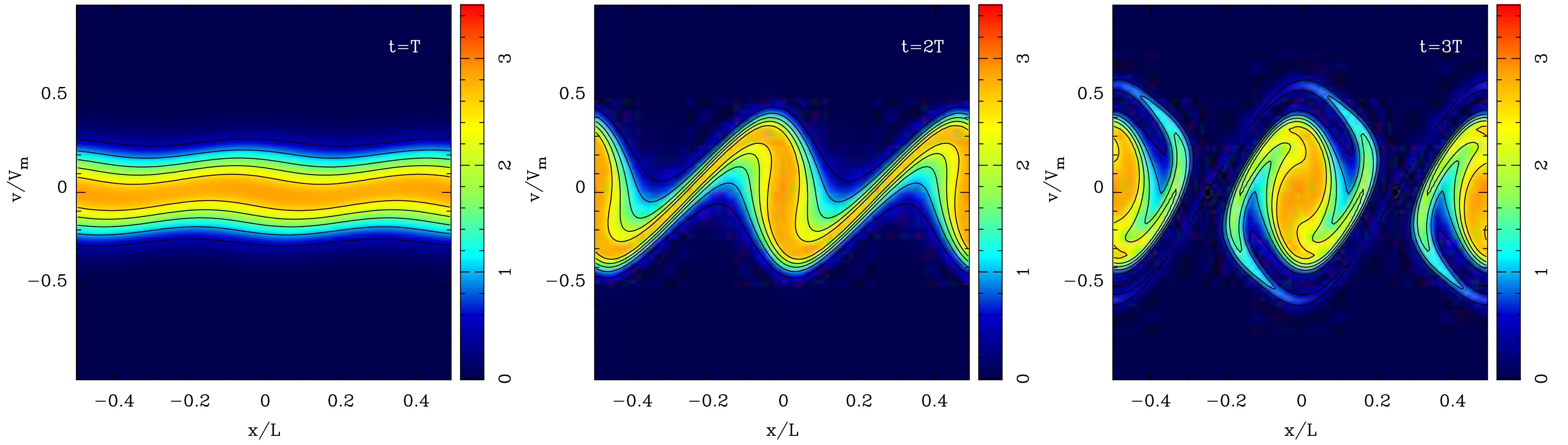
8. Rise of Vlasov-Poisson simulations

Parallel 6D Vlasov-Poisson solver (the only publicly available CDM Vlasov-Poisson simulation)

T.Sousbie and S.Colombi (2016), S.Colombi(2021)



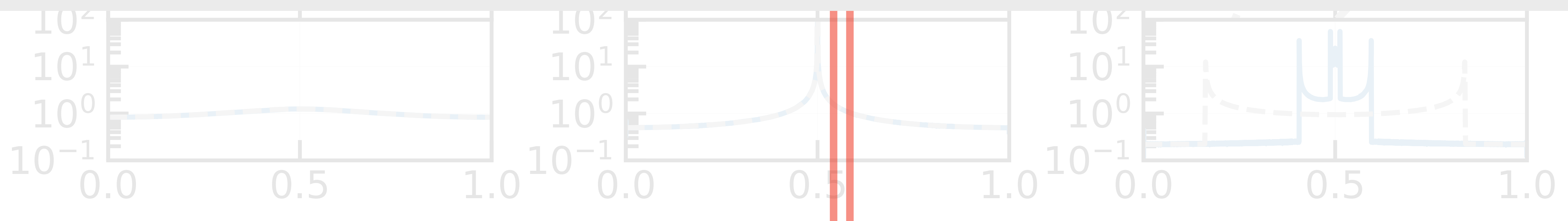
For “wam” case:

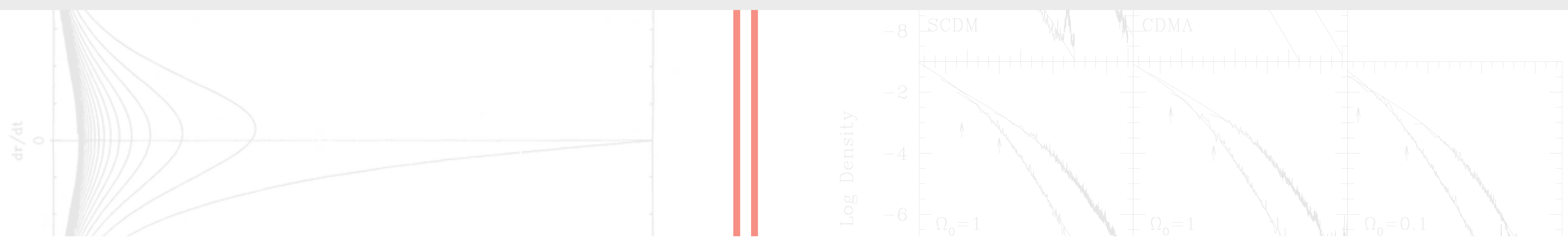


Yoshikawa, Yoshida & Umemura (2013), Yoshikawa et al. (2020)

9. Towards understanding CDM dynamics

The multi-stream phase is crucial stage not only for understanding the origin of the universal halo profile but also for accurately predicting the statistical quantity of large-scale structure





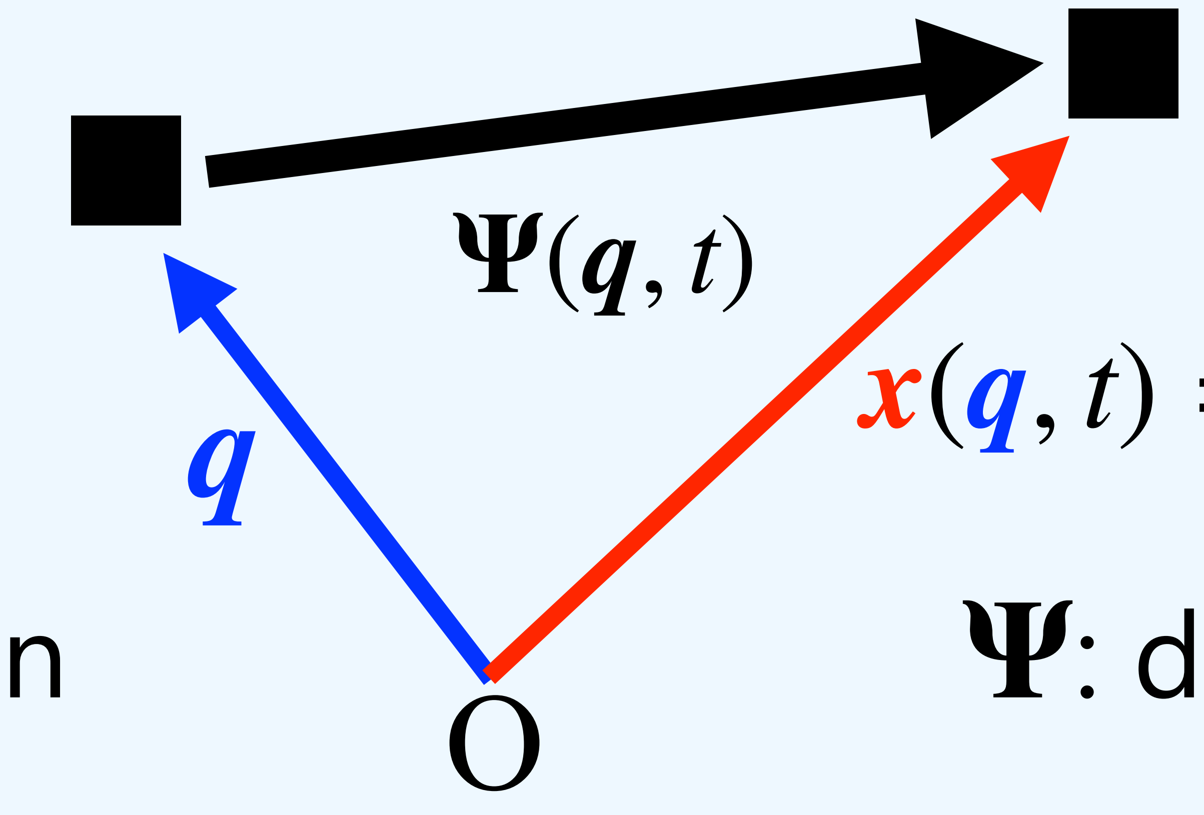
An analytical theory for consistently describing the multi-stream dynamics of CDM

– Refining with Vlasov-Poisson simulations

A. Pre-collapse phase

10. Lagrangian perturbation theory (LPT)

trajectory of a fluid element



Zel'dovich (1970), Shandarin and Zel'dovich (1989), Bouchet et al, (1992), Buchert (1992), Bouchet et al. (1994), Bernardeau (1994), ...

$$\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t)$$

$\boldsymbol{q}$: initial position

$\boldsymbol{\Psi}$: displacement field

Equation of Motion: $\ddot{\boldsymbol{\Psi}} + 2H\dot{\boldsymbol{\Psi}} = -\dfrac{1}{a^2}\boldsymbol{\nabla}_x\phi(\boldsymbol{x})$

Poisson equation: $\Delta_x\phi(\boldsymbol{x}) = 4\pi G\bar{\rho}a^2\delta(\boldsymbol{x})$

$$1+\delta = \frac{1}{J}$$

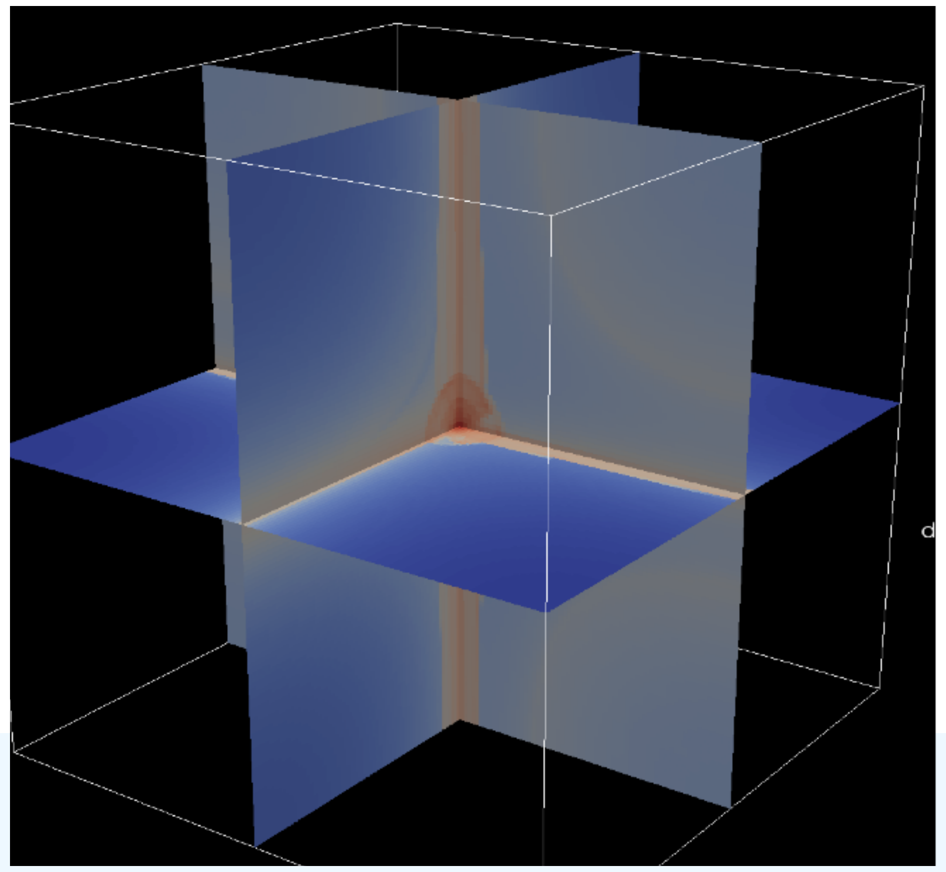
$$J = \det\left|\frac{\partial x_i(\boldsymbol{q})}{\partial q_j}\right|$$

Perturbative expansion : $\boldsymbol{\Psi}(\boldsymbol{q},t) = \sum_{n=1}^{\infty}(a(t))^n\,\boldsymbol{\Psi}^{(n)}(\boldsymbol{q})$

formally solved by LPT recurrence relaton

C.Rampf (2012), V.Zheligovsky & U.Frisch (2014), T.Matsubara (2015)

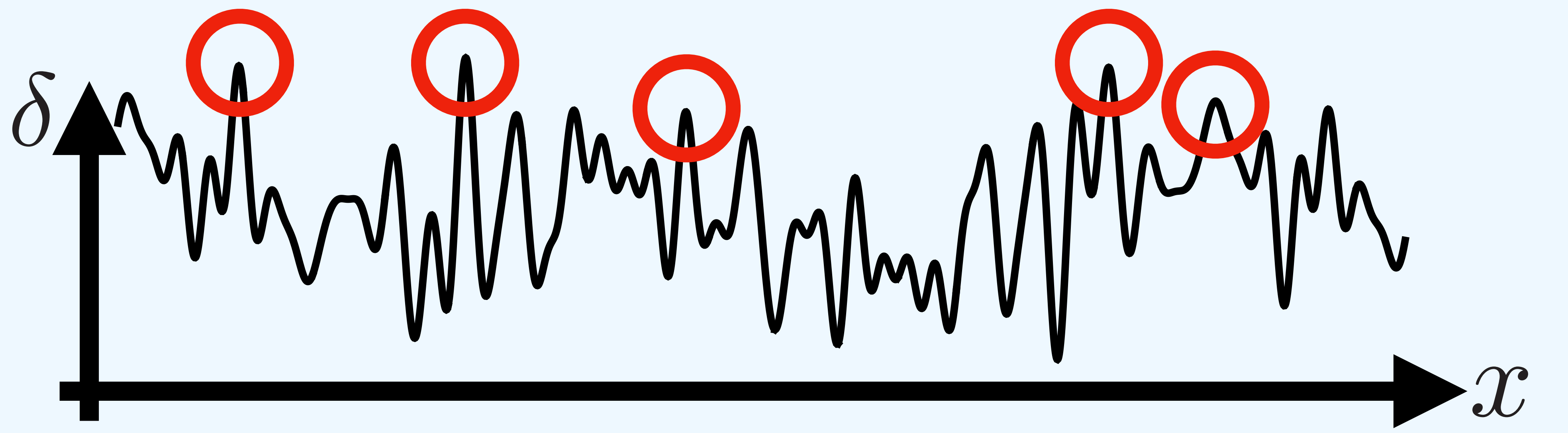
11. Tests of Lagrangian Perturbation Theory



Small density peak at the origin $\phi(\boldsymbol{q}) = -\epsilon_x \cos q_x - \epsilon_y \cos q_y - \epsilon_z \cos q_z$

F.Moutarde et al. (1991), T.Buchert et al. (1997)

[1] Representative of high density peak in the realistic universe



[2] Easy to solve the recurrence relation
→ LPT solutions up to eg, ~ 50LPT - 1000LPT

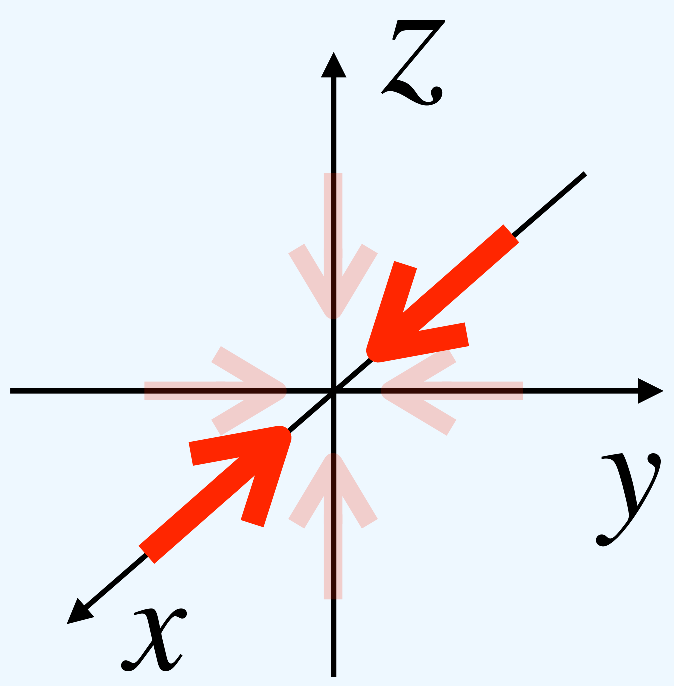
[3] Quantitative analysis of various collapse conditions

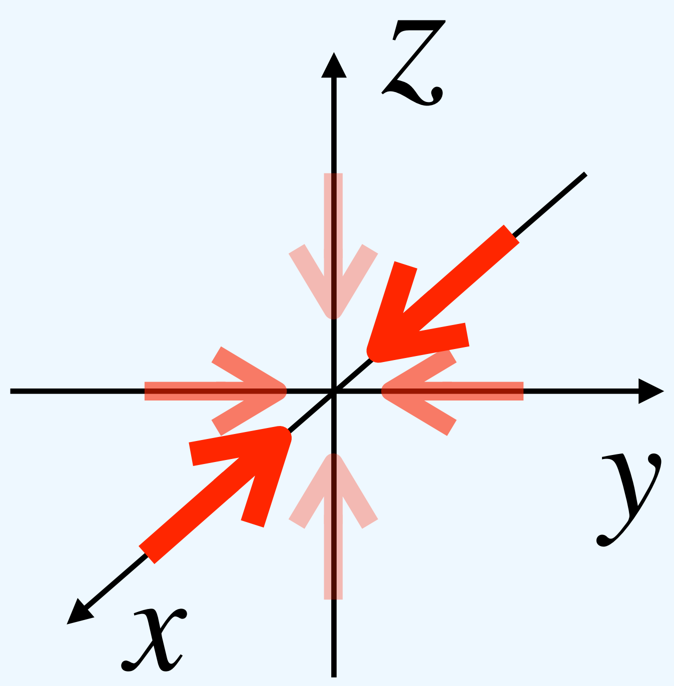
Q1D $\frac{\epsilon_z}{\epsilon_x}, \frac{\epsilon_y}{\epsilon_x} \ll 1$

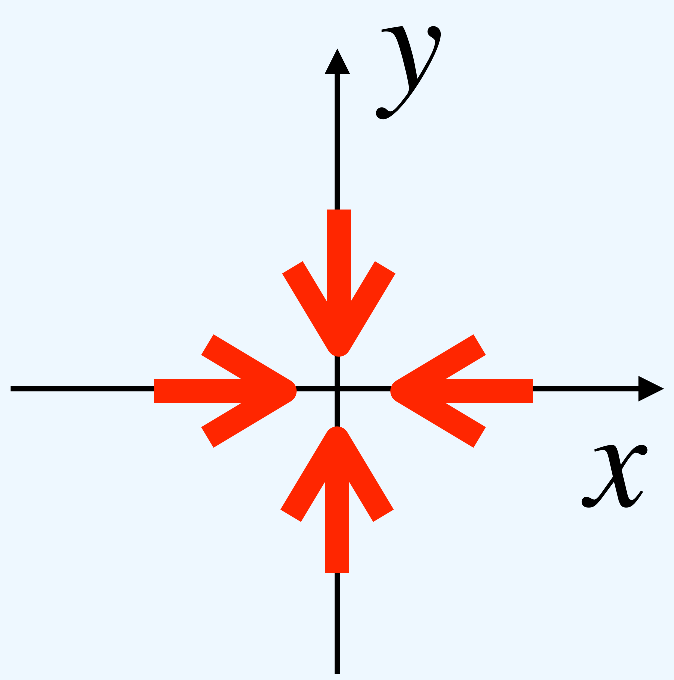
ANI $\frac{\epsilon_z}{\epsilon_x} \leq \frac{\epsilon_y}{\epsilon_x} \leq 1$

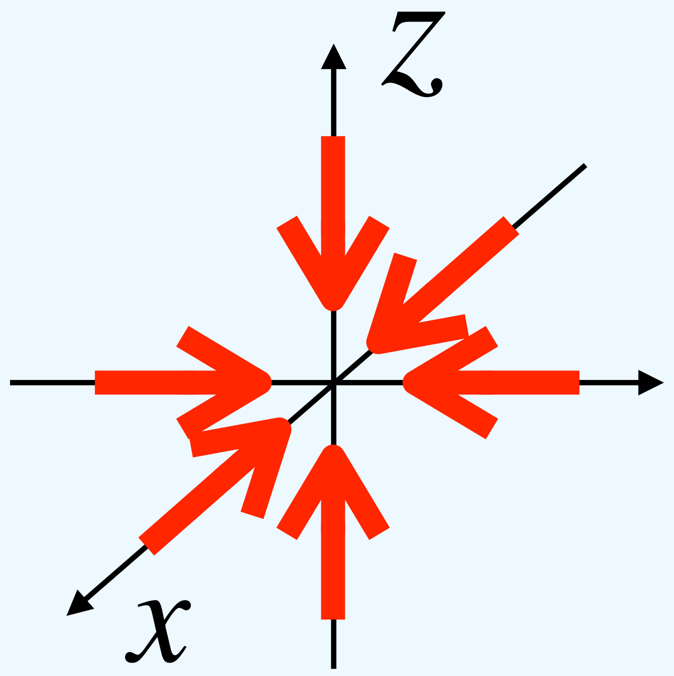
SYM (2D) $\frac{\epsilon_z}{\epsilon_x} = 0\,, \frac{\epsilon_y}{\epsilon_x} = 1$

SYM (3D) $\frac{\epsilon_z}{\epsilon_x} = \frac{\epsilon_y}{\epsilon_x} = 1$



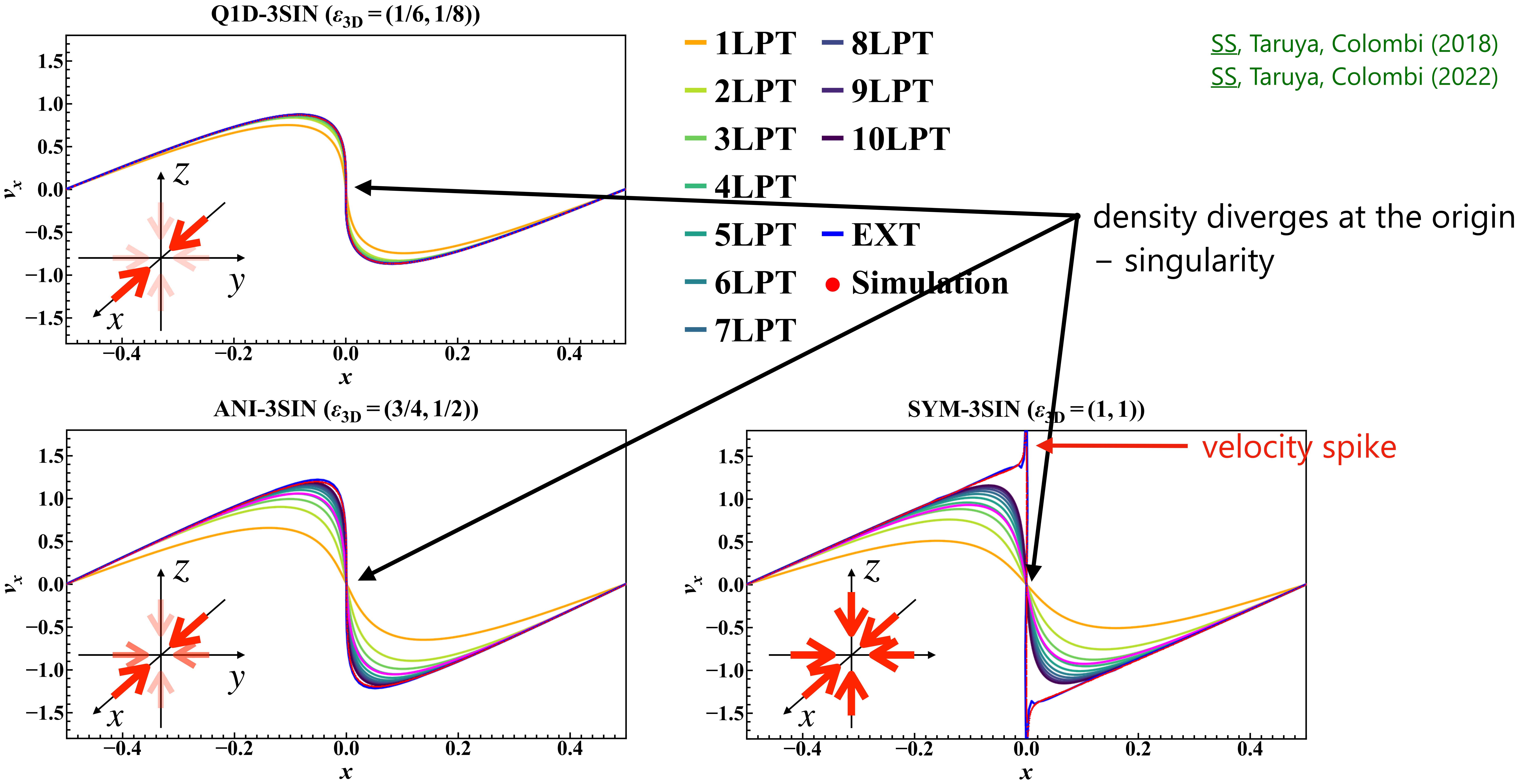






12. Phase-space structure *at* shell crossing

SS, Taruya, Colombi (2018)
SS, Taruya, Colombi (2022)

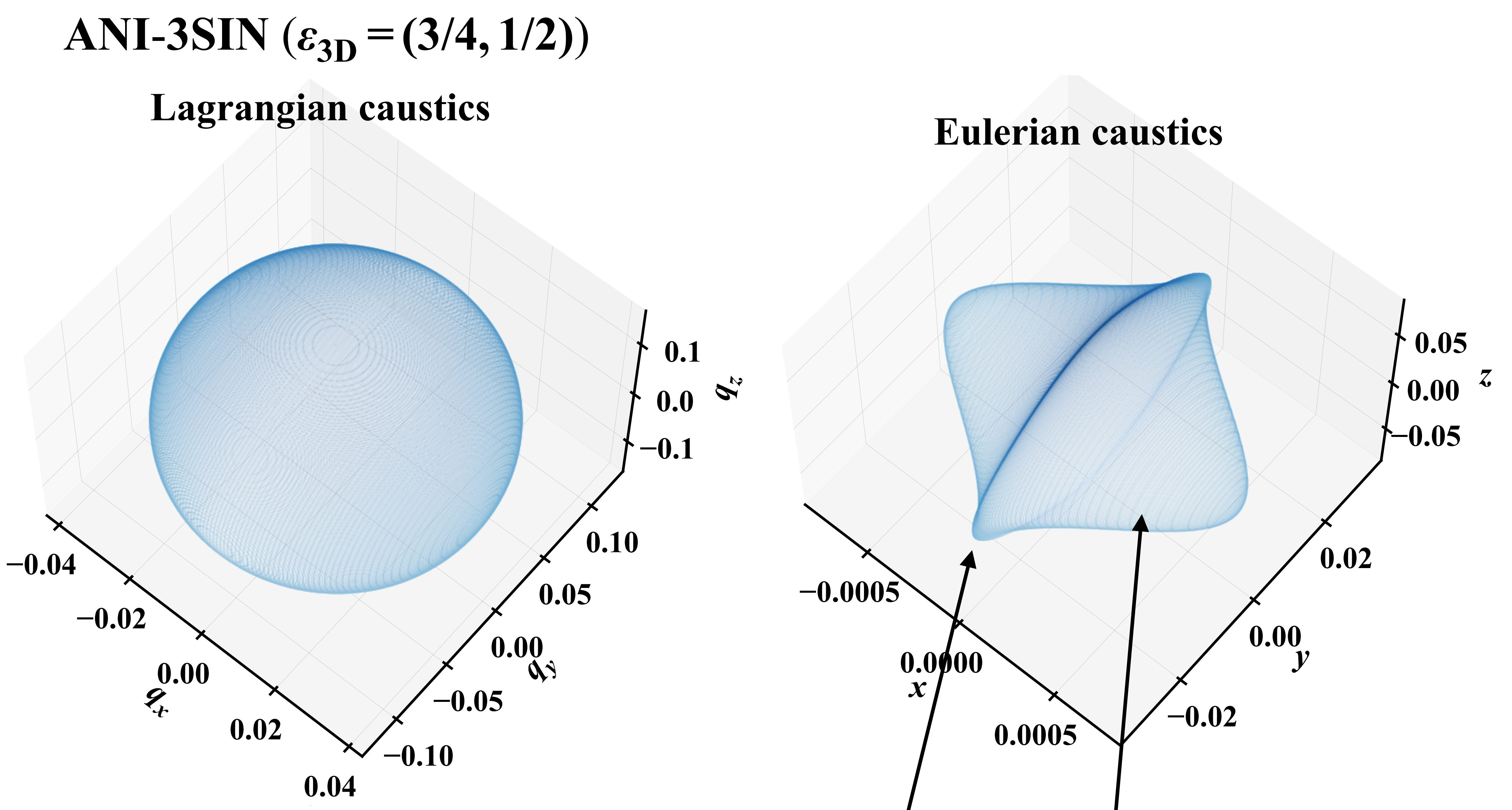


B. Post-collapse phase

13. Singularity

SS, Taruya, Colombi (2022)
Rampf, SS, Taruya, Colombi (2023)

Now focus on the *pancake collapse* (Q1D/ANI)
(collapse along x axis)



Caustics develop after shell-crossing

Singular surface $J = \det \left| \frac{\partial x_i(\boldsymbol{q})}{\partial q_j} \right| = 0$

Simple analysis of singularity:

Taylor expansion of the displacement field around the initial singularity point

$$x(\boldsymbol{q}) \simeq (1+\psi_{100})q_x + \frac{1}{2}\left(\psi_{120}\, q_y^2 + \psi_{102}\, q_z^2\right) q_x + \frac{1}{6}\psi_{300}\, q_x^3,$$
$$y(\boldsymbol{q}) \simeq (1+\psi_{010})q_y,$$
$$z(\boldsymbol{q}) \simeq (1+\psi_{001})q_z.$$

$$\frac{\partial x(\mathbf{0})}{\partial q_x} \equiv -h = 1+\psi_{100} < 0,$$
$$\frac{\partial y(\mathbf{0})}{\partial q_y} = 1+\psi_{010} > 0,$$
$$\frac{\partial z(\mathbf{0})}{\partial q_z} = 1+\psi_{001} > 0.$$

$$J \simeq \frac{1}{2}(1+\psi_{010})(1+\psi_{001})\left(-2h + \psi_{120}q_y^2 + \psi_{102}q_z^2 + \psi_{300}q_x^2\right)$$

Catastrophe theory

A_2 caustics (*fold*) $\rho \propto r^{-1/2}$

A_3 caustics (*cusp*) $\rho \propto r^{-2/3}$

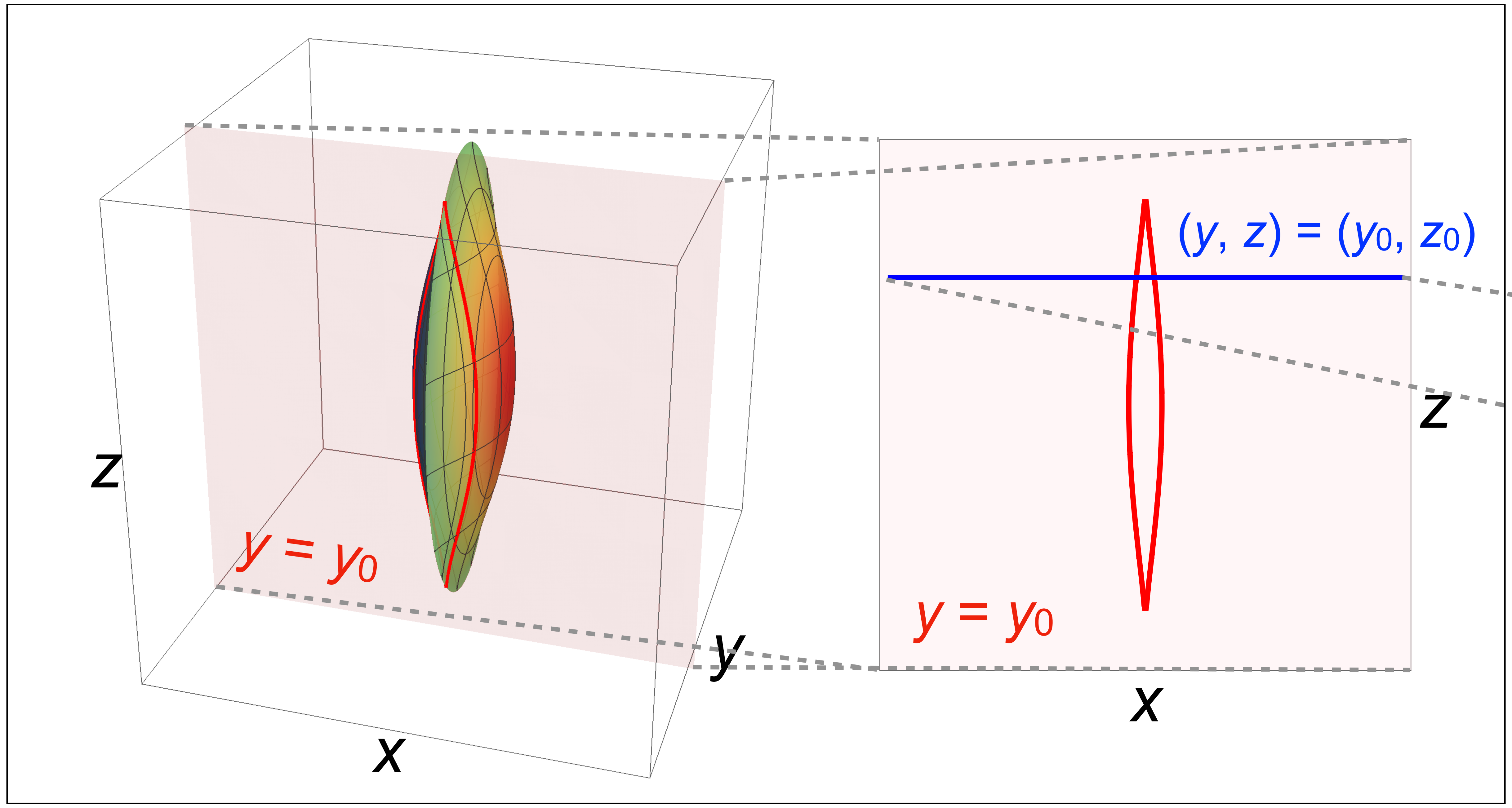
Arnold et al. (1982), Arnold (1983), Hidding (2014),
Feldbrugge et al. (2018), Feldbrugge & Weygaert (2024)

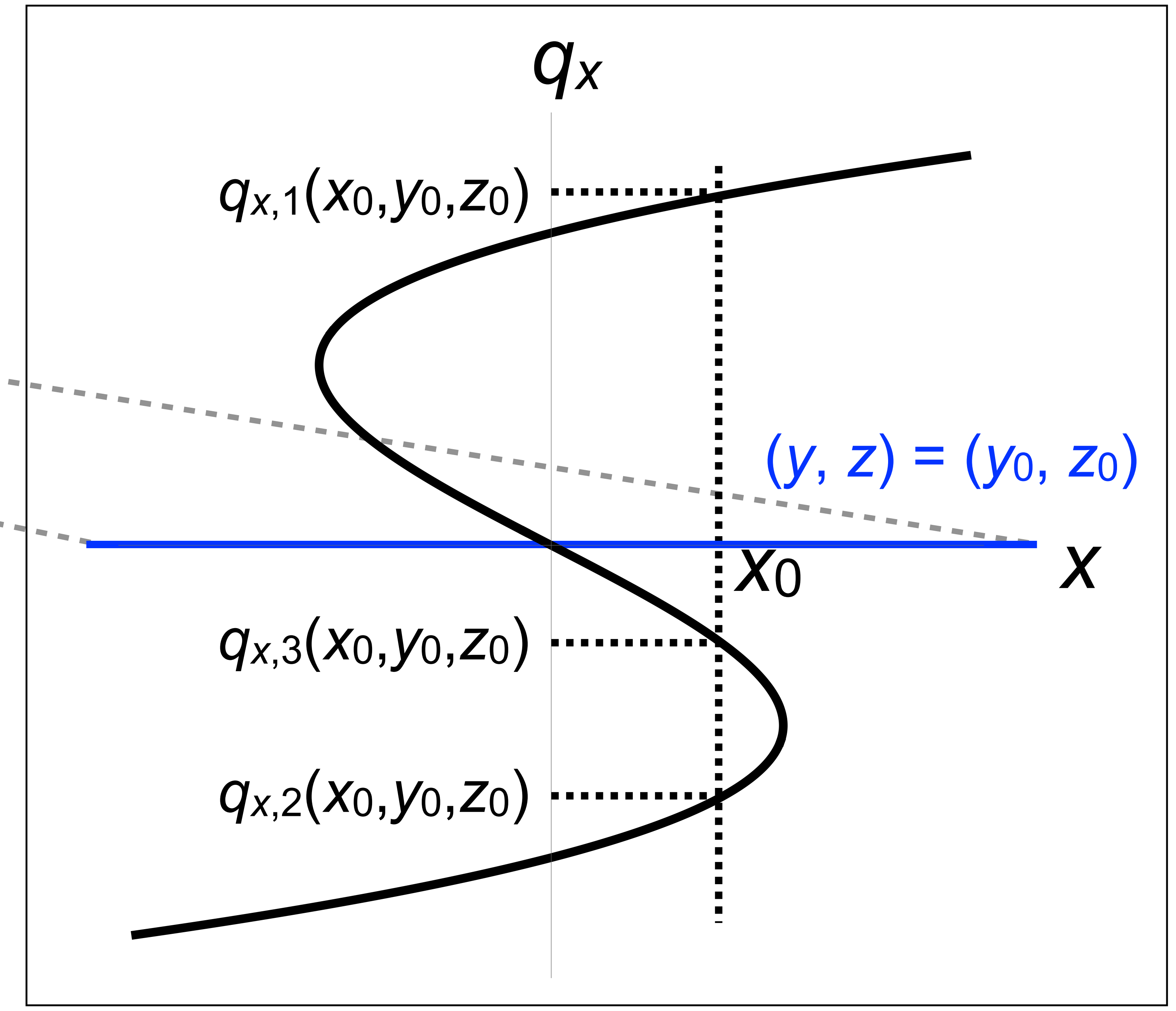
14. Towards post-collapse theory

SS, Colombi, Taruya (2023)
SS, Colombi, Taruya, Rampf, Parichha [in prep.]

Focus on the pancake collapse

Shortly after shell crossing, dynamics of CDM is reduced to solving cubic equation





We derive a simple analytic formula for the multi-stream force

$$F_x(\boldsymbol{x}) \simeq -\frac{4\pi G\bar{\rho}a^2}{(1+\psi_{010})(1+\psi_{001})}\left[q_{x,1}(\boldsymbol{x})+q_{x,2}(\boldsymbol{x})-q_{x,2}(\boldsymbol{x})\right]$$

✔ Good agreement with Vlasov-Poisson simulations

15. 3D post-collapse perturbation theory (PCPT)

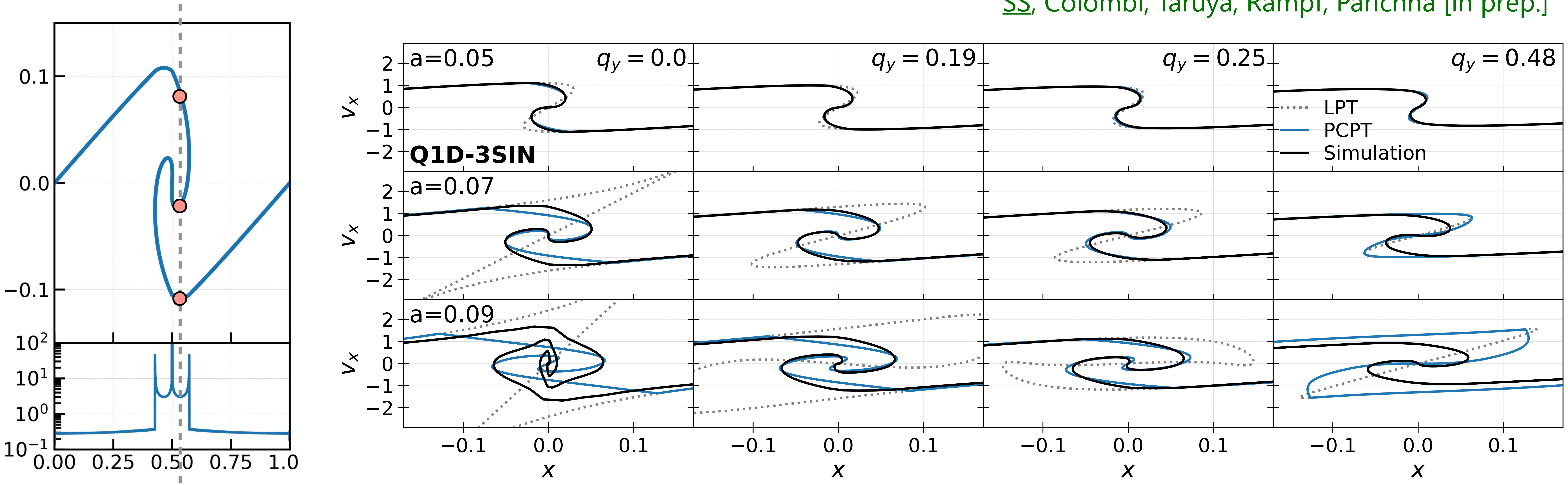
Assuming the collapse is *pancake*, we develop 3D post-collapse perturbation theory (PCPT)

see Colombi (2015), Taruya & Colombi (2017), Rampf, Frisch & Hahn (2021) for 1D case

We solve the cubic equation to incoorporate the multi-stream force
then, the LPT motion is perturbatively corrected

$$F_x(\boldsymbol{x}) \simeq -\frac{4\pi G\bar{\rho}a^2}{(1+\psi_{010})(1+\psi_{001})}\left[q_{x,1}(\boldsymbol{x})+q_{x,2}(\boldsymbol{x})-q_{x,2}(\boldsymbol{x})\right]$$

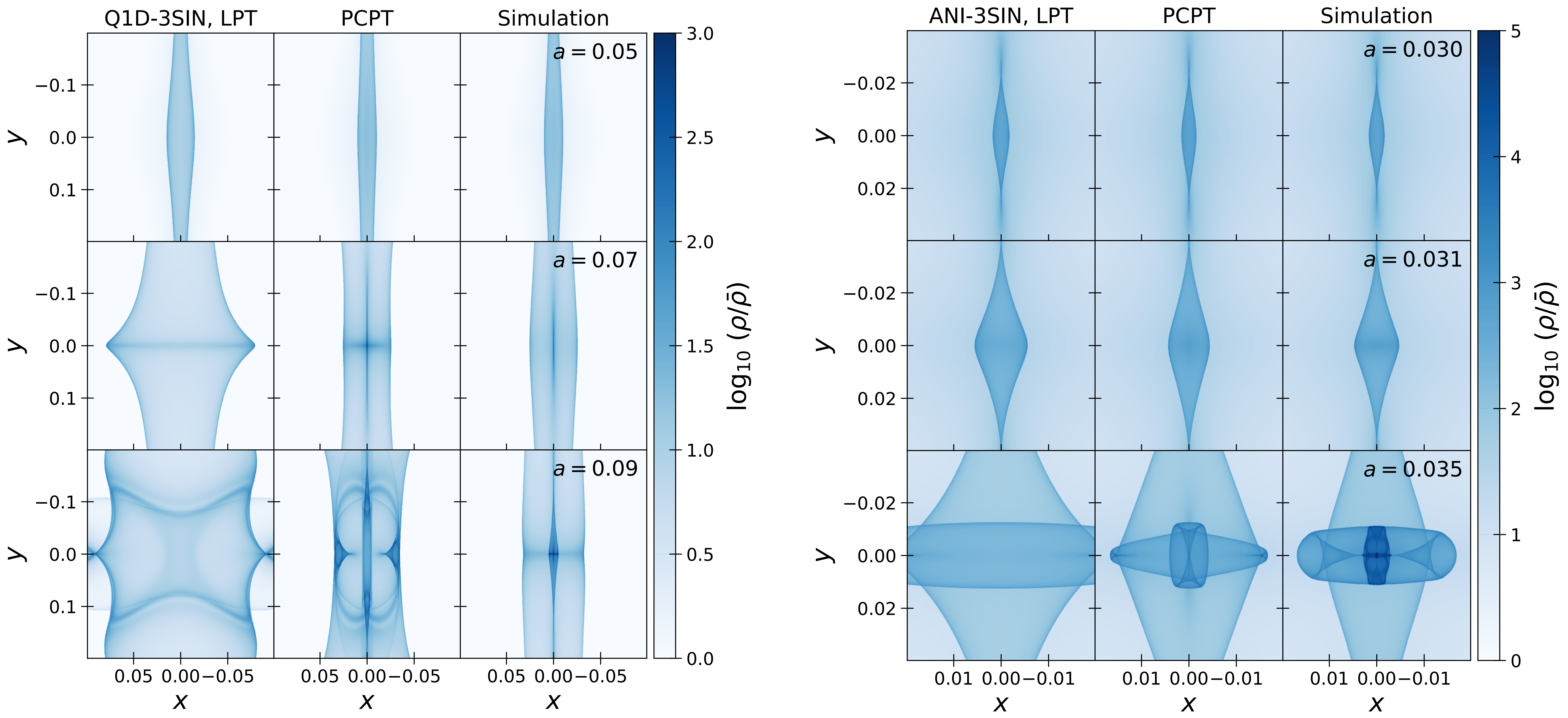
SS, Colombi, Taruya, Rampf, Parichha [in prep.]



✔ PCPT qualitatively improves the phase-space prediction

16. 3D PCPT (density)

SS, Colombi, Taruya, Rampf, Parichha [in prep.]



✔ PCPT also improves the prediction of density fields

C. Self-similar phase

17. Demo. 2D Vlasov-Poisson simulations

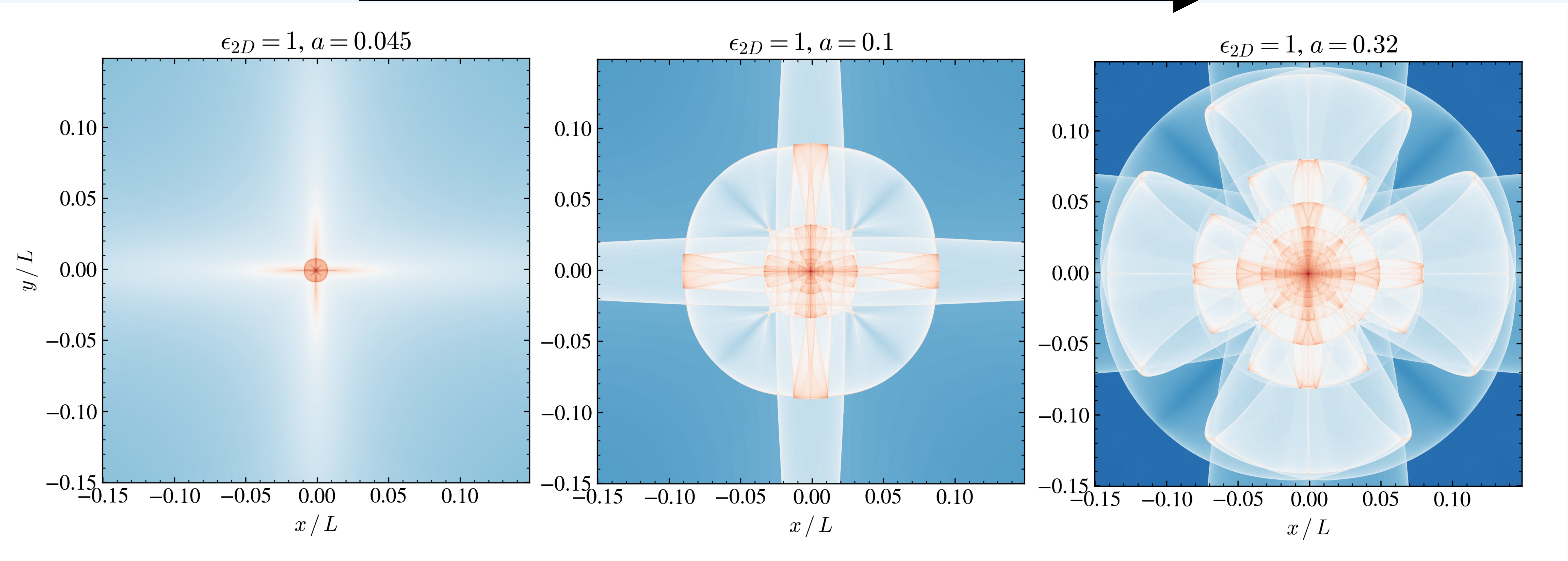
Parichha, Colombi, <u>SS</u>, Taruya (2025)

PCPT works very well just after shell-crossing

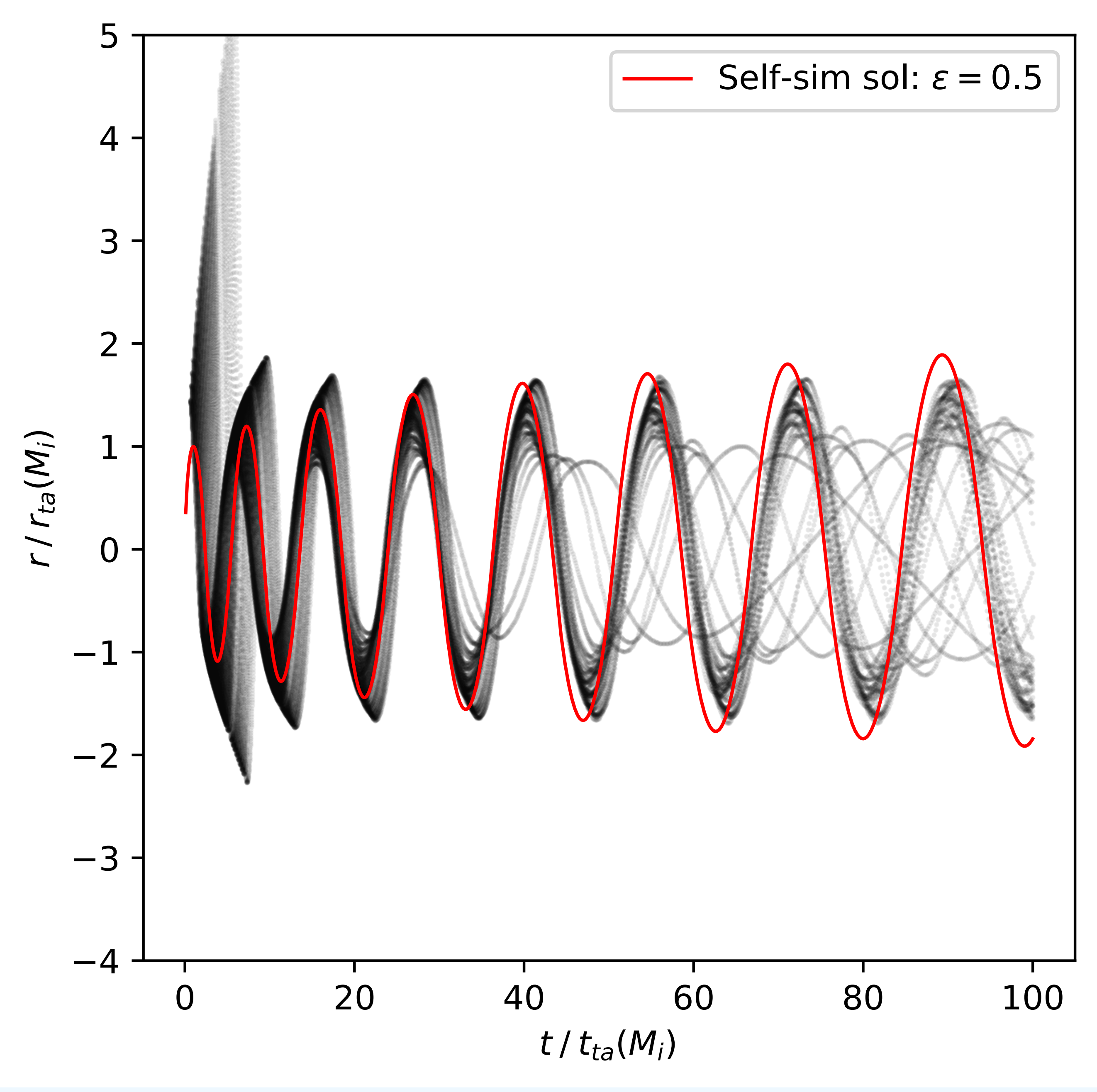
We want to connect **"B. Post-collapse phase"** to **"C. Self-similar phase"**

Density field

time evolution



Particle trajectory



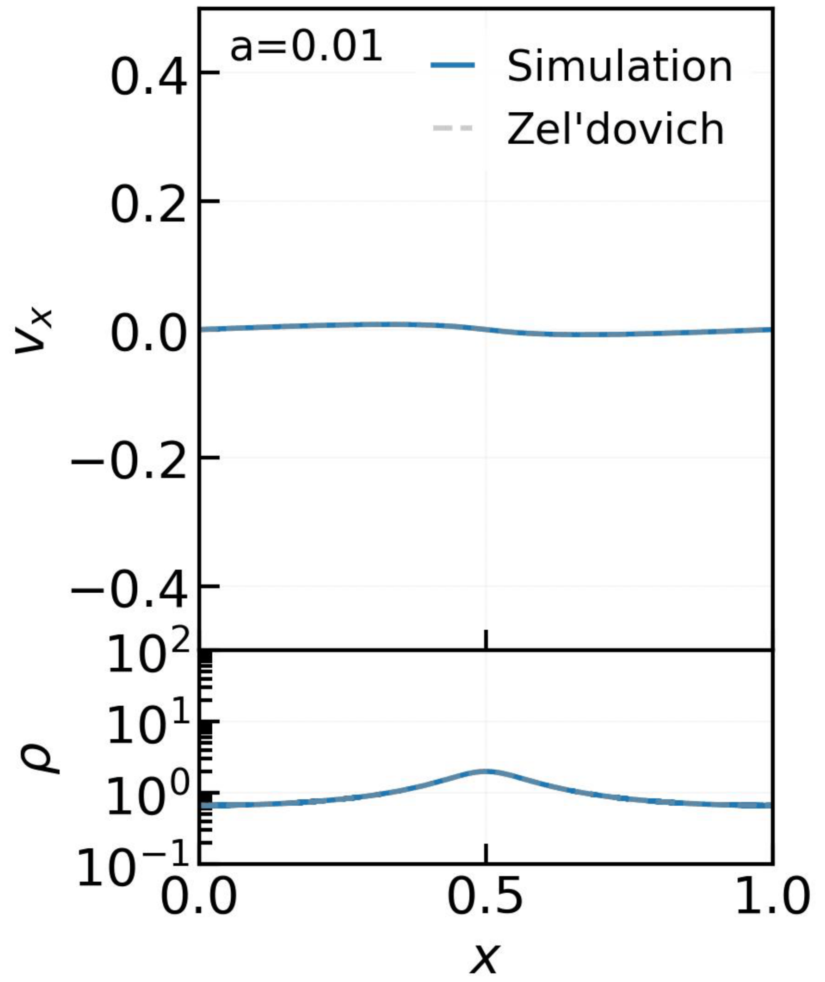
A self-similar structure becomes evident after 3–4 oscillations

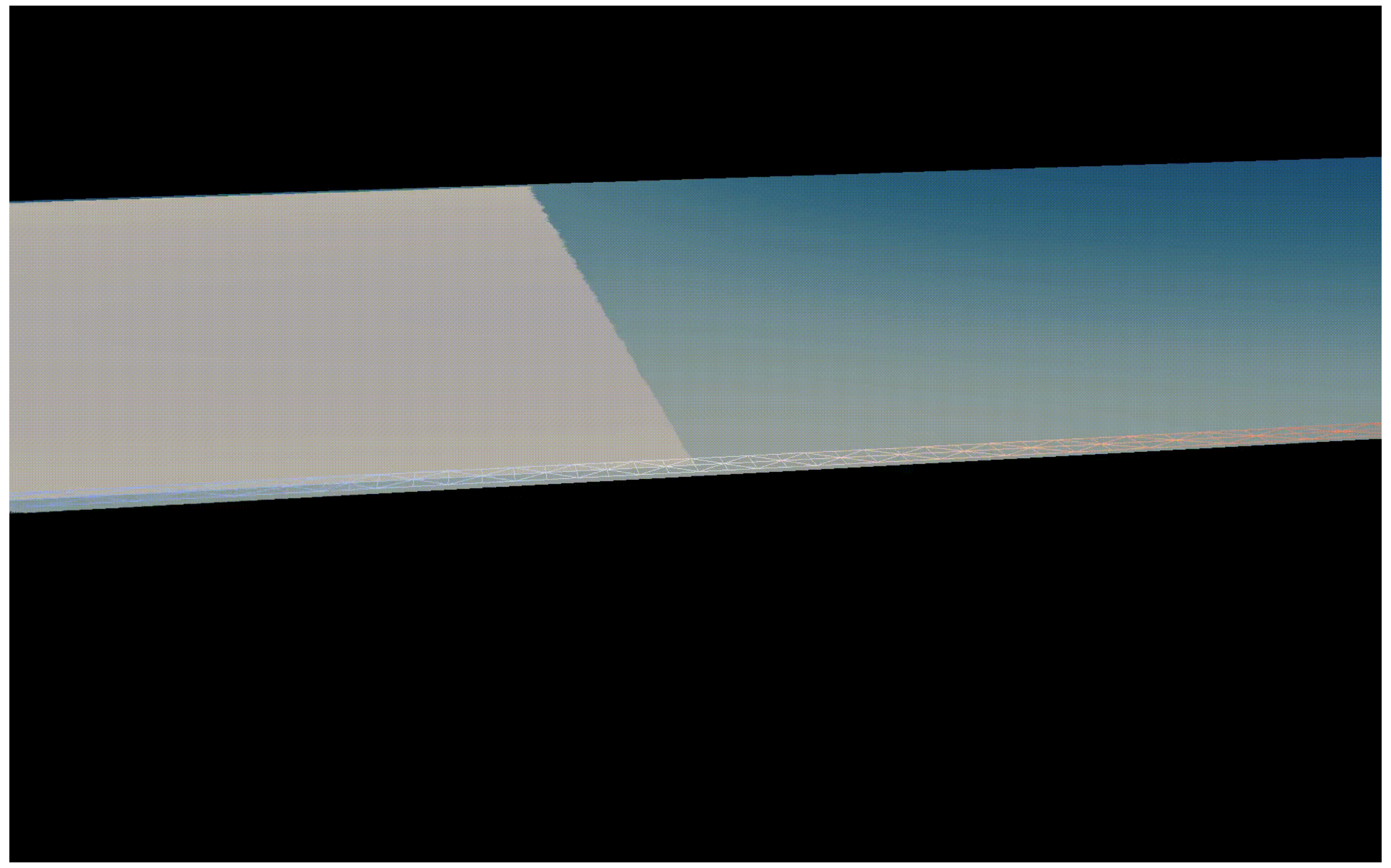
-> smooth connection between PCPT and the self-similar solution is expected!

Summary

single stream *shell crossing* multi stream

The rise of Vlasov-Poisson simulation -> Multi-stream dynamics will be further understood





Theoretical work will be further needed complementary to Vlasov-Poisson simulations -> `PCPT`

✔ Extending to ***specific dark matter candidates*** or ***warm*** case would be doable
-> comparing with **observations** in future for studying nature of dark matters

e.g., 2-point statistics, halo innner structure from lensing, annihilation signal from survived prompt cusp, and so on...