Detection of dark matter axions 3/3

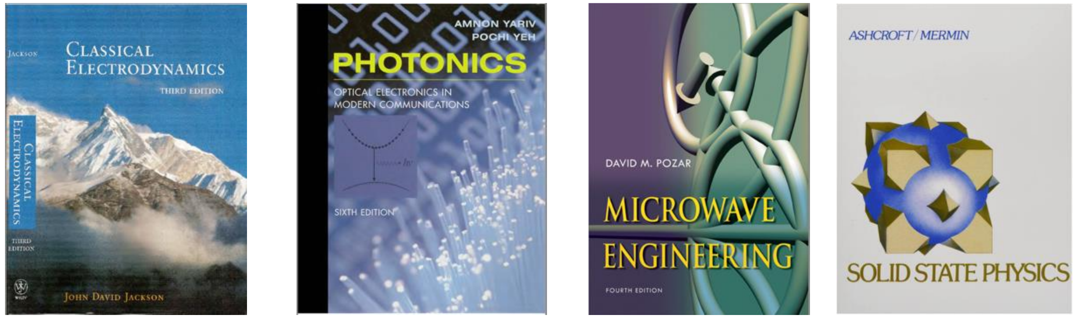
from classical microwaves to quantised photons

Akira Miyazaki

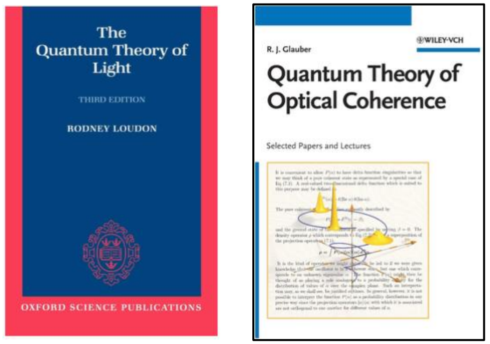
CNRS/IN2P3/IJCLab Université Paris-Saclay

Outline of the lecture courses and textbooks

- Part 1: overview on axion searches
  - Axion vs WIMPs
  - Various experiments
  - Non-DM axions
- Part 2: classical detection scheme
  - Boundary conditions
  - Microwave resonators
  - Analog and digital system
  - Data processing and noise



- Part 3: quantum detection scheme
  - Quantum coherent states
  - Glauber's theorem
  - Thermal noise and Standard Quantum Limit
  - Squeezing and photon counting



Revisit wave-particle duality

This lecture

Wave (field) states

$$\hat{\mathcal{E}} = \hat{a}\exp(+i\omega t) - \hat{a}^\dagger \exp(-i\omega t)$$

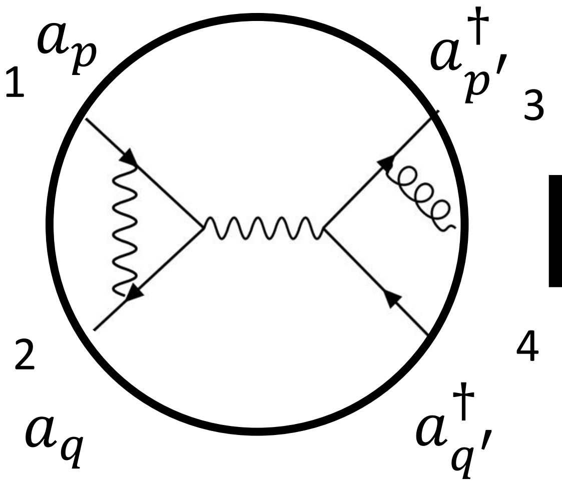
$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

Eigenstate of the annihilation operator

$$\rho = \int d\alpha^2 P(\alpha)|\alpha\rangle\langle\alpha|$$

projection

Quantum field



ρ

projection

Number (energy) states

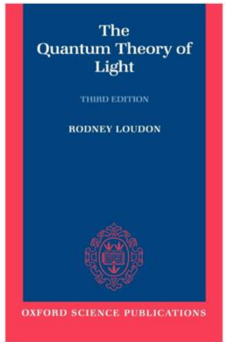
$$\hat{n} = \hat{a}^\dagger\hat{a}$$

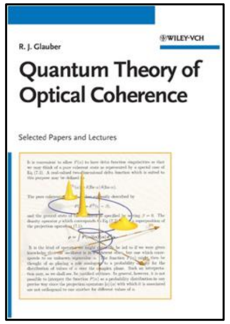
$$\hat{n}|n\rangle = n|n\rangle$$

Eigenstate of the number operator

$$\rho = \sum_n c_n |n\rangle\langle n|$$

- Typical particle physics focuses on number states
- We count numbers and measure energy NOT field
- We are not familiar with coherent states





Three pillars of modern physics

Lorentz invariance

Galilei invariance

Quantum Field Theory in Particle Physics

Quantum Optics for photon science

Quantum Many Body system for solid states

Ex) Standard Model

Ex) Laser

Ex) Superconductivity

number state

$$\hat{n}|n\rangle = n|n\rangle$$

→ Collider physics

Coherent state

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

→ Quantum information, ***axion***

Part 3: Quantum detection scheme

- Quantum coherent states
- Glauber's theorem
- Thermal noise and Standard Quantum Limit
- Squeezing and photon counting
- Conclusion of part 3

- Global conclusion of the lecture courses

Step by step

Part 3: Quantum detection scheme

- Quantum coherent states
- Glauber's theorem
- Thermal noise and Standard Quantum Limit
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Coherent state: eigen state of annihilation operator

Harmonic oscillator: $\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + (\hbar\omega/2)$

bosonic annihilation and creation operators: $\hat{a} \equiv \hat{x} + i\hat{p}$

Reminder: their algebra

$$[\hat{a}, \hat{a}^\dagger] = 1 \qquad [\hat{a}, \hat{a}] = [\hat{a}^\dagger, \hat{a}^\dagger] = 0$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \qquad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$\hat{a}$ is not Hermitian = not ***observable*** ($\in \mathbb{R}$) but can have an eigen state

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \; (\alpha \in \mathbb{C})$$

This is called a quantum coherent state

Electric field operator and expectation value with $|\alpha\rangle$

$$\hat{\mathcal{E}}(\boldsymbol{r},t) = i\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}\left[\hat{a}e^{-i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})} - \hat{a}^\dagger e^{i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})}\right]$$

$$E \equiv \langle\alpha|\hat{\mathcal{E}}(\boldsymbol{r},t)|\alpha\rangle = i\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}\left[\alpha e^{-i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})} - \alpha^* e^{+i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})}\right]$$

$$= E^+ e^{-i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})} - E^- e^{+i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})}$$

This is a solution of classical Maxwell equation with a dispersion relation $\omega^2 = |\boldsymbol{k}|^2$

$$\left(\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2\right)E\ (t,\boldsymbol{r}) = 0$$

Exercise 1: $\hat{a} \equiv \hat{x} + i\hat{p}$

This is not your 1st time to see the coherent state

Wavefunction: projected onto the x-axis

$$\Psi(x) \equiv \langle x|\alpha\rangle = ?$$

Hint…

$$1 = \int dx' \, |x'\rangle\langle x'| \qquad [\hat{x}, \hat{p}] = \frac{i}{2}$$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \qquad \hat{p} \rightarrow \frac{1}{2i}\frac{d}{dx}$$

Answer 1: $\Psi(x) \equiv \langle x|\alpha\rangle$

Gaussian!

$$\langle x|\alpha\rangle = \frac{1}{\alpha}\langle x|\alpha|\alpha\rangle = \frac{1}{\alpha}\langle x|\hat{a}|\alpha\rangle = \frac{1}{\alpha}\langle x|(\hat{x}+i\hat{p})|\alpha\rangle = \frac{1}{\alpha}\int dx'\,\langle x|(\hat{x}+i\hat{p})|x'\rangle\langle x'|\alpha\rangle$$

$$\rightarrow \alpha\Psi(x) = \int dx'\,\boxed{\langle x|\hat{x}|x'\rangle}\Psi(x') + i\int dx'\,\boxed{\langle x|\hat{p}|x'\rangle}\Psi(x')$$

$= x\delta(x-x')$ $\quad$ $= \frac{1}{2i}\frac{d}{dx}\delta(x-x')$ $\quad$ (partial integral)

$$= \int dx'\,x\delta(x-x')\Psi(x') + \frac{1}{2}\int dx'\,\delta(x-x')\frac{d\Psi(x')}{dx}$$

$$\rightarrow \alpha\Psi(x) = \left[x + \frac{1}{2}\frac{d}{dx}\right]\Psi(x) \quad \rightarrow \boxed{\frac{d\Psi(x)}{dx} + 2x\Psi(x) - 2\alpha\Psi(x) = 0}$$

Solution

$$\Psi(x) = \exp(Ax^2 + Bx + C)$$

$$\rightarrow \frac{d\Psi(x)}{dx} = [2Ax + B]\Psi(x)$$

$$\begin{cases} A = -1 \\ B = 2\alpha \end{cases}$$

$$\rightarrow \Psi(x) = \frac{1}{\sqrt{2\pi}}e^{-(x-\alpha)^2}$$

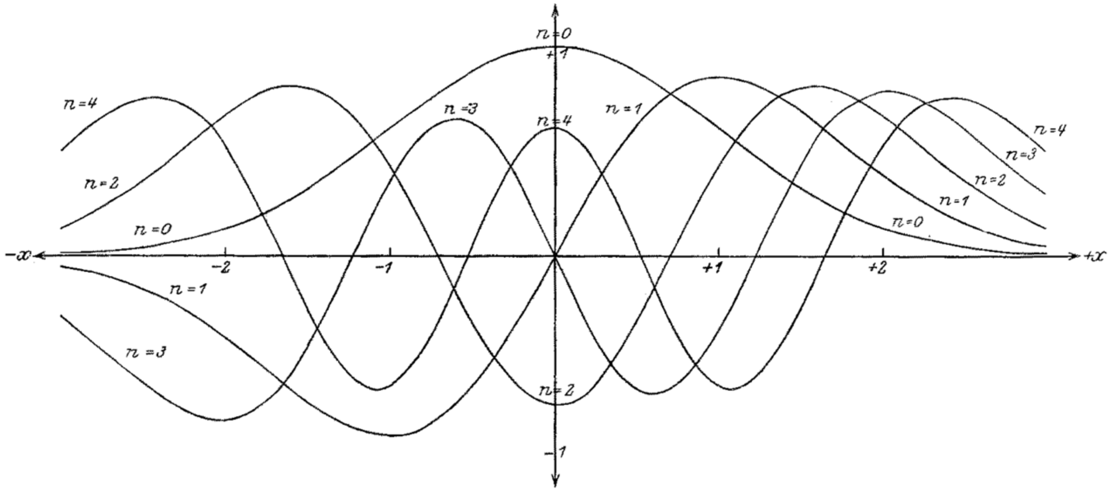
Der stetige Übergang von der Mikro- zur Makromechanik.

Von E. SCHRÖDINGER, Zürich.

$$\frac{m}{2}\left(\frac{dq}{dt}\right)^2 + 2\pi^2\nu_0^2 m q^2$$

$$\left.\begin{array}{l} \psi_n = e^{-\frac{x^2}{2}} H_n(x)\, e^{2\pi i \nu_n t} \\ \left(\nu_n = \frac{2n+1}{2}\nu_0;\; n = 0, 1, 2, 3 \ldots.\right) \end{array}\right\} \quad (3)$$

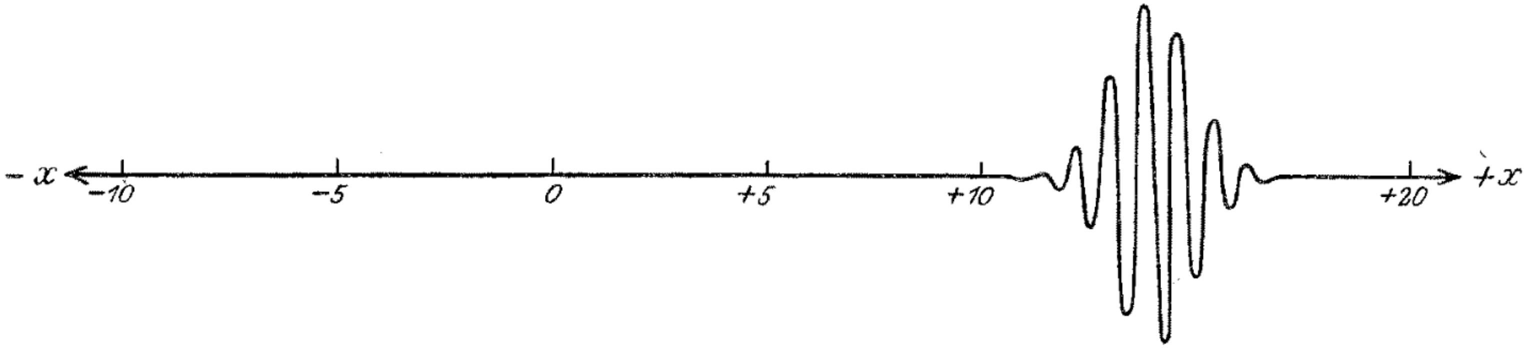
Die H_n sind die nach HERMITE benannten



$$\psi = \sum_{n=0}^{\infty}\left(\frac{A}{2}\right)^n \frac{\psi_n}{n!}$$

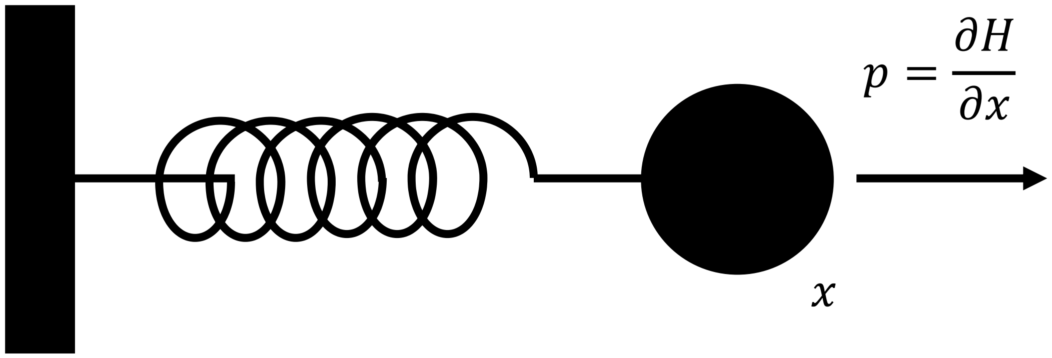
$$= e^{\pi i \nu_0 t}\sum_{n=0}^{\infty}\left(\frac{A}{2}e^{2\pi i \nu_0 t}\right)^n \frac{1}{n!}\, e^{-\frac{x^2}{2}} H_n(x).$$

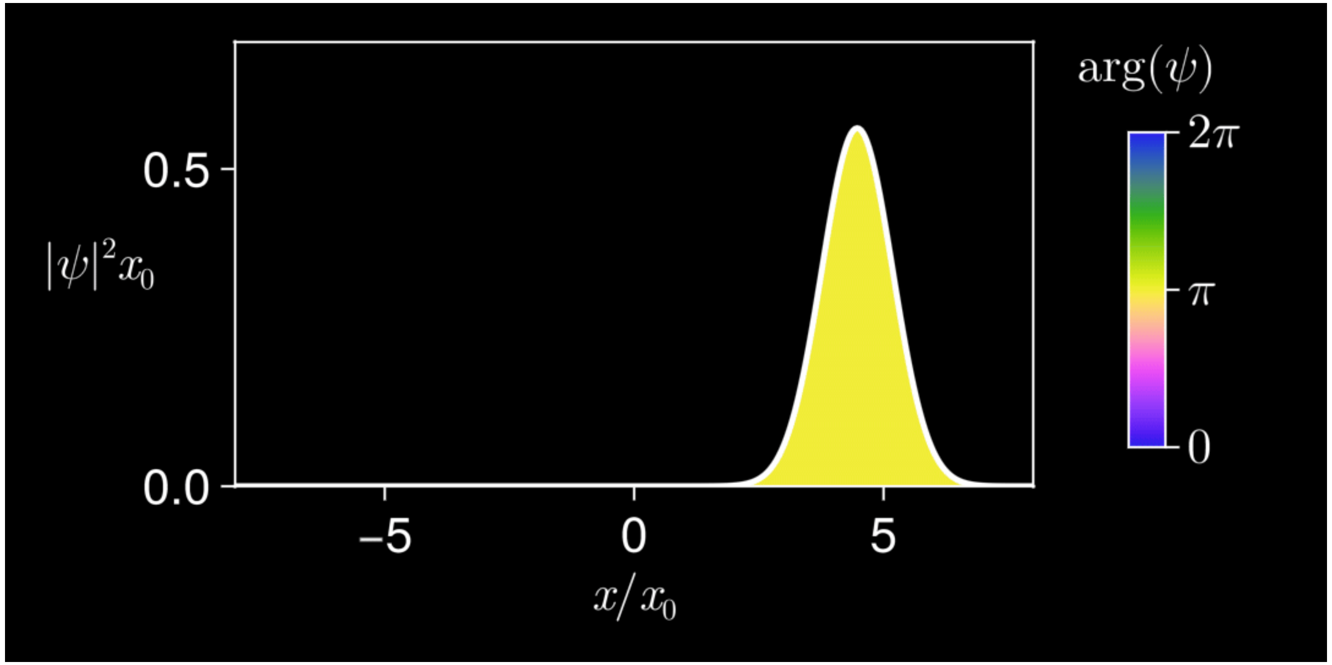
$$\psi = e^{\frac{A^2}{4} - \frac{1}{2}(x - A\cos 2\pi\nu_0 t)^2} \cdot \cos\left[\pi\nu_0 t + (A\sin 2\pi\nu_0 t)\cdot\left(x - \frac{A}{2}\cos 2\pi\nu_0 t\right)\right]$$



Schrödinger, E. (1926). "Der stetige Übergang von der Mikro- zur Makromechanik". Die Naturwissenschaften 14 (28)

Coherent state = Schrödinger's wave packet





- Harmonic oscillator with Gaussian quantum fluctuation
- Standard deviation around the mean
- The center is exactly the classical harmonic oscillator

Exercise 2: displacement operator

$$\widehat{D}(\alpha) \equiv \exp(\alpha \hat{a}^{\dagger} - \alpha^{*} \hat{a})$$

$$\widehat{D}^{\dagger}(\alpha) \equiv \exp\left(\alpha^{*} \hat{a} - \alpha \hat{a}^{\dagger}\right) = \widehat{D}(-\alpha)$$

$$\hat{a}\widehat{D}(\alpha)|0\rangle = ?$$

Hint…

$$\widehat{D}^{\dagger}(\alpha)\hat{a}\widehat{D}(\alpha) = ?$$

$$\exp(\hat{A})\,\hat{B}\,\exp(-\hat{A}) = \hat{B} + [\hat{A}, \hat{B}] + \frac{1}{2!}\left[\hat{A}, [\hat{A}, \hat{B}]\right] \ldots$$

Baker-Campbell-Hausdorff lemma

Answer 2

This plays a central role in axion detection

$$\hat{A} = \alpha^* \hat{a} - \alpha \hat{a}^\dagger \qquad [\hat{A}, \hat{B}] = [\alpha^* \hat{a} - \alpha \hat{a}^\dagger, \hat{a}] = -\alpha[\hat{a}^\dagger, \hat{a}] = \alpha$$

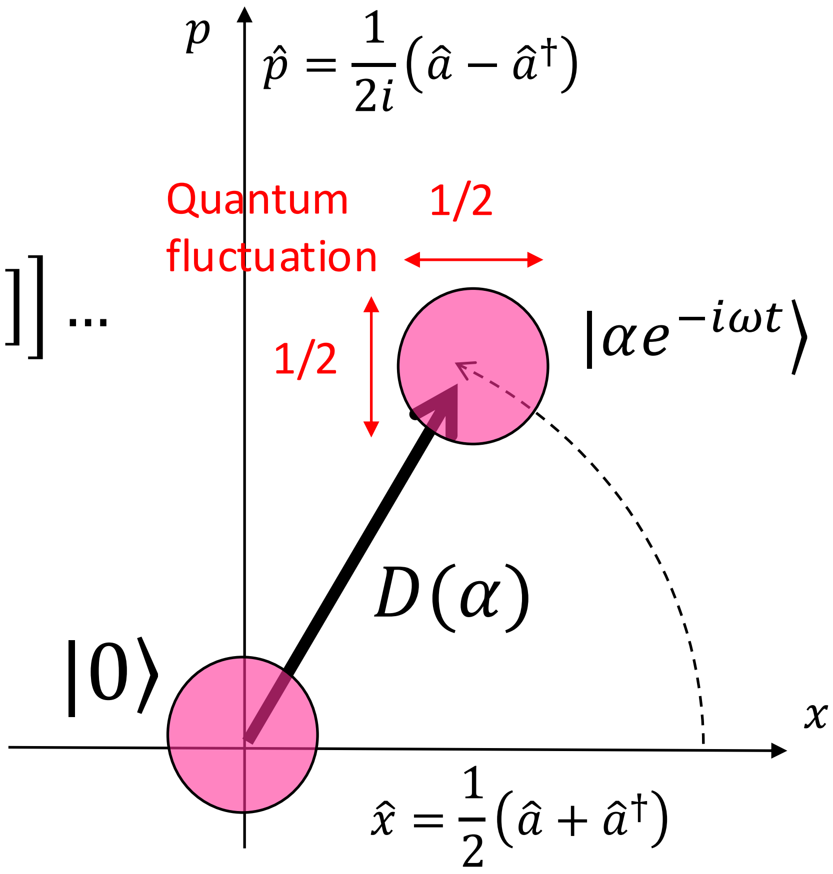
$$\hat{B} = \hat{a} \qquad [\hat{A}, [\hat{A}, \hat{B}]] = [\alpha^* \hat{a} - \alpha \hat{a}^\dagger, \alpha] = 0$$

$$\hat{D}^\dagger(\alpha)\hat{a}\hat{D}(\alpha) = \exp(\hat{A})\,\hat{B}\exp(-\hat{A}) = \hat{B} + [\hat{A}, \hat{B}] + \frac{1}{2!}[\hat{A}, [\hat{A}, \hat{B}]] \dots$$

$$= \hat{a} + \alpha$$

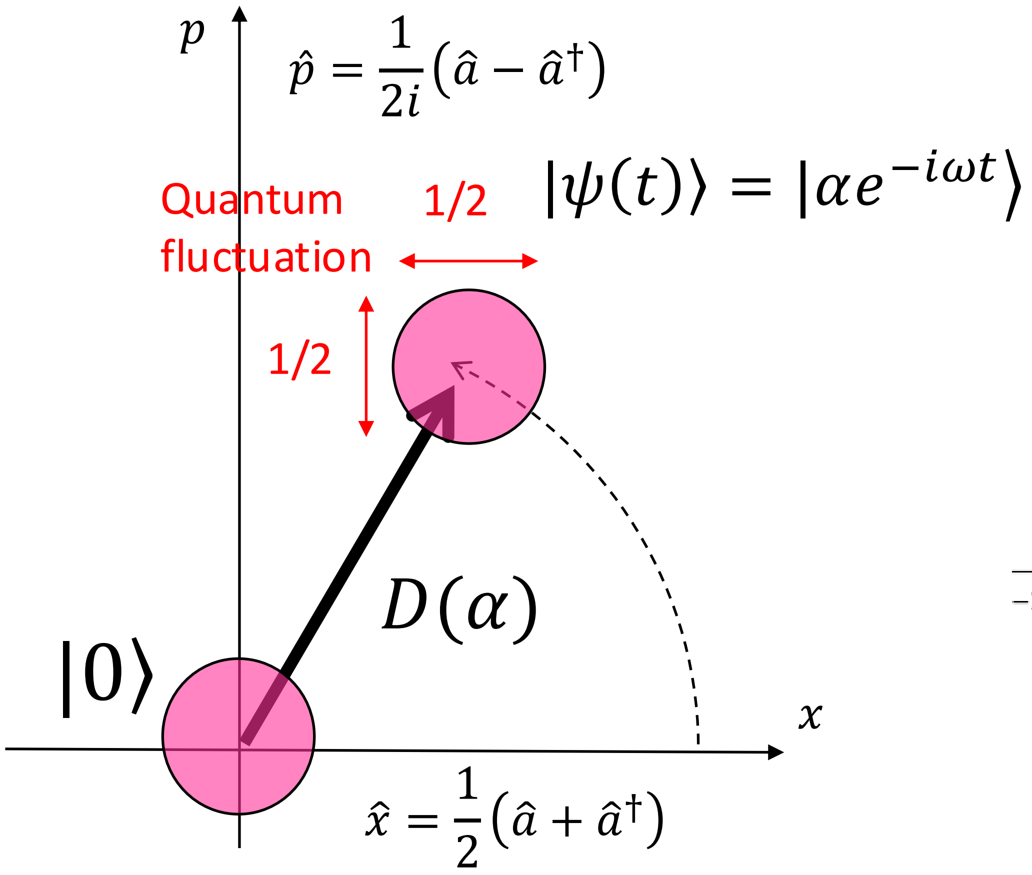
$$\rightarrow \hat{a}\boxed{\hat{D}(\alpha)|0\rangle} = \hat{D}(\alpha)(\hat{a} + \alpha)|0\rangle = \alpha\boxed{\hat{D}(\alpha)|0\rangle}$$

$$\hat{D}(\alpha)|0\rangle = |\alpha\rangle$$

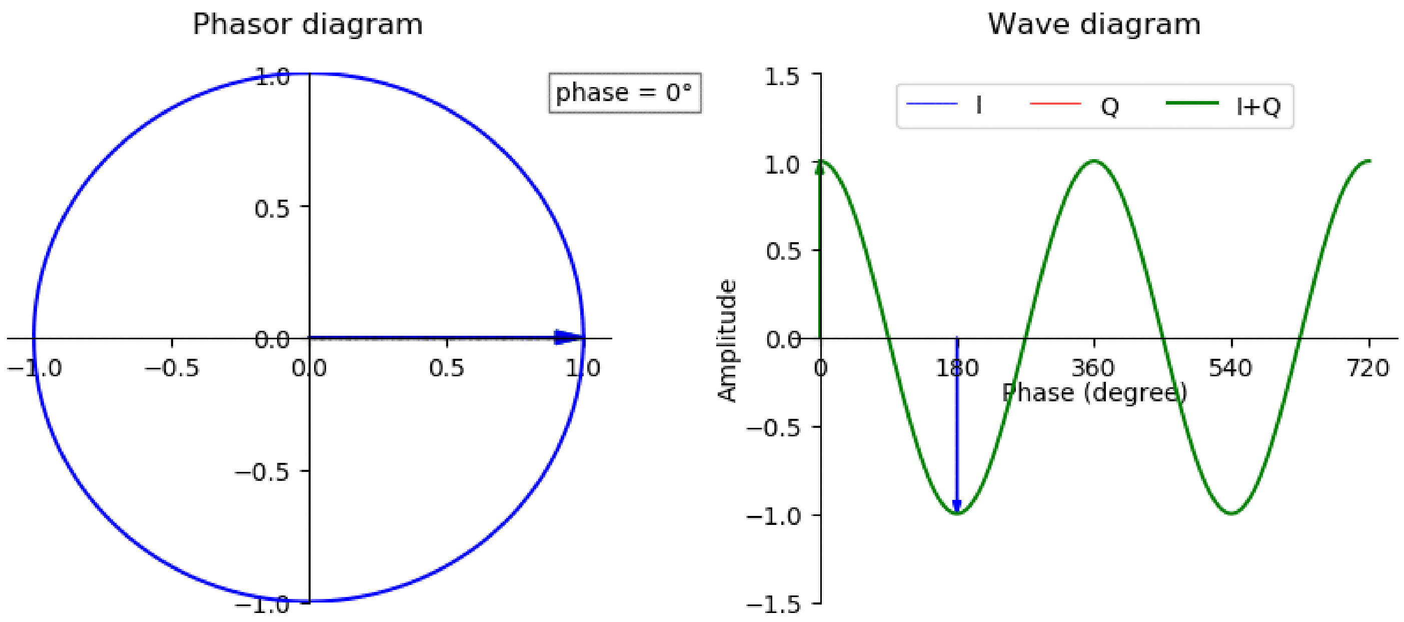


A displacement operator generates a coherent state out of vacuum

Coherent state = classical Maxwell + quantum fluctuation



Classical I/Q sampling



- Two quadrants: $x \sim I\ ,\ q \sim Q$
- $\hat{a}$ is ***not an observable*** at a time but is an ***operational observable*** with measurement over time to get an amplitude and phase

Exercise 3: expansion in number states

$$|\alpha\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$$

$$c_n = ?$$

Hint…

$$\alpha \sum_{n=0}^{\infty} c_n |n\rangle = \alpha|\alpha\rangle = \hat{a}|\alpha\rangle = \sum_{n=0}^{\infty} c_n \hat{a}|n\rangle = ?$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

Answer 3: $|\alpha\rangle$ expanded by $|n\rangle$

$|\alpha\rangle$ is called coherent state. Its explicit representation with number states can be given by

$$|\alpha\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$$

and applying $\hat{a}$ gives

$$\alpha \sum_{n=0}^{\infty} c_n |n\rangle = \alpha|\alpha\rangle = \hat{a}|\alpha\rangle = \sum_{n=0}^{\infty} c_n \hat{a}|n\rangle = \sum_{n=1}^{\infty} c_n \sqrt{n}|n-1\rangle = \sum_{n=0}^{\infty} c_{n+1}\sqrt{n+1}|n\rangle$$

Comparing the coefficients gives

$$c_{n+1} = \frac{\alpha}{\sqrt{n+1}} c_n$$

The coefficient can be recursively determined

$$c_n = \frac{\alpha}{\sqrt{n}} c_{n-1} = \frac{\alpha}{\sqrt{n}} \frac{\alpha}{\sqrt{n-1}} c_{n-2} = \frac{\alpha}{\sqrt{n}} \frac{\alpha}{\sqrt{n-1}} \dots \frac{\alpha}{\sqrt{1}} c_0 = \frac{\alpha^n}{\sqrt{n!}} c_0$$

The normalization condition gives

$$1 = \langle\alpha|\alpha\rangle = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} c_m^* c_n \langle m|n\rangle = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{\alpha^{*m}}{\sqrt{m!}} \frac{\alpha^n}{\sqrt{n!}} c_0^* c_0 \delta_{mn} = |c_0|^2 \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} = |c_0|^2 e^{|\alpha|^2} \rightarrow |c_0| = e^{-|\alpha|^2/2}$$

Selecting one phase $c_0 = e^{-|\alpha|^2/2}$ leads to

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

Similar to the well-known stuff

Projection of $|\alpha\rangle$ onto $|n\rangle$ returns a Poisson distribution

Probability of observing n photon in photon counting is

$$P(n) = |\langle n|\alpha\rangle|^2 = \left| e^{-|\alpha|^2/2} \sum_{m=0}^{\infty} \frac{\alpha^m}{\sqrt{m!}} \underbrace{\langle n|m\rangle}_{\delta_{nm}} \right|^2 = \left| e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} \right|^2 = \frac{(|\alpha|^2)^n}{n!} e^{-|\alpha|^2}$$

This is a Poisson distribution with mean number of photons $\mu = |\alpha|^2$

$$P(n;\mu) = \frac{\mu^n}{n!} e^{-\mu}$$

<u>Remark</u>

- Coherent state → Poisson is true
- Poisson → coherent state is not necessarily true eg) thermal state with degenerated multi-modes

Part 3: Quantum detection scheme

- Quantum coherent states
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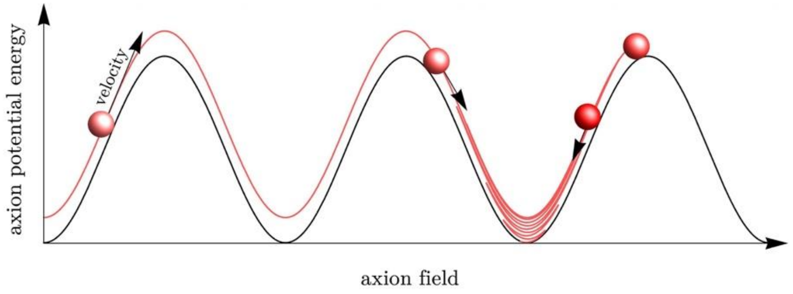
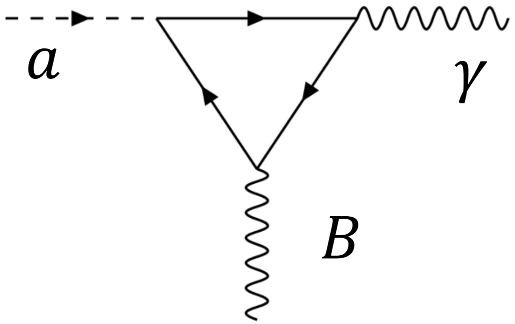
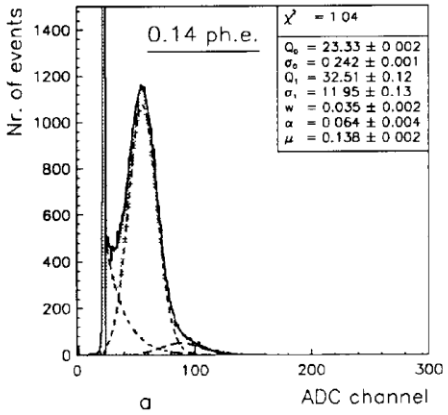
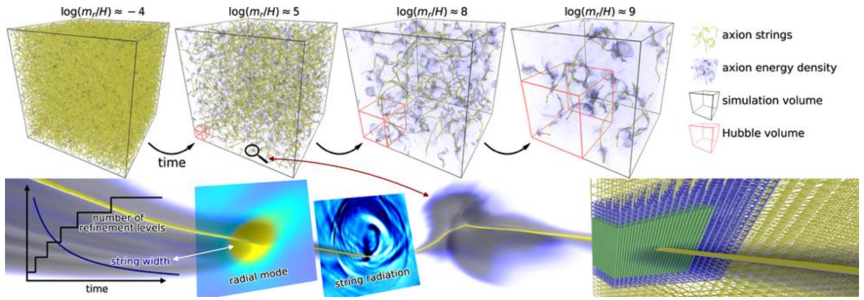
Do we know a quantum state of axion?







Quantum axion → **Quantum** microwaves → **Quantum** detection



$$\hat{H} = \omega_a \hat{a}_a^\dagger \hat{a}_a + \omega_\gamma \hat{a}_\gamma^\dagger \hat{a}_\gamma + \omega_m \hat{a}_a^\dagger \hat{a}_\gamma + \omega_m^* \hat{a}_\gamma^\dagger \hat{a}_a$$

$$\rightarrow |\alpha_a, \alpha_\gamma\rangle = (1 + U_{a\gamma}\alpha_a \alpha_\gamma^\dagger)|\alpha_a, 0\,\rangle$$

G. Rafelt et al, "Quantum statistics in particle mixinv phenomena" PRD 45 1782 (1992)

- If the **dark matter axion is in the coherent state**, microwave photons are in a coherent state
- But we are not sure if axions are in such a quantum state…DM axions are classical objects

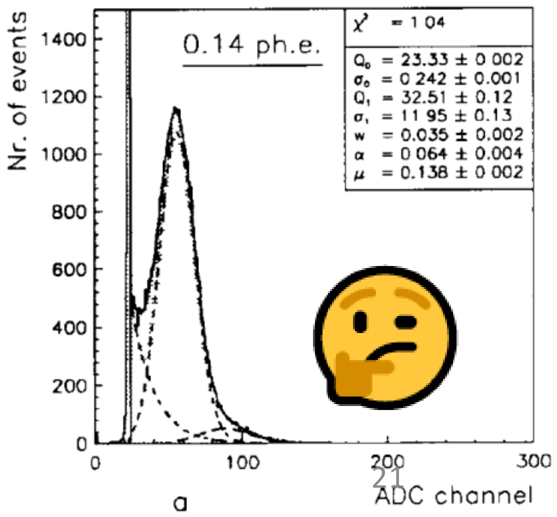
Question: classical → quantum

Classical axion → **Classical** microwaves → **Quantum** detection

$$\epsilon \nabla \cdot \mathbf{E} = \rho - g_{a\gamma} \mathbf{B}_\mathrm{e} \cdot \nabla a \,,$$
$$\nabla \times \mathbf{H} - \dot{\mathbf{E}} = \mathbf{J} + g_{a\gamma} \mathbf{B}_\mathrm{e} \dot{a} \,,$$
$$\ddot{a} - \nabla^2 a + m_a^2 a = g_{a\gamma} \mathbf{E} \cdot \mathbf{B}_\mathrm{e} \,,$$

What is the quantum statistical distribution of detected photons?

- Can we obtain quantum statistical distribution **without assuming quantum nature in the source**?
- Can we assume a Poisson distribution?



Glauber's theorem: classical → quantum

PHYSICAL REVIEW VOLUME 131, NUMBER 6 15 SEPTEMBER 1963

Coherent and Incoherent States of the Radiation Field*

ROY J. GLAUBER
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 29 April 1963)

$$H_1(t) = -\frac{1}{c}\int \mathbf{j}(\mathbf{r},t)\cdot\mathbf{A}(\mathbf{r},t)d\mathbf{r}. \qquad (9.16)$$

The introduction of an explicitly time-dependent interaction of this type means that the state vector for the field, $|\,\rangle$, which previously was fixed (corresponding to the Heisenberg picture) will begin to change with time in accordance with the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\,\rangle = H_1(t)|\,\rangle, \qquad (9.17)$$

which is the one appropriate to the interaction representation. The solution of this equation is easily found.[20] If we assume that the initial state of the field at time $t=-\infty$ is one empty of all photons, then the state of the field at time t may be written in the form

$$|t\rangle = \exp\left\{\frac{i}{\hbar c}\int_{-\infty}^{t}dt'\int \mathbf{j}(\mathbf{r},t')\cdot\mathbf{A}(\mathbf{r},t')d\mathbf{r}+i\varphi(t)\right\}|\text{vac}\rangle. \qquad (9.18)$$

$$D_k(\beta_k) = \exp[\beta_k a_k^\dagger - \beta_k^* a_k]. \qquad (9.19)$$

Then it is clear from the expansion (2.10) for the vector potential that we may write

$$\exp\left\{\frac{i}{\hbar c}\int_{-\infty}^{t}dt'\int \mathbf{j}(\mathbf{r},t')\cdot\mathbf{A}(\mathbf{r},t')d\mathbf{r}\right\} = \prod_k D_k[\alpha_k(t)], \qquad (9.20)$$

where the time-dependent amplitudes $\alpha_k(t)$ are given by

$$\alpha_k(t) = \frac{i}{(2\hbar\omega)^{1/2}}\int_{-\infty}^{t}dt'\int d\mathbf{r}\,\mathbf{u}_k^*(\mathbf{r})\cdot\mathbf{j}(\mathbf{r},t')e^{i\omega t'}. \qquad (9.21)$$

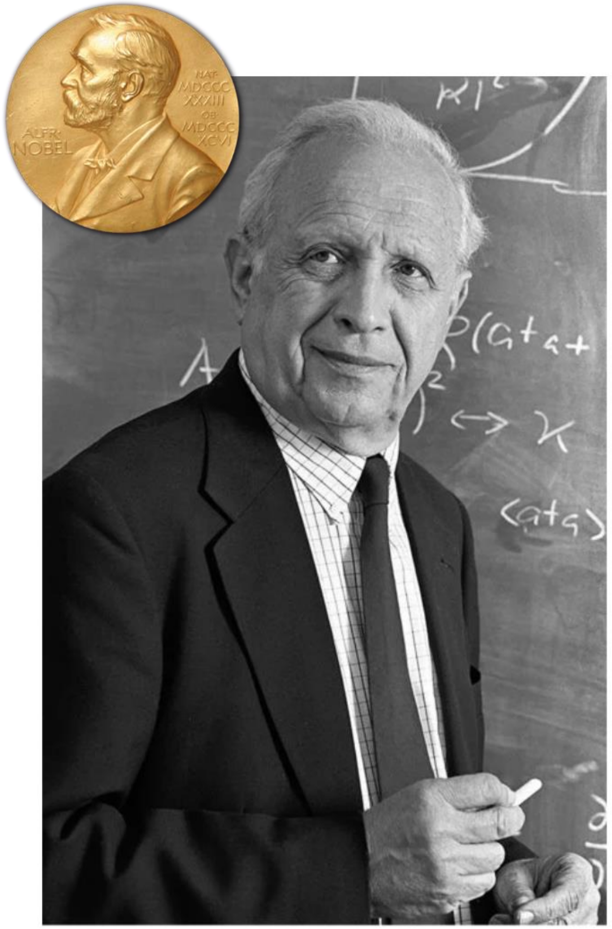
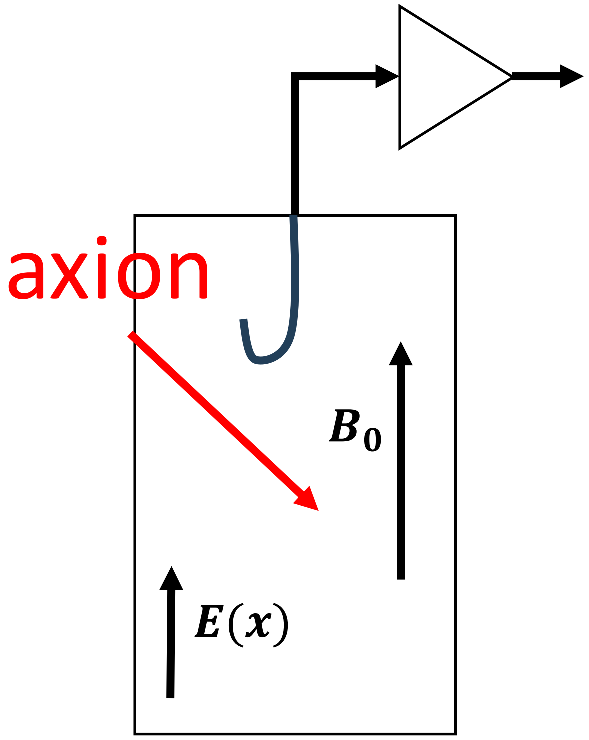


Photo: J.Reed

Classical current generates quantized photons in a quantum coherent state
→ **Let's apply this to classical axions**

Classical axion in a MW resonant cavity



Quantized MW photons

$$[\hat{b}, \hat{b}^\dagger] = 1$$

$$[\hat{b}, \hat{b}] = [\hat{b}^\dagger, \hat{b}^\dagger] = 0$$

$$\hat{n}|n\rangle \equiv \hat{b}^\dagger \hat{b}|n\rangle = n|n\rangle$$

$$\hat{b}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\hat{b}|n\rangle = \sqrt{n}|n-1\rangle$$

$$i\hbar \frac{\mathrm{d}}{dt}|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle$$

$$\hat{H} = \underbrace{\hbar\omega\left(\hat{b}^\dagger \hat{b} + \frac{1}{2}\right)}_{\hat{H}_0} + \underbrace{\hbar[f(t)\hat{b}^\dagger + f^*(t)\hat{b}]}_{\hat{V}(t)}$$

$$f(t) = g_{a\gamma\gamma}\, a(t) \int (\boldsymbol{E}(x)\cdot \boldsymbol{B}_0) d^3x$$

Any classical axion field

<u>In the interaction picture</u>

$$|\psi_I(t)\rangle = \exp(i\hat{H}_0 t/\hbar)\,|\psi(t)\rangle$$

$$\hat{V}_I(t) = \exp(i\hat{H}_0 t/\hbar)\,\hat{V}(t)\,\exp(-i\hat{H}_0 t/\hbar)$$

$$i\hbar \frac{\mathrm{d}}{dt}|\psi_I(t)\rangle = \hat{V}_I(t)|\psi_I(t)\rangle$$

This is usually solved via time-dependent **perturbation theory**…

Solution by Ayuki Kamada 1/2

The perturbation term in the interaction picture can be dramatically simplified

Linear **Linear**

$$\hat{V} = \hbar\left[f(t)\hat{b}^\dagger + f^*(t)\hat{b}\right] \rightarrow \hat{V}_I(t) = \exp\left(i\hat{H}_0 t/\hbar\right)\hat{V}(t)\exp\left(-i\hat{H}_0 t/\hbar\right) = ?$$

$$\rightarrow \hat{b}_I^\dagger = \exp\left(i\omega t\hat{b}^\dagger\hat{b}\right)\hat{b}^\dagger\exp\left(-i\omega t\hat{b}^\dagger\hat{b}\right) = ?$$

BCH lemma again

$$\exp(\hat{A})\,\hat{B}\exp(-\hat{A}) = \hat{B} + [\hat{A},\hat{B}] + \frac{1}{2!}\left[\hat{A},[\hat{A},\hat{B}]\right]\dots \qquad \begin{cases}\hat{A} = (i\omega t)\hat{b}^\dagger\hat{b}\\ \hat{B} = \hat{b}^\dagger\end{cases}$$

$$[\hat{A},\hat{B}] = \left[(i\omega t)\hat{b}^\dagger\hat{b},\hat{b}^\dagger\right] = (i\omega t)\hat{b}^\dagger \qquad \left[\hat{A},[\hat{A},\hat{B}]\right] = \left[(i\omega t)\hat{b}^\dagger\hat{b},(i\omega t)\hat{b}^\dagger\right] = (i\omega t)^2\left[\hat{b}^\dagger\hat{b},\hat{b}^\dagger\right] = (i\omega t)^2\hat{b}^\dagger$$

$$\hat{b}_I^\dagger = \exp\left(i\omega t\hat{b}^\dagger\hat{b}\right)\hat{b}^\dagger\exp\left(-i\omega t\hat{b}^\dagger\hat{b}\right) = \hat{b}^\dagger + (i\omega t)\hat{b}^\dagger + \frac{1}{2!}(i\omega t)^2\hat{b}^\dagger + \cdots = \exp(i\omega t)\,\hat{b}^\dagger$$

$$\rightarrow \hat{V}_I(t) = \hbar\left[f(t)\exp(i\omega t)\,\hat{b}^\dagger + f^*(t)\exp(-i\omega t)\,\hat{b}\right]$$

Solution by Ayuki Kamada 2/2

Exceptionally, **an exact solution** can be obtained in this special perturbative term

$$i\hbar\frac{\mathrm{d}}{dt}|\psi_I(t)\rangle = \hat{V}_I(t)|\psi_I(t)\rangle = \left[\hbar f^*(t)\exp(-i\omega t)\,\hat{b} + \hbar f(t)\exp(i\omega t)\,\hat{b}^\dagger\right]|\psi_I(t)\rangle$$

Linear ***Linear***

A solution can be

$$|\psi_I(t)\rangle \propto \exp[\eta(t)\hat{b}^\dagger - \eta^*(t)\hat{b}]\,|\psi_I(0)\rangle$$

cheating

← You probably still remember this guy ☺

To find the coefficients

$$\frac{\mathrm{d}}{dt}|\psi_I(t)\rangle \propto \left[\frac{d\eta(t)}{dt}\hat{b}^\dagger + \frac{d\eta^*(t)}{dt}\hat{b}\right]|\psi_I(t)\rangle$$

cheating $= f(t)\exp(i\omega t)$ $= f^*(t)\exp(-i\omega t)$

$$\rightarrow \eta(t) \sim \int_0^t dt'\, f(t')\exp(i\omega t')$$

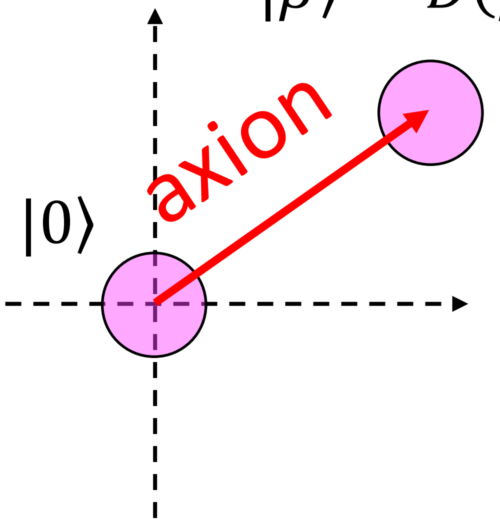
Axion induced microwave photon states inside a cavity

General solution including a phase factor

$$
\begin{cases}
|\psi(t)\rangle = \exp\left(-iC_1(t)\right)\boxed{\hat{D}\left(\beta(t)\right)}\exp(-iH_0 t/\hbar)\,|\psi(0)\rangle \\
\beta(t) = C_2(t)\exp(-i\omega t) \\
C_1(t) = -\frac{i}{2}\int_0^t dt' f^*(t')\exp(i\omega t')\int_0^{t'} dt'' f(t'')\exp(i\omega t'') \\
C_2(t) = \int_0^t dt' f(t')\exp(i\omega t')
\end{cases}
$$

Complete publication being prepared with A. Kamada

$|\beta\rangle = \hat{D}(\beta)|0\rangle$

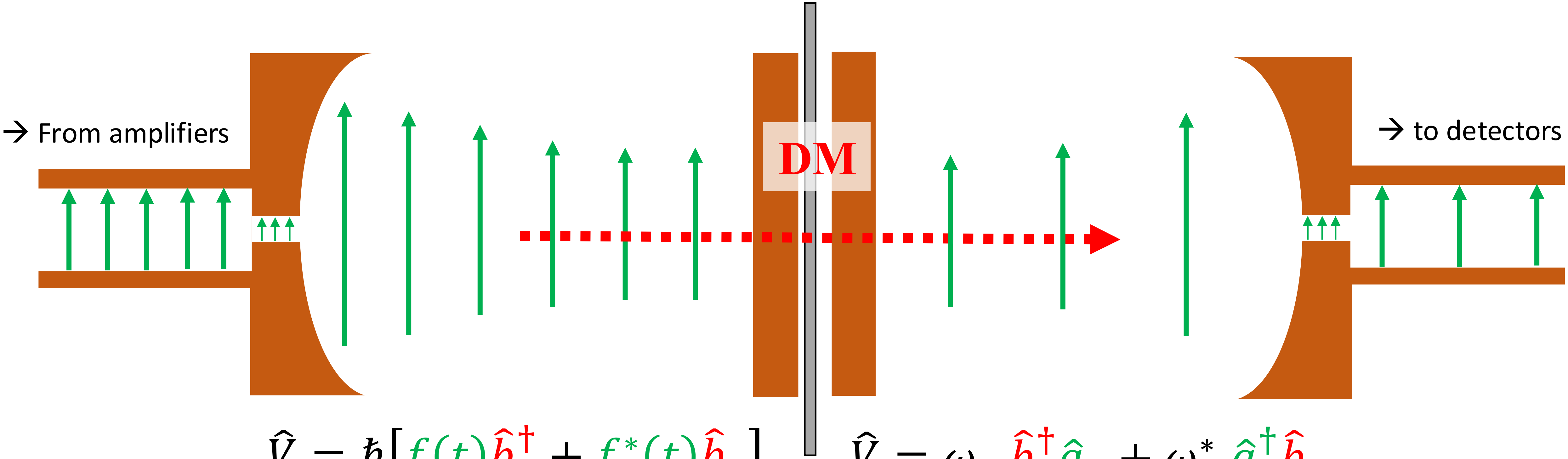


$\hat{D}(\beta) = \exp(\beta\hat{b}^\dagger - \beta^*\hat{b})$

Classical DM axion fields is a displacement operator that generates a quantum coherent state of microwave photons out of the vacuum state inside a microwave cavity under a static magnetic field

→ **This is the key to understand quantum detection of DM axions**

Eg) Quantum interpretation of LSW experiments



$$\hat{V} = \hbar\left[f(t)\hat{b}_a^\dagger + f^*(t)\hat{b}_a\right]$$

$$|\psi(t)\rangle \sim \hat{D}\big(\beta_a(t)\big)|0\rangle = |\beta_a\rangle$$

Classical MW from an amplifier drives DM in the coherent state

$$\hat{V} = \omega_m \hat{b}_a^\dagger \hat{a}_\gamma + \omega_m^* \hat{a}_\gamma^\dagger \hat{b}_a$$

$$|\beta_a'\rangle \otimes |\alpha_\gamma\rangle = (1 + U_{a\gamma}\beta_a\alpha_\gamma^\dagger)|\beta_a\rangle \otimes |0\rangle$$

DM in the coherent state is converted to microwave photons in a coherent state

→ We need a detector which can measure microwave coherent state via waves or photons

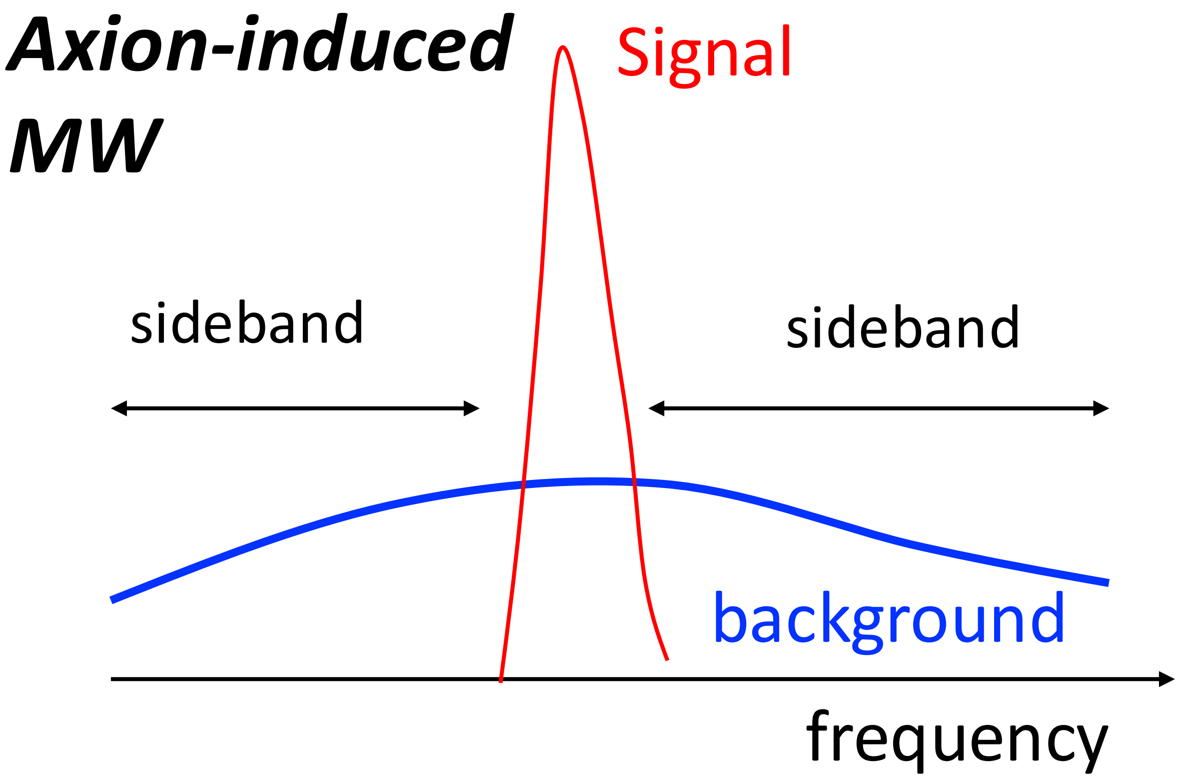
Part 3: Quantum detection scheme

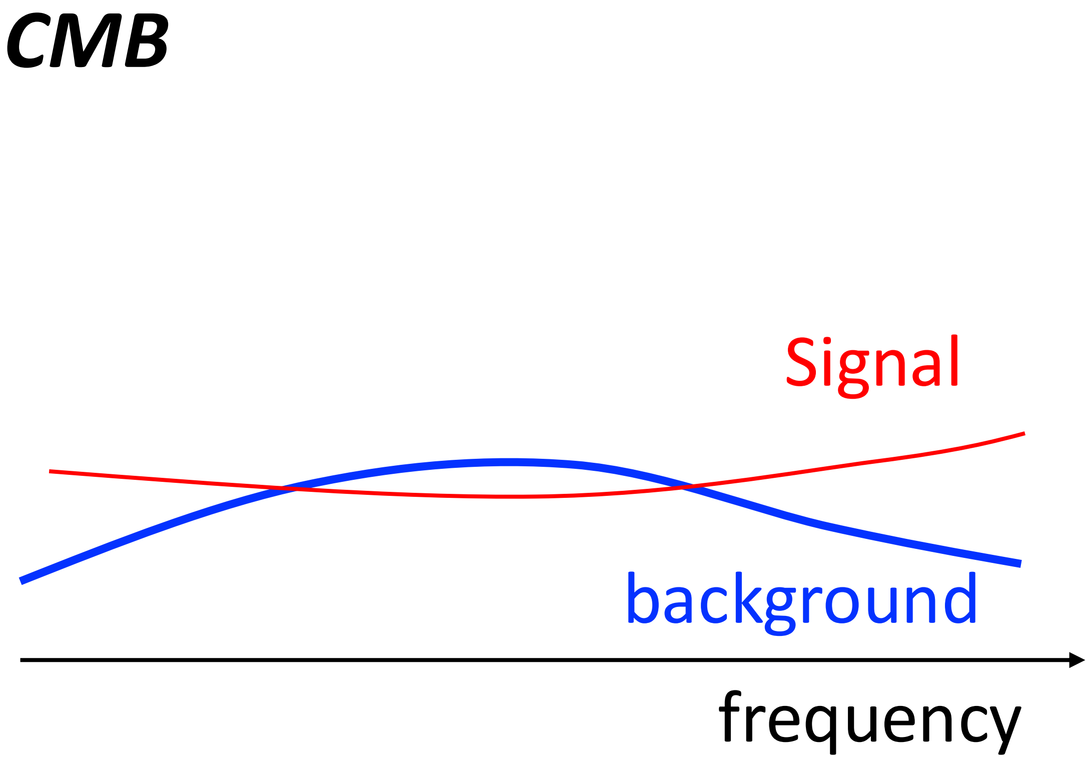
- Quantum coherent states
- Glauber's theorem
- Thermal noise and Standard Quantum Limit
- Squeezing and photon counting
- Conclusion of part 3

- Global conclusion of the lecture courses

~~Narrow-band~~ coherent vs ~~broad-band~~ incoherent background

With finite coherence time





- The axion-induced MW is described as a quantum coherent state
- The broad-band background was model as a classical wave of random phases
- What is the quantum optical description of **incoherent** background?

Density matrix $\hat{\rho} = \sum_n \lambda_n |\phi_n\rangle\langle\phi_n|$

Density matrix is a mean to express mixed states (quantum + classical statistics)

Pure state $\hat{\rho}_{pure} = |\phi\rangle\langle\phi|$

mixed state $\hat{\rho}_{mix} = \sum_n \lambda_n |\phi_n\rangle\langle\phi_n|$

Incoherent sum

<u>Expectation value of $\hat{O}$</u>

$$\langle\hat{O}\rangle = Tr\{\hat{O}\hat{\rho}\}$$

<u>Von Neumann Entropy</u>

$$S = -\sum_n \lambda_n \ln \lambda_n = -Tr\{\hat{\rho}\ln\hat{\rho}\}$$

<u>Von Neumann equation</u>

$$i\hbar \frac{d}{dt}\hat{\rho} = [\hat{H}, \hat{\rho}]$$

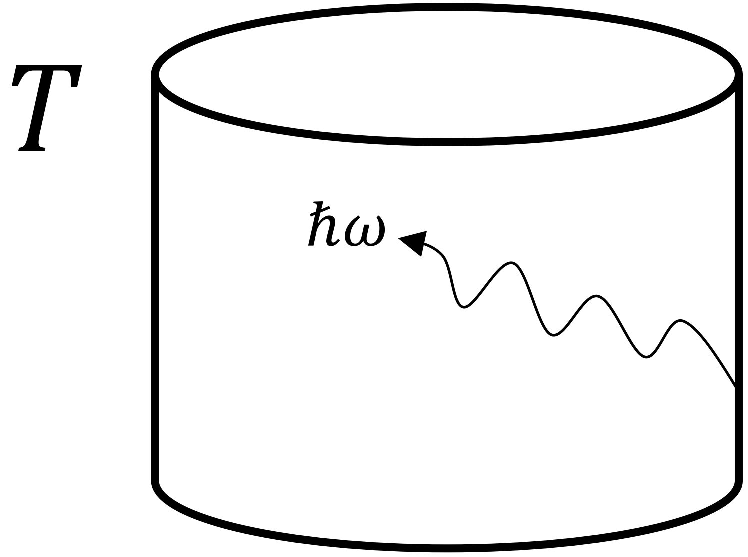
- Open quantum system to describe thermalization / decoherence process
- Typical particle physics experiment is adiabatic
- Axion-induced microwaves: coherent signal vs incoherent background

Blackbody radiation

$$\bar{n} = \frac{1}{\exp(\hbar\omega/k_B T) - 1}$$

$$\hat{\rho}_{th} = \frac{1}{1+\bar{n}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{1+\bar{n}}\right)^n |n\rangle\langle n|$$

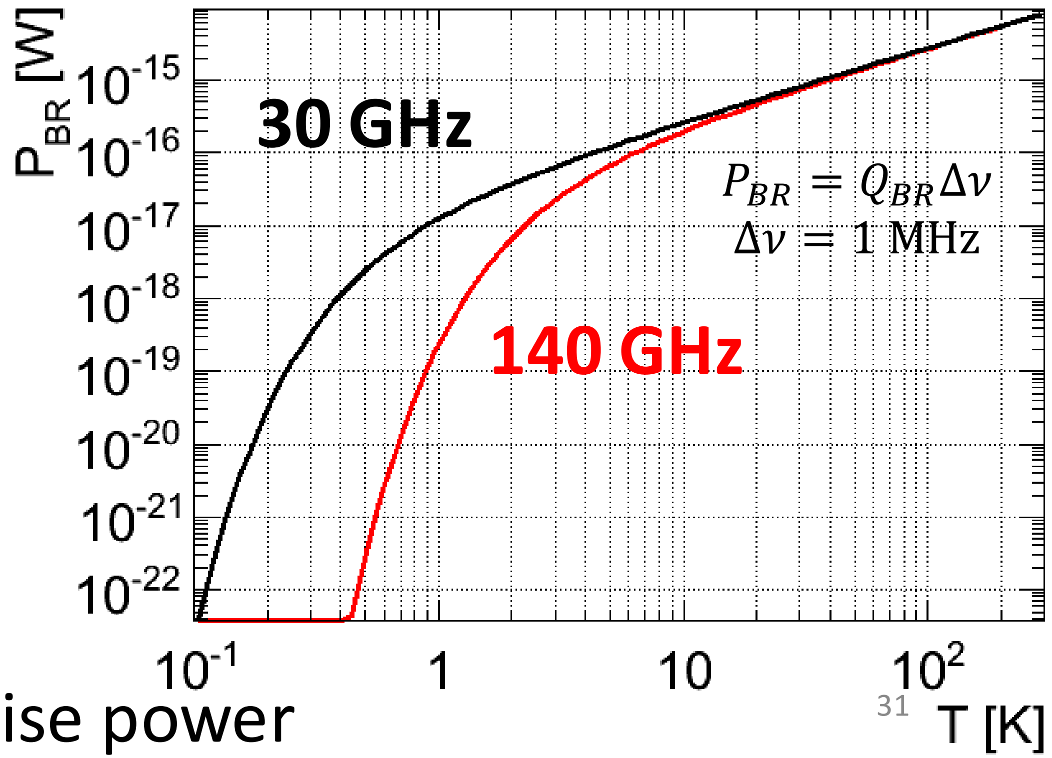
Mixed state



Noise power spectral density

$$Q_{BR} = \frac{1}{2}\Omega A \frac{2h\nu^3}{\exp\left(\frac{h\nu}{k_B T}\right) - 1} \text{ [W/Hz]}$$

Solid angle Ω [st]
Sensor area A $[m^2]$
→ Usually, $\Omega A \sim \lambda^2$



Cooling down the system can decrease the noise power

Exercise 4 (homework?):

$$\hat{\rho}_{co} = |\alpha\rangle\langle\alpha|$$

$$\hat{\rho}_{th} = \frac{1}{1+\bar{n}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{1+\bar{n}}\right)^{n} |n\rangle\langle n| = \int d^2\alpha \frac{\exp(-|\alpha|^2/\bar{n})}{\pi\bar{n}} |\alpha\rangle\langle\alpha|$$

$$\hat{n} = \hat{a}^\dagger \hat{a}$$

$$\hat{\mathcal{E}}(\boldsymbol{r},t) = i\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}} \left[\hat{a} e^{-i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})} - \hat{a}^\dagger e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}\right]$$

$\langle n\rangle \equiv Tr\{\hat{n}\hat{\rho}_{co}\} =?$

$\langle n\rangle \equiv Tr\{\hat{n}\hat{\rho}_{th}\} =?$

$\langle E\rangle \equiv Tr\{\hat{\mathcal{E}}(\boldsymbol{r},t)\hat{\rho}_{co}\} =?$

$\langle E\rangle \equiv Tr\{\hat{\mathcal{E}}(\boldsymbol{r},t)\hat{\rho}_{th}\} =?$

$\langle E^2\rangle \equiv Tr\{\hat{\mathcal{E}}^2(\boldsymbol{r},t)\hat{\rho}_{co}\} =?$

$\langle E^2\rangle \equiv Tr\{\hat{\mathcal{E}}^2(\boldsymbol{r},t)\hat{\rho}_{th}\} =?$

Blackbody radiation → classical chaos field

arXiv:2408.04696

$$\hat{\phi}(t,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\sqrt{2\omega_\mathbf{k}}}\left(\hat{a}_\mathbf{k}e^{-ik\cdot x} + \hat{a}^\dagger_\mathbf{k}e^{ik\cdot x}\right) \qquad [\hat{a}_\mathbf{k},\, \hat{a}^\dagger_\mathbf{q}] = (2\pi)^3\delta(\mathbf{k}-\mathbf{q}) \qquad \hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

Coherent state

$$\hat{\rho}_{th} \equiv \frac{\exp\left(-\hat{H}_0/k_B T\right)}{Z} = \frac{1}{1+\bar{n}_{th}}\sum_{n=0}^{\infty}\left(\frac{\bar{n}_{th}}{1+\bar{n}_{th}}\right)^n |n\rangle\langle n| = \int d^2\alpha \frac{\exp\left(-|\alpha|^2/\bar{n}_{th}\right)}{\pi\bar{n}_{th}}|\alpha\rangle\langle\alpha|$$

→ Incoherent mixed state with max entropy (thermal equilibrium)→ Stefan Boltzmann, Planck law

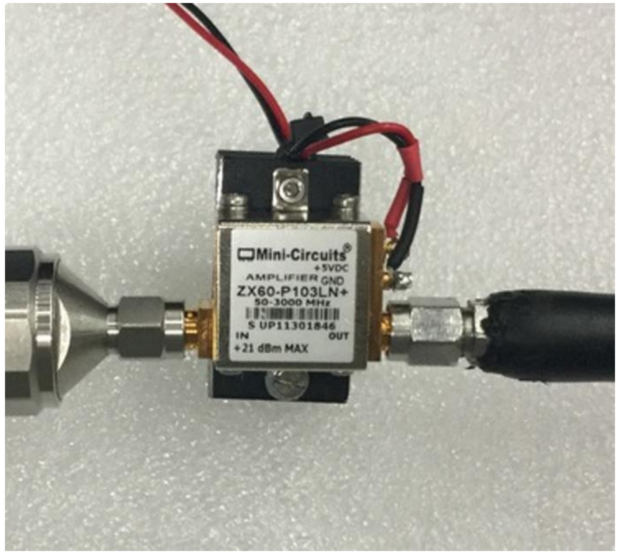
$$\boxed{N \gg 1} \qquad \langle \hat{A}(\hat{a},\hat{a}^\dagger)\rangle = \int d\alpha\, P(\alpha)A(\alpha) + \mathcal{O}(1/N)$$

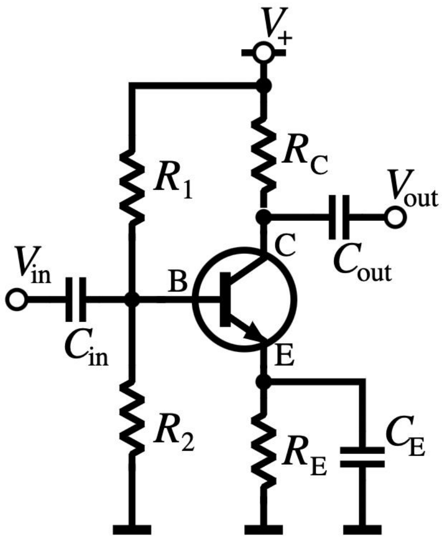
$$\hat{\phi}(t,\mathbf{x}) \simeq \phi(t,\mathbf{x}) = \sum_\mathbf{k}\sqrt{\frac{2}{V\omega_\mathbf{k}}}\,\mathrm{Re}\left[\alpha_\mathbf{k}e^{-ik\cdot x}\right] \longrightarrow \sum_\mathbf{k}\sqrt{\frac{2N_\mathbf{k}}{V\omega_\mathbf{k}}}\cos(\omega_\mathbf{k}t - \mathbf{k}\cdot\mathbf{x} + \boxed{\varphi_\mathbf{k}})$$

Quantum field — classical field

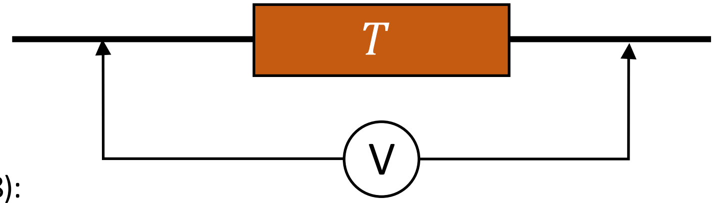
A bosonic field of a high occupation number becomes classical chaos field

Johnson Nyquist Thermal Noise

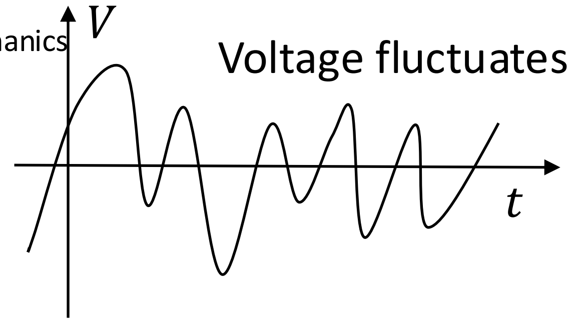




Any conductor at temperature T



J. B. Johnson Phys Rev 32 97 (1928): Experimental discovery of the relation
H. Nyquist Phys Rev 32 110 (1928): Thermodynamics + statistical mechanics of **bosonic** modes



Voltage fluctuates

$$P_N = \frac{\langle V^2 \rangle}{4R}$$

"Blackbody radiation" of electromagnetic waves inside a 1D conductor

$$\langle V^2 \rangle \Delta\nu \sim 4R\Delta\nu \boxed{\frac{h\nu}{e^{h\nu/k_B T} - 1}} \xrightarrow[\text{Rayleigh Jeans}]{h\nu \ll k_B T} 4R \boxed{k_B T} \Delta\nu$$

Average number of thermal photons

$$\bar{n} = \frac{1}{e^{h\nu/k_B T} - 1}$$

was derived by Planck via quantum statistics

Noise PSD: $N = \frac{\langle V^2 \rangle}{4R\Delta\nu} \sim k_B T$ [W/Hz]

Corresponding classical waves are incoherent chaos waves

A thermal state + classical axion → coherent thermal state

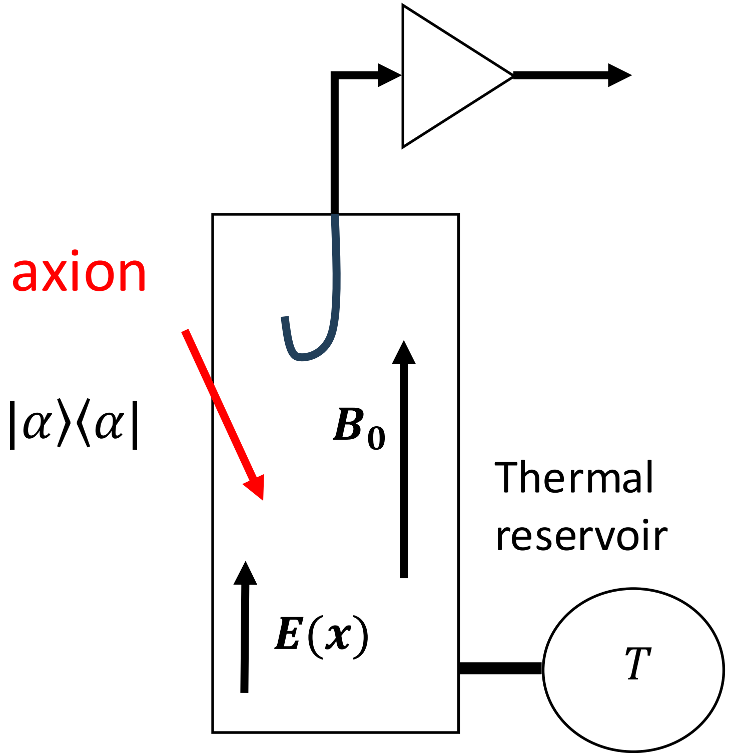
Unitary operator of time evolution

$$U(t) = \exp(-iC_1(t))\hat{D}(\beta(t))$$

$$\bar{n}_{th} = \frac{1}{\exp(\hbar\omega/k_B T) - 1}$$

$$\hat{\rho}_{\rm th} \equiv \frac{\exp(-\hat{H}_0/k_B T)}{Z} = \frac{1}{1+\bar{n}_{th}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}_{th}}{1+\bar{n}_{th}}\right)^n |n\rangle\langle n| = \int d^2\alpha \frac{\exp(-|\alpha|^2/\bar{n})}{\pi\bar{n}} |\alpha\rangle\langle\alpha|$$

$$Z = \mathrm{Tr}\left[\exp(-\hat{H}_0/k_B T)\right]$$



Coherent thermal state

Realistic quantum model of axion + noise

$$\hat{\rho}(t) = U(t)\hat{\rho}_0 U^\dagger(t) = \frac{1}{Z}\hat{D}(\beta(t)) \exp(-\hat{H}_0/k_B T)\, \hat{D}^\dagger(\beta(t))$$

Statistics of photons

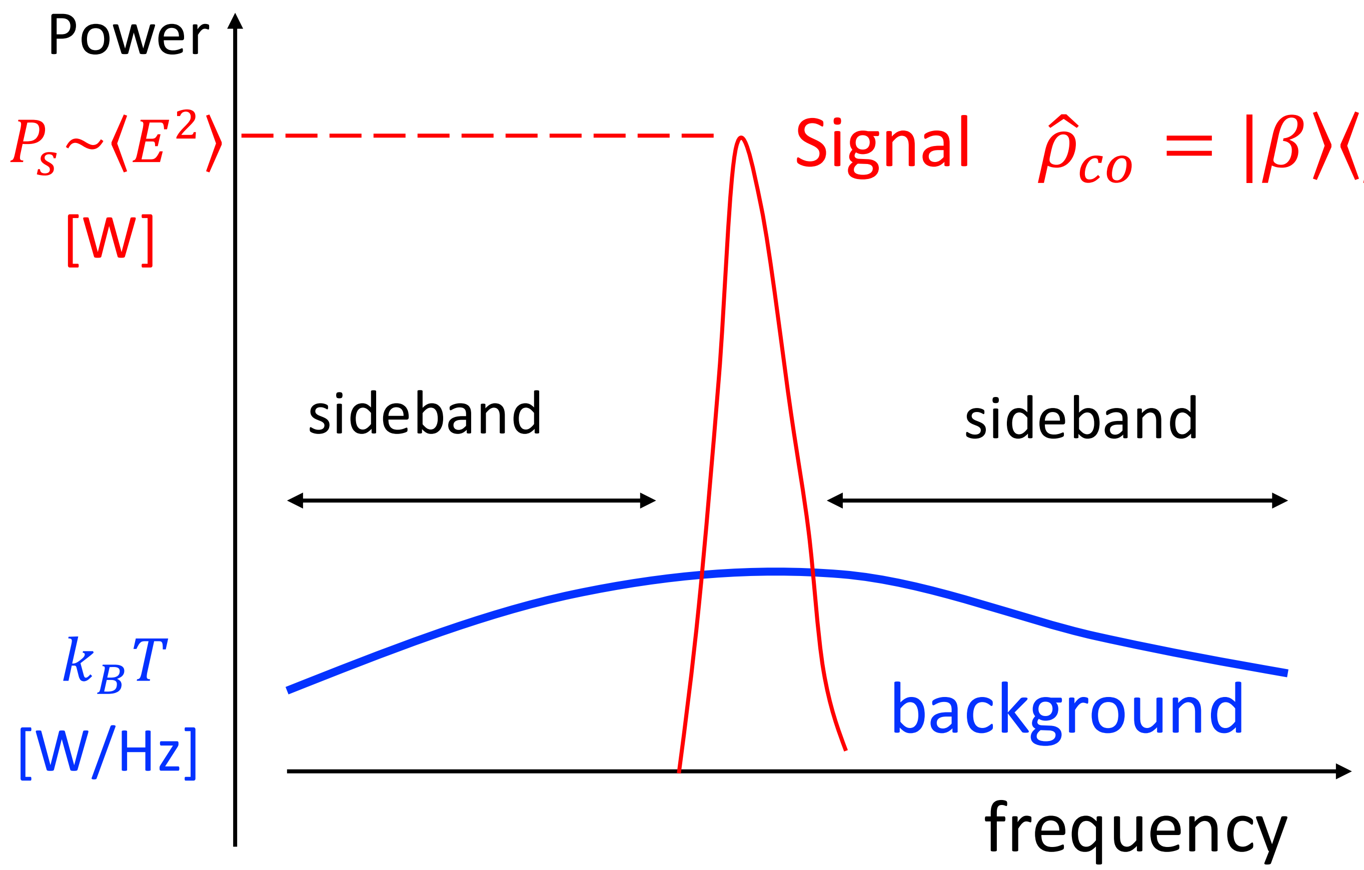
$$\langle A\rangle_{th} = \frac{1}{Z} Tr\{e^{-\hbar\omega b^\dagger b/k_B T} A\}$$

$$\bar{n} = |\beta(t)|^2 + \bar{n}_{th}$$ **= Poisson + Planck**

$$\sigma^2 = \sigma_{th}^2 + |\beta|^2 \left(1 + 2\langle b^\dagger b\rangle_{th}\right)$$ **≠ Poisson + Planck**

→ **Naïve additive Monte Carlo (Poissonian signal + thermal noise) may be wrong!**

Quantum mechanical interpretation of axion signal / background



Remark in standard deviation

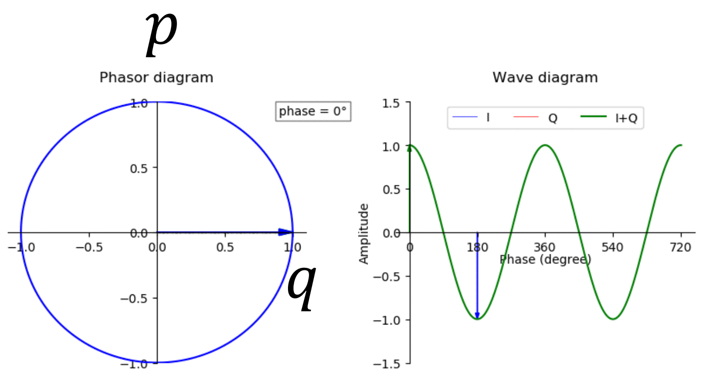
$$\bar{n} = \frac{1}{\exp(\hbar\omega/k_B T) - 1}$$

$$\hat{\rho}_{th} = \int d^2\alpha \frac{\exp(-|\alpha|^2/\bar{n})}{\pi\bar{n}} |\alpha\rangle\langle\alpha|$$

- What is the minimum possible power to measure?
- **Can we improve S/N forever by cooling down T?**

Standard Quantum Limit from the Kennard inequality

$$[p,q] = \frac{i\hbar}{2} \rightarrow \langle \Delta p^2 \rangle \langle \Delta q^2 \rangle \geq |\langle [p,q] \rangle|^2 = \frac{\hbar}{4} \quad \text{(Kennard)}$$



$$[p_i, q_i] = \frac{i\hbar}{2} = [p_f, q_f]$$: before and after the amplifier chain

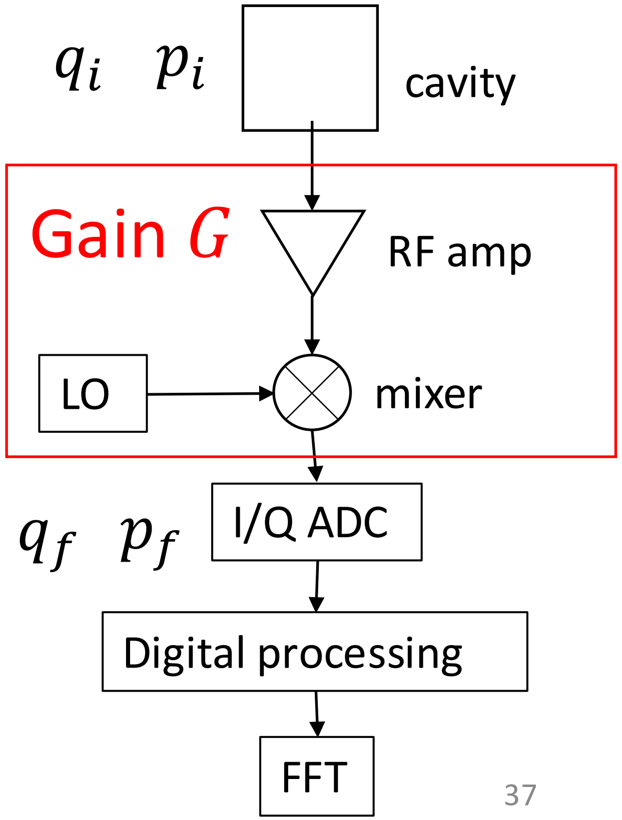
$$q_f = G q_i + q_g \qquad p_f = G p_i + p_g$$

$$[p_f, q_f] = [G p_i, G q_i] + [p_g, q_g] = \frac{i G^2 \hbar}{2} + [p_g, q_g] = \frac{i\hbar}{2}$$

$$\rightarrow [p_g, q_g] = \frac{i(1-G^2)\hbar}{2} \rightarrow \langle \Delta p_g^2 \rangle \langle \Delta q_g^2 \rangle \geq \frac{(G^2 - 1)\hbar^2}{4}$$

Amplifier uncertainty principle

$$P \geq \frac{1}{G^2}\left[\frac{G^2 h\nu}{2} + \frac{(G^2-1)h\nu}{2}\right] \xrightarrow{G \gg 1} 2 \times \frac{h\nu}{2} = h\nu$$



$P_{SQL} = h\nu$: standard quantum limit

Ex) $h \times 1\ \text{GHz} = 6.6 \times 10^{-25}$ W/Hz

S.K. Lamoreaux et al Phys Rev D 98 035020 (2013)
C. M. Caves PRD 26 8 1817 1982

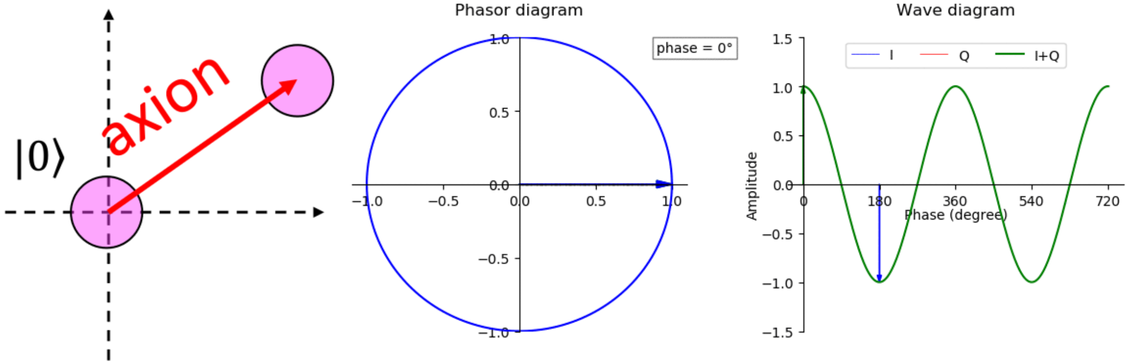
Part 3: Quantum detection scheme

- Quantum coherent states
- Glauber’s theorem
- Thermal noise and Standard Quantum Limit
- Squeezing and photon counting
- Conclusion of part 3

- Global conclusion of the lecture courses

Arguments around quantum detection

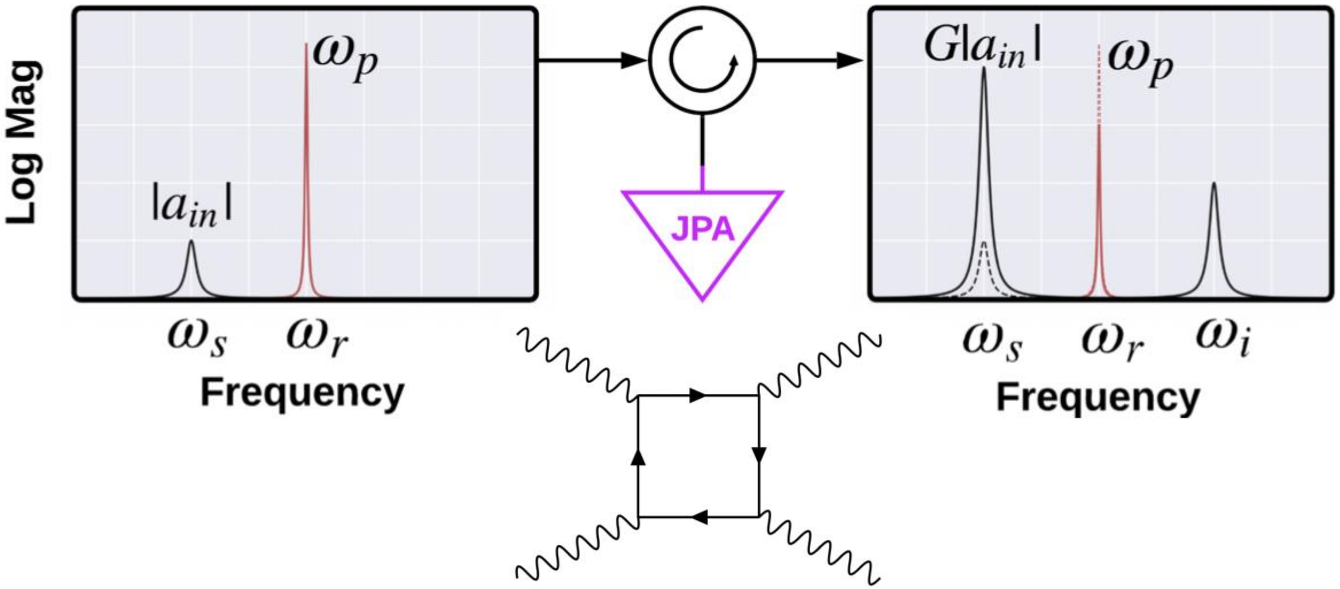
- How to reach SQL (quantum limited amplifier)?
  - Standard HEMT LNA is lossy → noise temperature of a few Kelvin (7 K for 20 GHz)
  - Super-low-loss Josephson Parametric Amplifier (JPA) in phase insensitive mode
- Is SQL truly fundamental?
  - $[p, q] = i\hbar/2$
  - $|0\rangle$ is not an eigenstate of either p or q

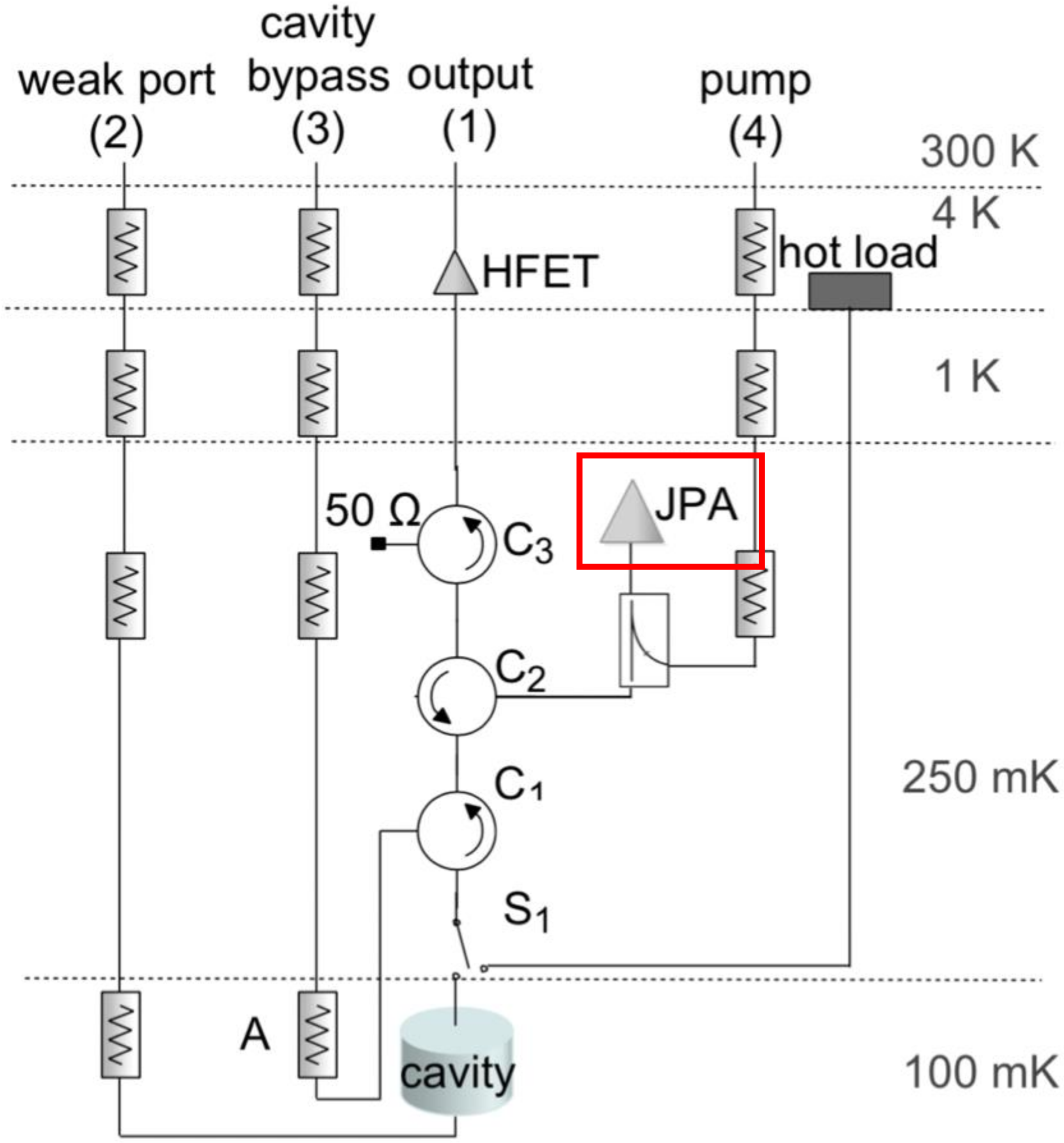


- How to overcome SQL?
  - Manipulate $|0\rangle$ with squeezing: JPA in phase-sensitive operation (HAYSTAC)
  - Directly read out $\hat{n}$ ($|0\rangle$ is its eigen state): photon counting / bolometer
- How about intrinsic quantum nature of axion dark matter?

Parametric Amplifier (in ADMX)

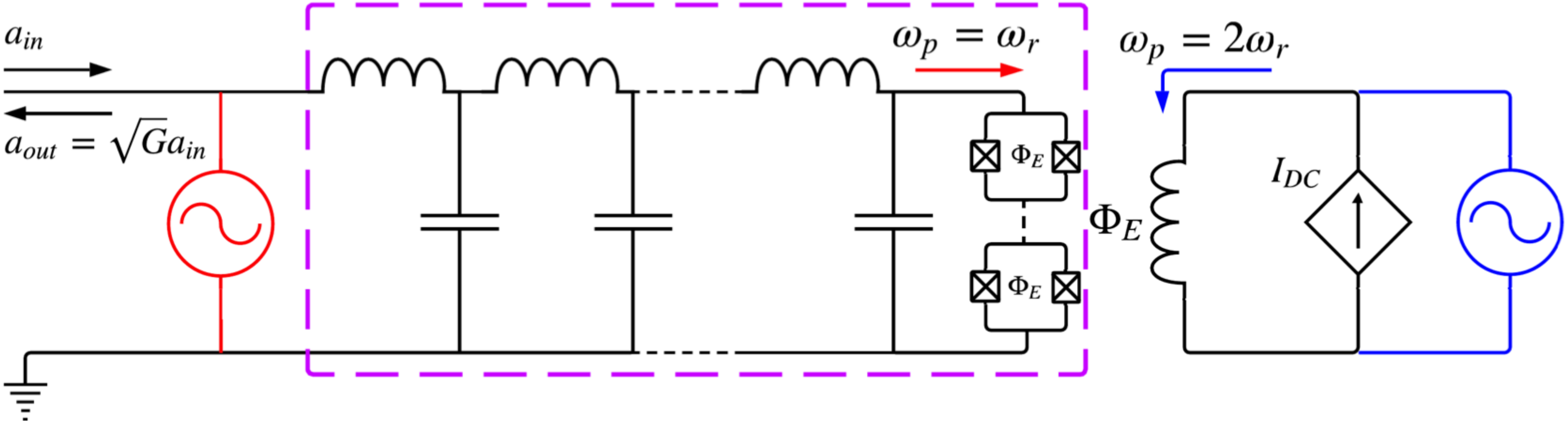
arXiv:2010.00169



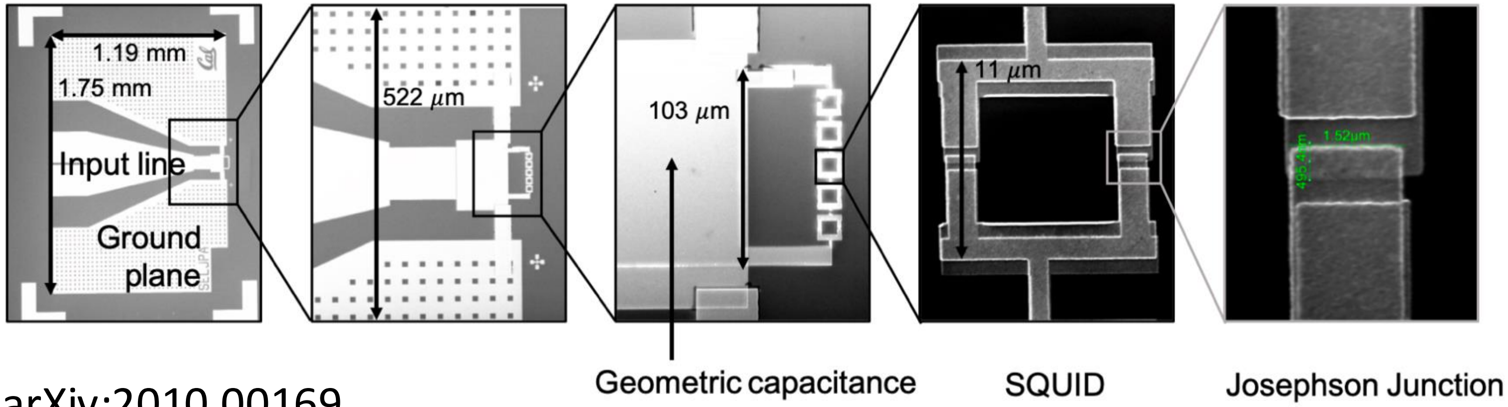


- Amplification of microwave signal at ω_s via pump microwaves at ω_p
- Nonlinear optics (Kerr effect) for frequency mixing
- No real electron/hole current
  - → Free from the noise source of transistors
  - → One can reach $k_B T_{SQL} = h\nu$ by cooling down

Implementation: Josephson Parametric Amplifier



$$L_J = \frac{L_{J0}}{\sqrt{1-(I/I_0)^2}} = L_{J0}\left(1 + \frac{1}{2}(I/I_0)^2 + \dots\right)$$



arXiv:2010.00169

- The nonlinearity is induced from Josephson junctions inside SQUID
- Although SQUID is a superconducting quantum device, **microwave's behavior is classical** (→ semi-classical)

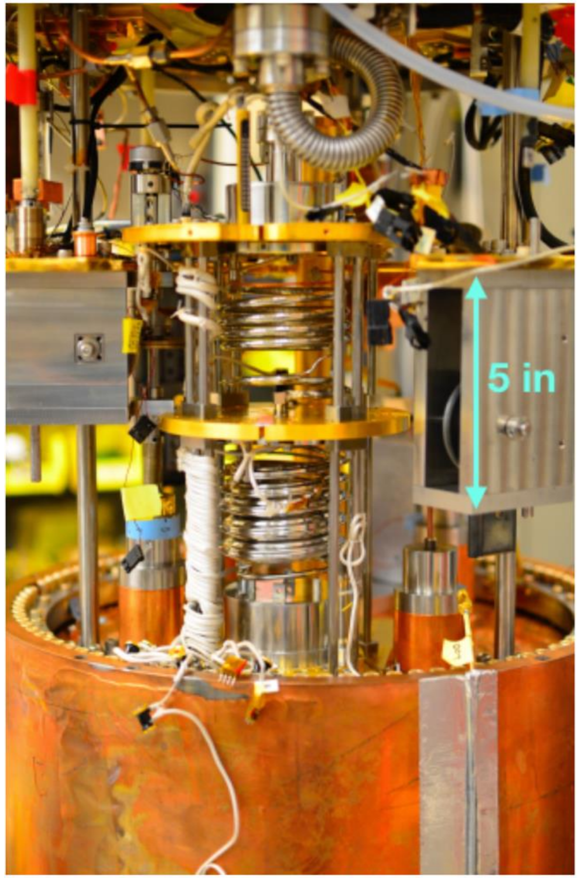
arXiv:2010.00169

ADMX: $T_{SQL} = h\nu/k_B \sim 40\ \text{mK}$

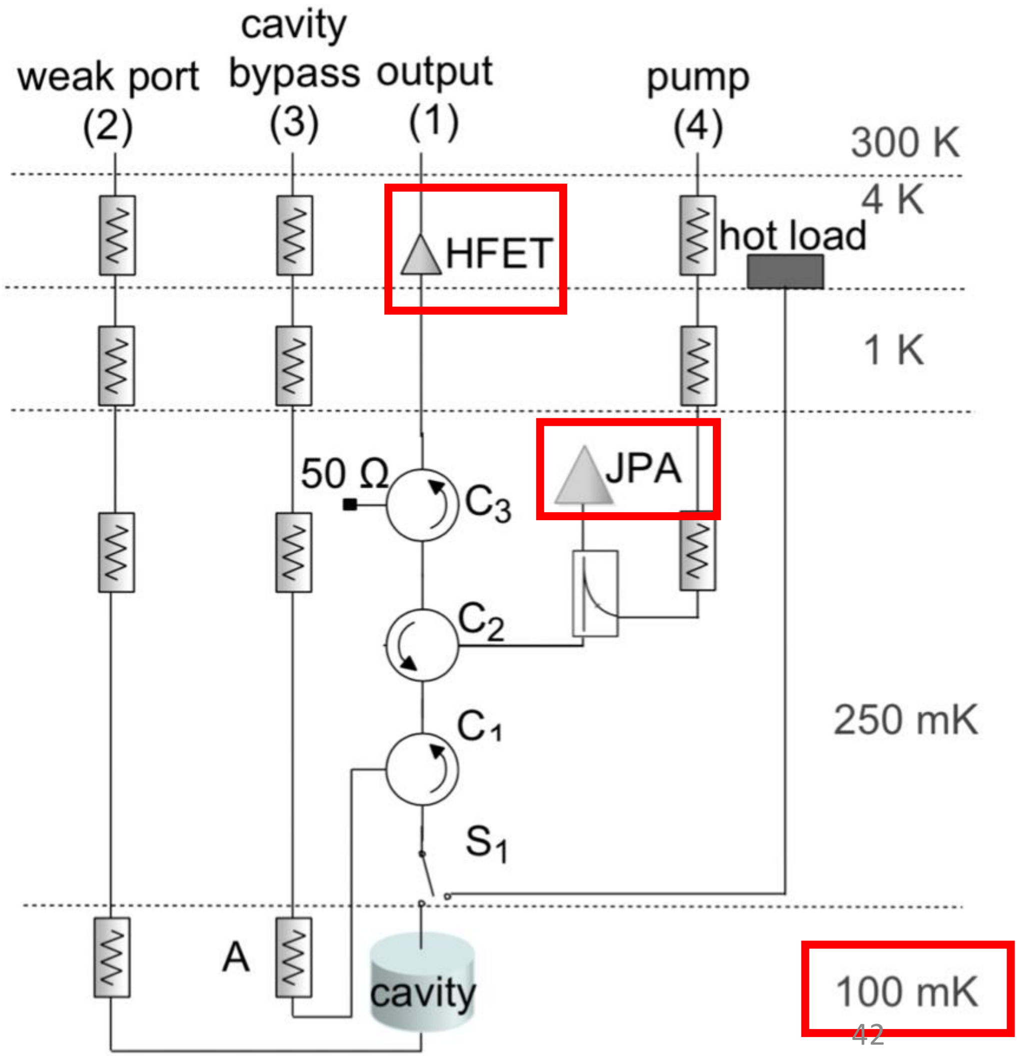
With a dilution refrigerator, the noise level is approaching to the quantum limit

→ Quantum devices have been developed and implemented

→ Toward quantum microwave optics of coherent signals



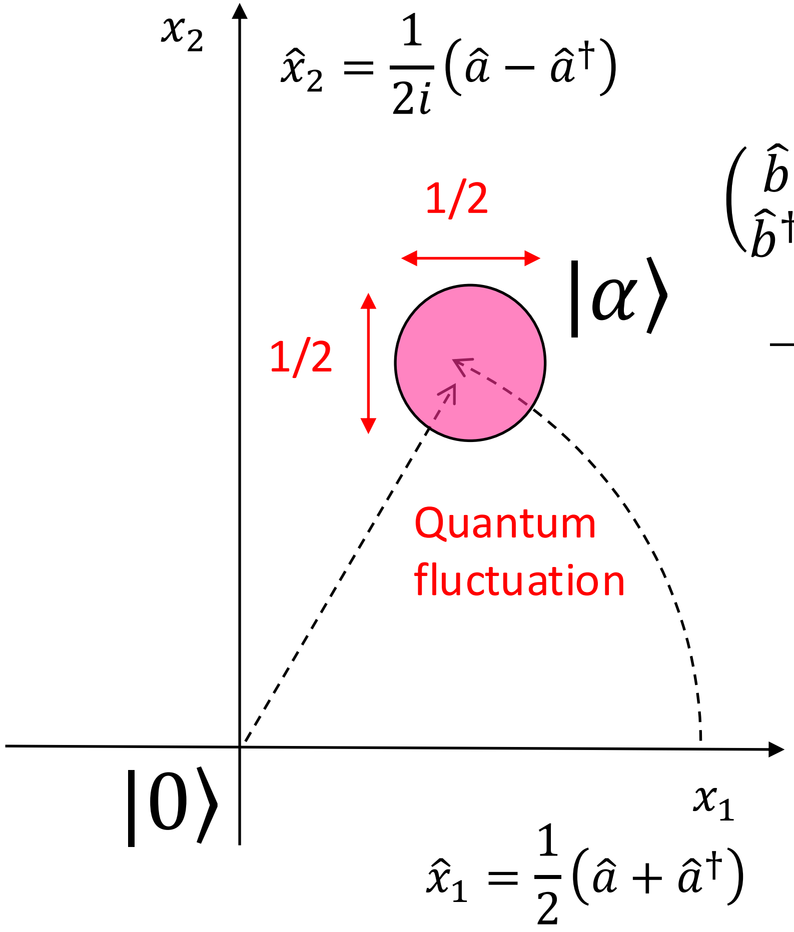
Run 1B



Another use of parametric amplifier

$$\hat{H} = \left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right)\hbar\omega + \hbar\left(\frac{E^*}{2}\hat{a}^2 + \frac{E}{2}\hat{a}^{\dagger 2}\right)$$

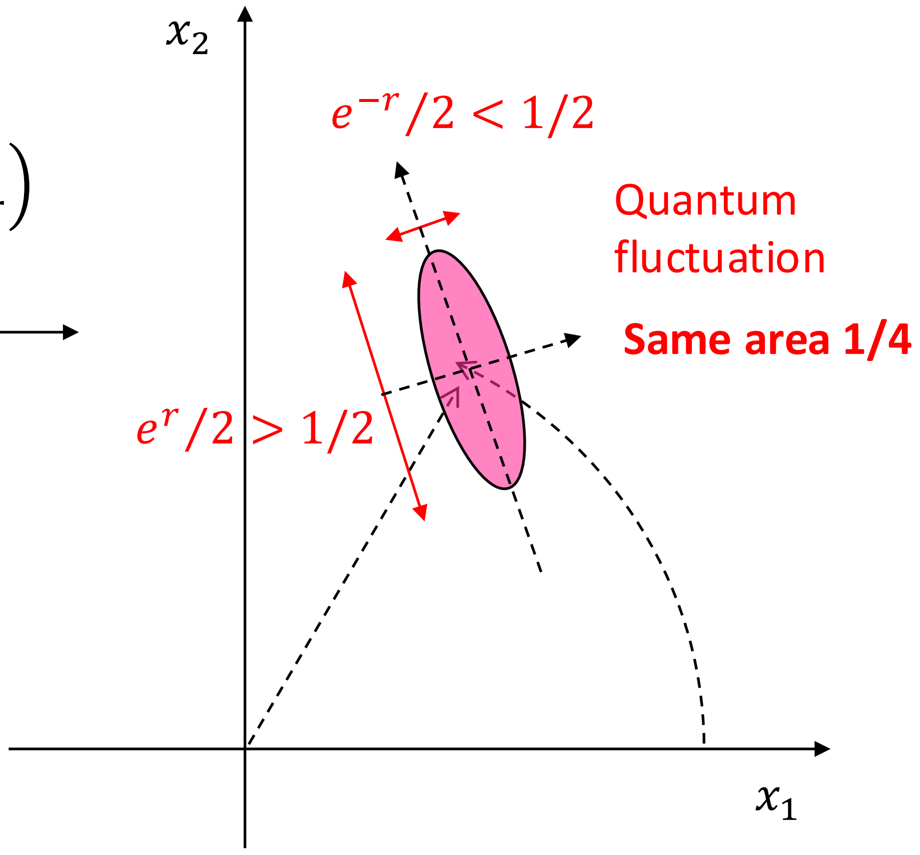
Nonlinear term added by parametric oscillation



$$\begin{pmatrix} \hat{b} \\ \hat{b}^\dagger \end{pmatrix} = \begin{pmatrix} \cosh r & e^{2i\phi}\sinh r \\ e^{-2i\phi}\sinh r & \cosh r \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{a}^\dagger \end{pmatrix}$$

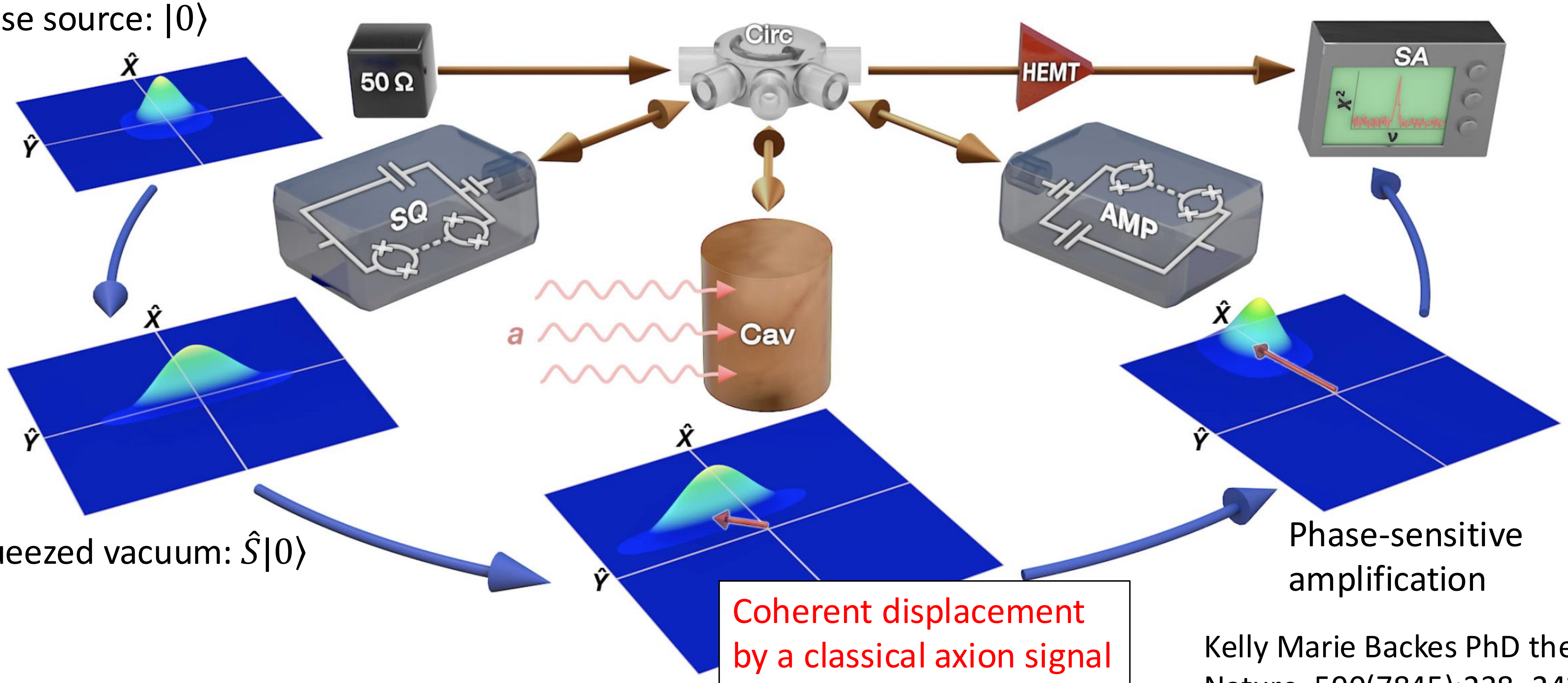
$$\hat{H} = \hbar\lambda\hat{b}^\dagger\hat{b} + \frac{\lambda - \omega}{2}$$

$$\begin{cases} \lambda = \sqrt{\omega^2 - |E|^2} \\ r = \frac{1}{2}\tanh^{-1}(|E|/\omega) \\ e^{2i\phi} = E/|E| \end{cases}$$



Squeezed vacuum state + axion signal

Noise source: $|0\rangle$



Squeezed vacuum: $\hat{S}|0\rangle$

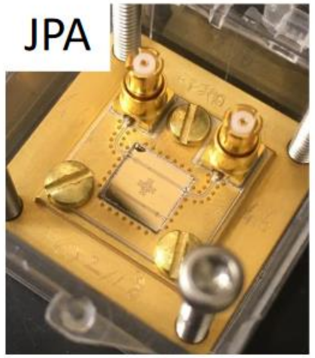
Coherent displacement by a classical axion signal

Phase-sensitive amplification

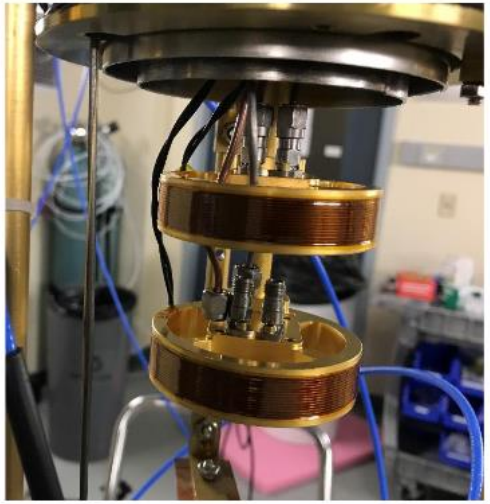
Kelly Marie Backes PhD thesis
Nature, 590(7845):238–242

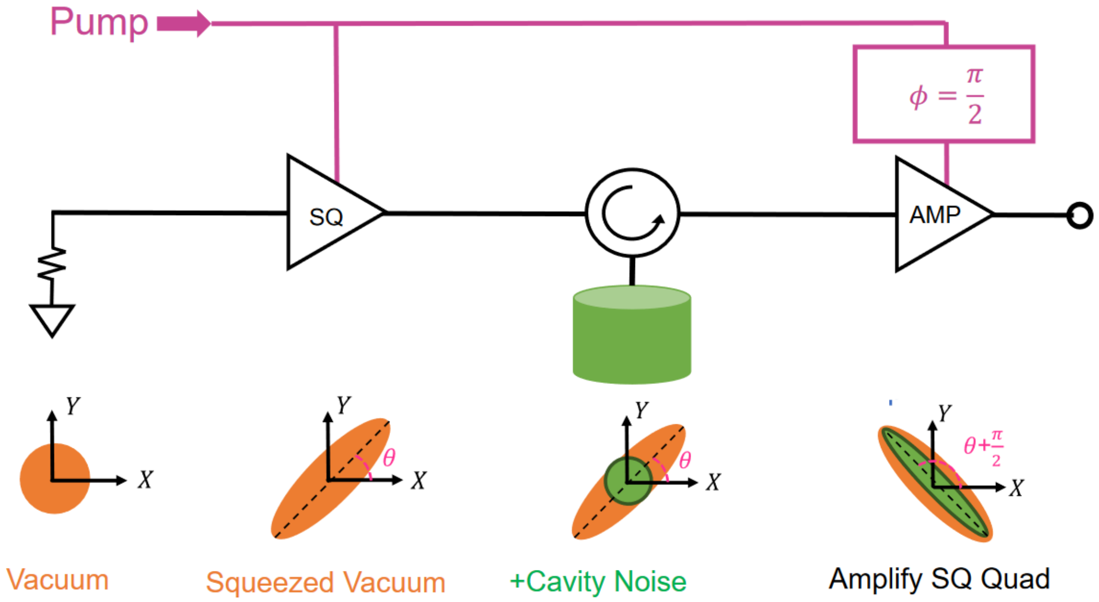
→ SQL was overcome and S/N was improved by factor two

HAYSTAC (Yale University + Berkley)



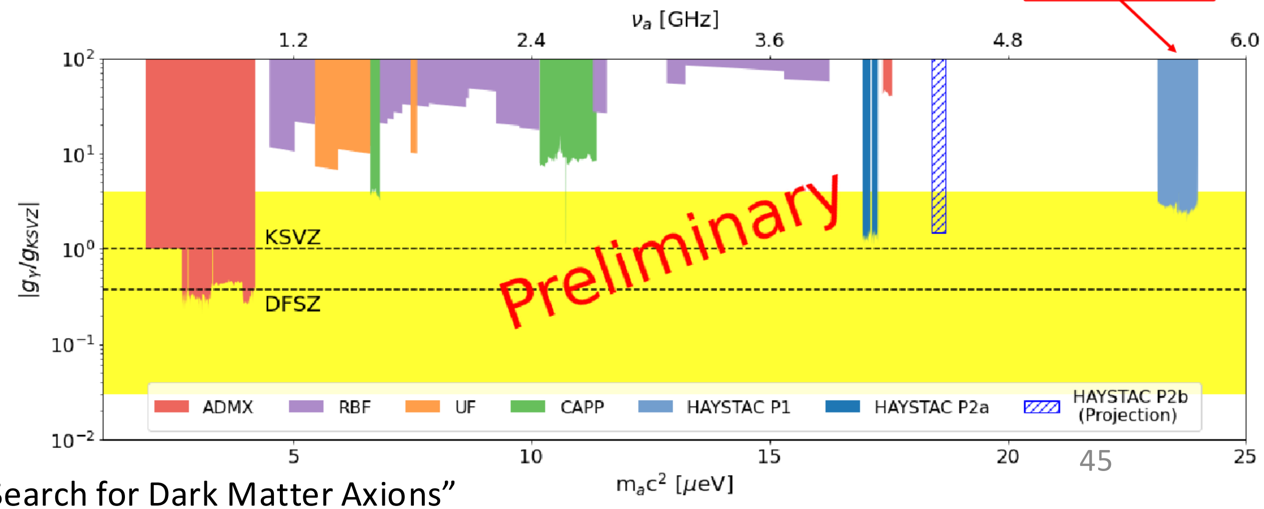
Squeezed State Receiver





- JPAs achieve near SQL when Phase-Insensitive
- JPAs are Phase-Sensitive Amplifiers
- Each Phase alone is not limited by SQL
- Can produce “Squeezed” States
  - Dump all uncertainty/noise into a single quadrature

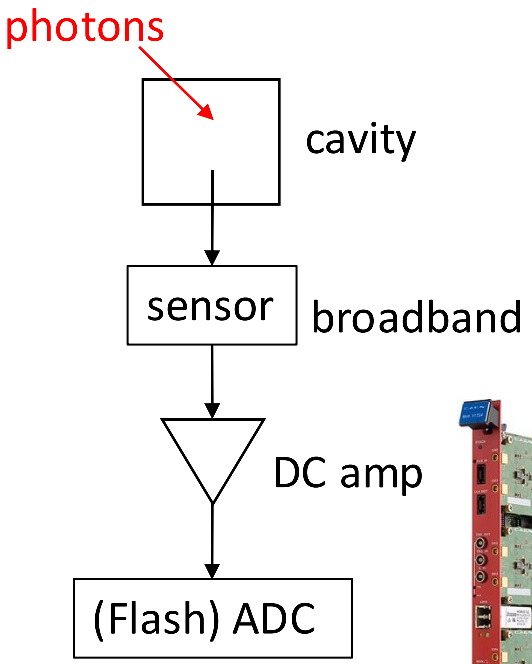
- Results are still being analyzed

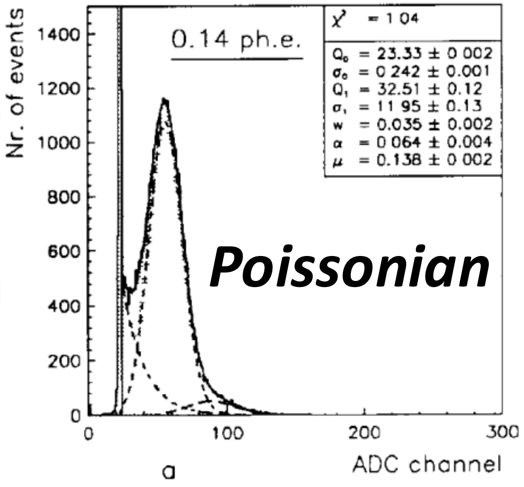


Courtesy: Michael Jewell, “Updated Results from HAYSTAC's Quantum-Enhanced Search for Dark Matter Axions”

Photon (energy) detection is free from SQL

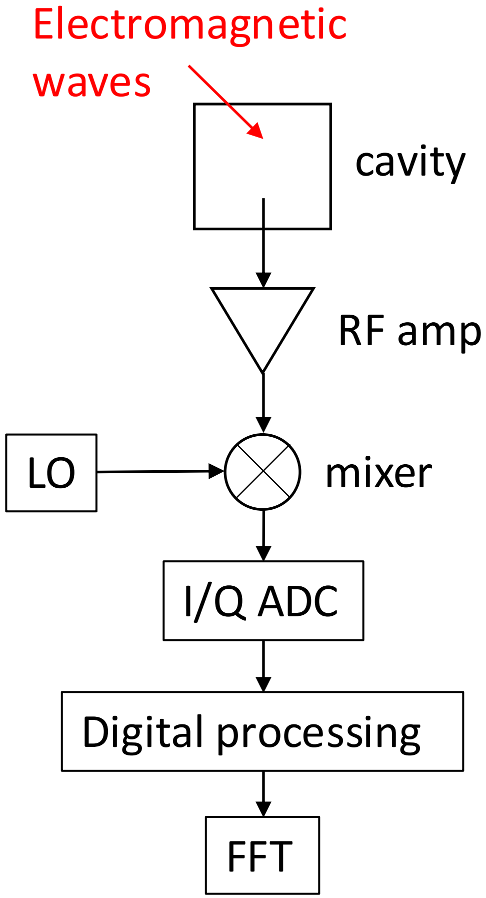
Welcome back to particle physics ☺



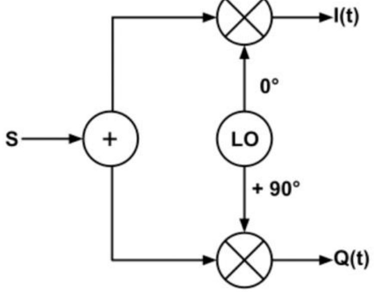


$$P(t) = n \times \hbar\omega \propto V_{ADC}(t)$$

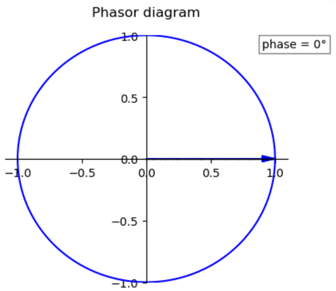
$$\Delta\phi\Delta n > 1$$

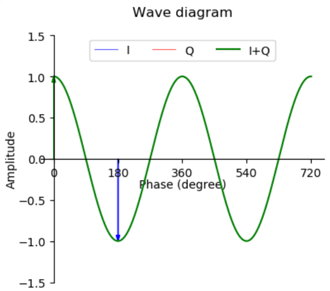


I/Q Demodulator



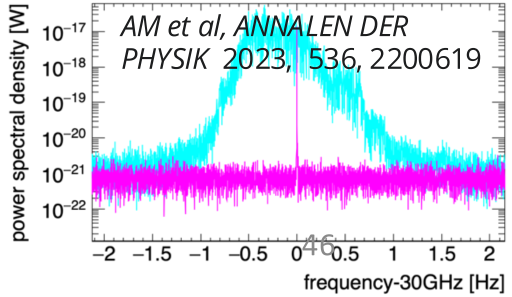
From wikipedia





$$RF(t) = I(t)\cos(\omega t) + Q(t)\sin(\omega t)$$

$$\rightarrow P(\omega) = \tilde{I}^2(\omega) + \tilde{Q}^2(\omega)$$



Single photon sensors can also overcome SQL

<u>Noise power of wave detection</u>

$$P_l = h\nu(\bar{n} + 1)\sqrt{\frac{\Delta\nu}{t}} \propto \nu$$

<u>Noise power of photon detection</u>

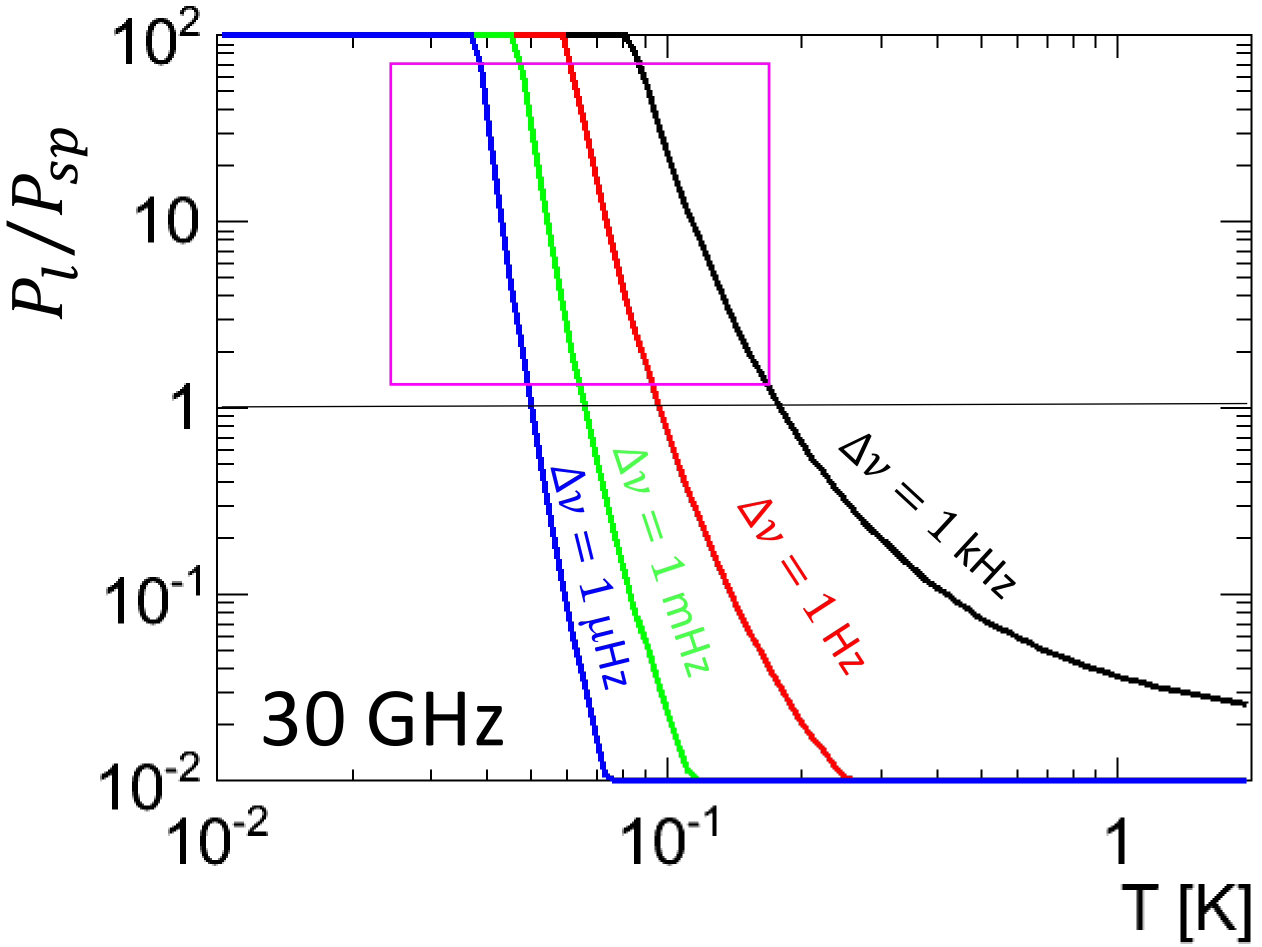
$$P_{sp} = h\nu\sqrt{\frac{\eta\bar{n}Q_c}{2\pi\nu t}} \propto \sqrt{\nu}$$

$$\bar{n} = \frac{1}{e^{h\nu/k_B T} - 1}$$

Q_c: cavity quality factor

η: quantum efficiency of the sensor

S.K. Lamoreaux et al Phys Rev D 98 035020 (2013)



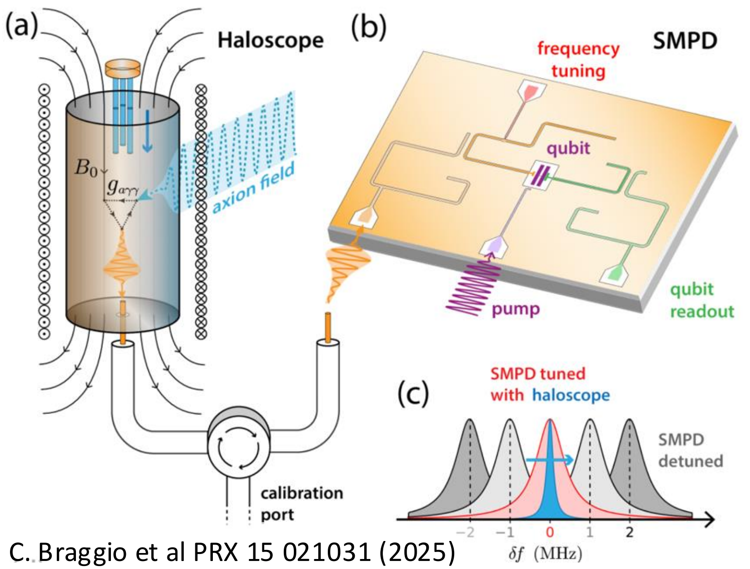
Single photon sensors may be a solution in the future

→ Although one loses phase information, zero background at cold may be better

→ Lower noise in **higher frequency** → where is the cross-over? 10 GHz? 100 GHz??

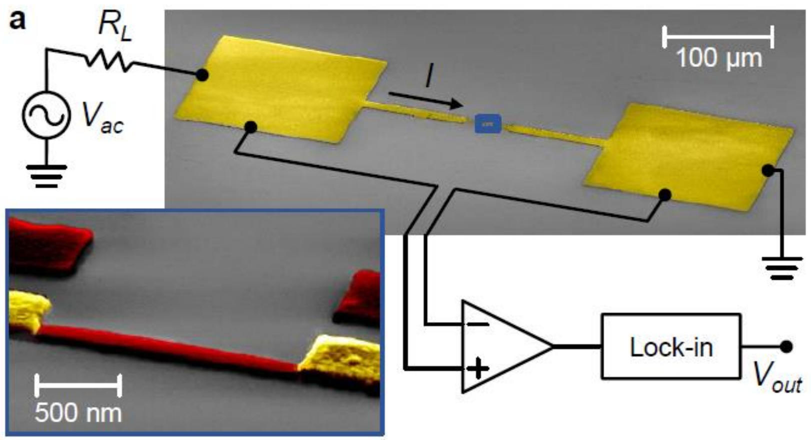
Examples of MW photon sensors

Transman qubit-based photon counting



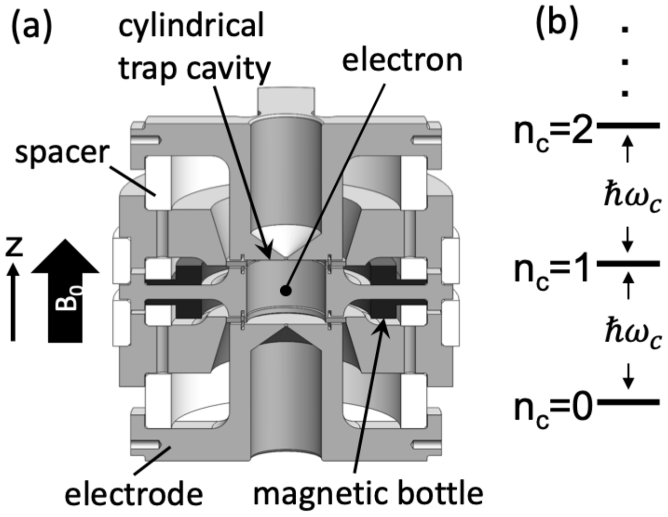
C. Braggio et al PRX 15 021031 (2025)

Biased superconducting nano-wire Transition Edge Sensor (TES)



F. Paolucci et al Phys Rev Appl. 14 034055 (2020)

Landau levels of a trapped electron under cyclotron motion



X. Fan et al, PRL129, 261801, 2022

- Photon counting device counts photons $|n\rangle$ without energy resolution
- Calorimeter resolves single-photon energy $NEP \sim 10^{-25}\ W/\sqrt{Hz}$
- Bolometer (intermediate step) measures power of the microwave's photon flow
  - TES: $NEP \sim 10^{-16}\ W/\sqrt{Hz}$ → CMB measurement
  - Graphene-based ***Josephson*** Junction: $NEP \sim 10^{-18}\ W/\sqrt{Hz}$ [Nature 586 42 2020]

Noise equivalent power

$$\text{NEP}\ [\text{W}/\sqrt{\text{Hz}}] \equiv \frac{P[W]}{SNR\sqrt{\Delta\nu}}$$

Parameter of RF→DC sensors

- Photon sensors
- Bolometer / calorimeter

$$v_{out} \propto v_0^2 = P_{in}$$

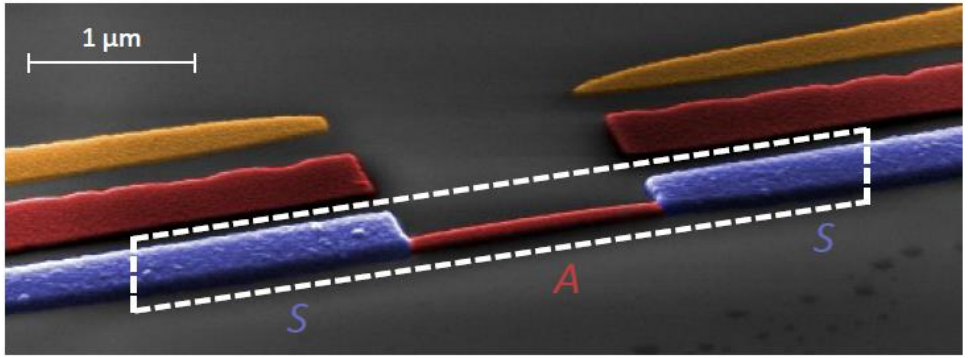
<u>Minimum "detectable" power</u>

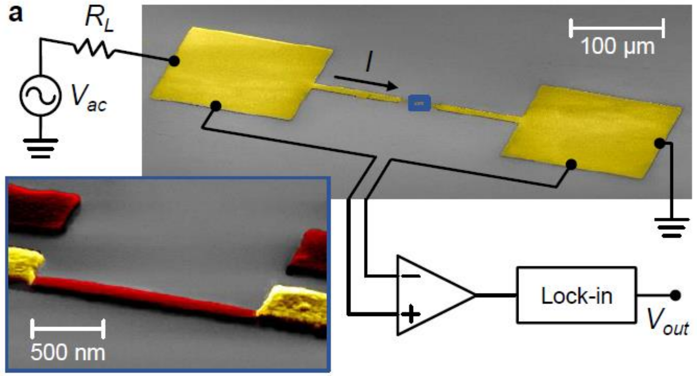
"Detectable": $S/N > 1$

$$P_{min} = \text{NEP} \times \sqrt{\Delta\nu}$$

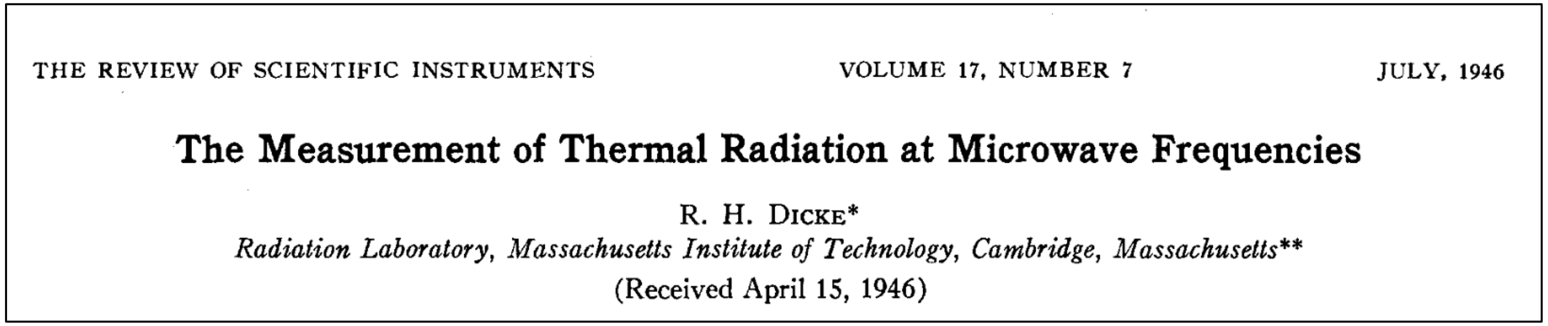
What is this BandWidth $\sqrt{\Delta\nu}$?

Where is integration time?





Dicke radiometer equation and noise power

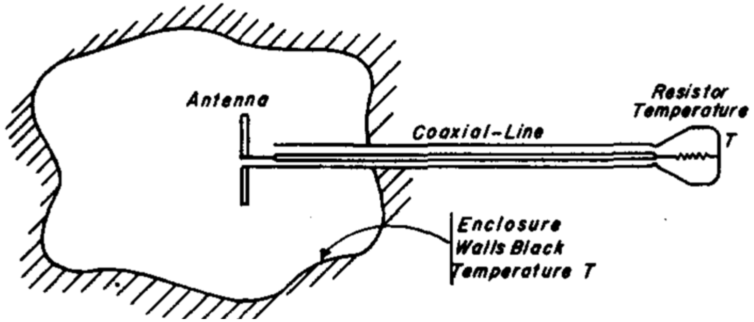


THE REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 17, NUMBER 7 JULY, 1946

The Measurement of Thermal Radiation at Microwave Frequencies

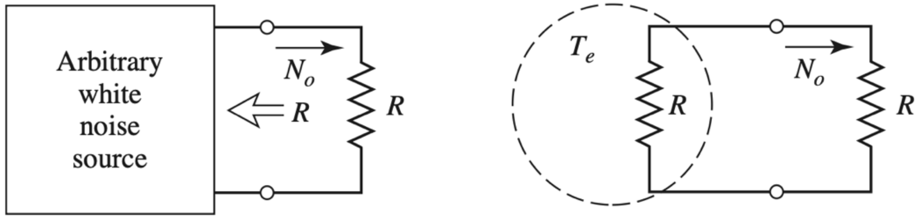
R. H. DICKE*
*Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts***

(Received April 15, 1946)



$$SNR = \frac{P_S}{k_B T}\frac{\sqrt{t}}{\sqrt{\Delta\nu}}$$

Equivalent noise temperature T



Noise power $P_N = k_B T \frac{\sqrt{\Delta\nu}}{\sqrt{t}}$

Increase noise from side bands $\Delta\nu$

Reduce noise by integration time t

In the linear amplifier-mixer-I/Q-FFT chain

$v_{out} \propto v_0^1$ $\quad \Delta\nu = \frac{1}{t}$

$$\rightarrow P_N = \frac{k_B T[\mathrm{W/Hz}]}{t} \quad (t < T_{FFT})$$

$t > T_{FFT}$

In the photon sensor / bolometer

$\Delta\nu$: from filter before sensor

$$\rightarrow P_N = k_B T \frac{\sqrt{\Delta\nu}}{\sqrt{t}} \equiv \frac{\mathrm{NEP}\ [\mathrm{W}/\sqrt{\mathrm{Hz}}]}{\sqrt{t}}$$

NEP definition seems implicitly assuming 1 s integration time or 1 Hz BW

Fundamental question

- We introduced quantum sensors to overcome SQL and enhance S/N
- This is quantum sensing of **microwave photons** induced from **classical axion dark matter**
- It is NOT a quantum sensing of quantum axion
- We still rely on the argument that dark matter axion is classical due to the large occupation number
- **Can we see any fundamental quantum nature of dark matter axions?**

Quantum nature of axion DM: natural squeezing

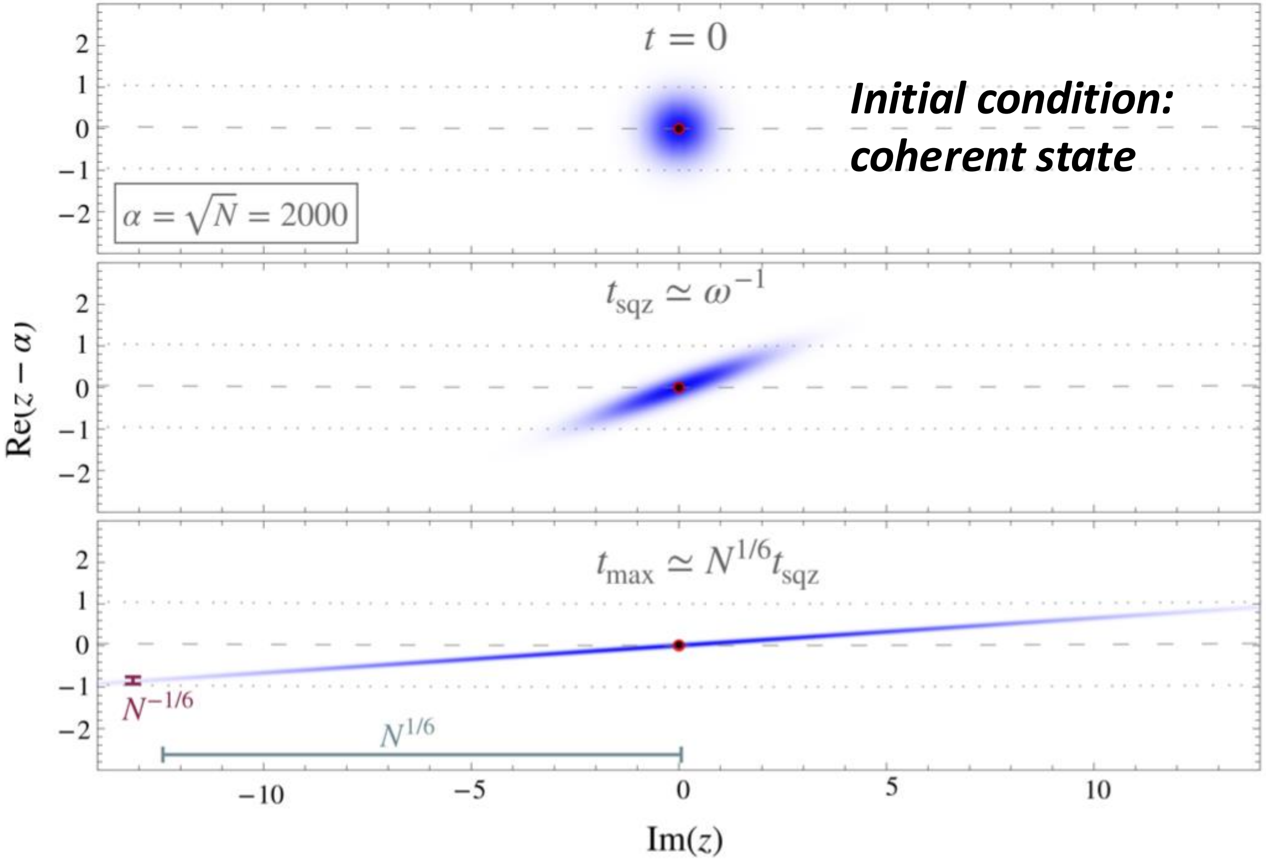
$$i\hbar\partial_t\hat{\psi}(\boldsymbol{x},t) = -\frac{\hbar^2}{2mA(t)^2}\boldsymbol{\nabla}^2\hat{\psi}(\boldsymbol{x},t) + m\boxed{\hat{\Phi}(\boldsymbol{x},t)}\hat{\psi}(\boldsymbol{x},t) \tag{1}$$

$$\boxed{\boldsymbol{\nabla}^2\hat{\Phi}(\boldsymbol{x},t)} = \frac{4\pi G m}{A(t)}\left(\hat{\psi}^\dagger(\boldsymbol{x},t)\hat{\psi}(\boldsymbol{x},t) - \overline{\hat{\psi}^\dagger(\boldsymbol{x},t)\hat{\psi}(\boldsymbol{x},t)}\right),$$

Gravitational potential

$$i\partial_t\hat{a}(t) = \omega(t)\hat{a}(t) + \boxed{2\chi(t)\hat{a}^\dagger(t)\hat{a}(t)\hat{a}(t)}$$

Kerr-type nonlinearity



	Cosmology	Solitonic core	Milkyway
$t_{\rm sqz}[\mu s]$	$66\left(\frac{10^{-5}\mathrm{eV}}{m}\right)$	$1400\left(\frac{10^{-5}\mathrm{eV}}{m}\right)$	$33\left(\frac{10^{-5}\mathrm{eV}}{m}\right)$
$t_{\rm max}$ [yr]	$3500\left(\frac{10^{-5}\mathrm{eV}}{m}\right)^{\frac{7}{6}}$	$0.5\left(\frac{10^{-5}\mathrm{eV}}{m}\right)^{\frac{4}{3}}$	$33000\left(\frac{10^{-5}\mathrm{eV}}{m}\right)^{\frac{7}{6}}$
$r_{\rm max}$	$36 + \frac{1}{6}\ln\left(\frac{10^{-5}\mathrm{eV}}{m}\right)$	$24 + \frac{1}{3}\ln\left(\frac{10^{-5}\mathrm{eV}}{m}\right)$	$32 + \frac{1}{6}\ln\left(\frac{10^{-5}\mathrm{eV}}{m}\right)$
$t_{\rm Ehr}$ [yr]	$10^{35}\left(\frac{10^{-5}\mathrm{eV}}{m}\right)^{\frac{3}{2}}$	$10^{21}\left(\frac{10^{-5}\mathrm{eV}}{m}\right)^{2}$	$10^{32}\left(\frac{10^{-5}\mathrm{eV}}{m}\right)^{\frac{3}{2}}$

- Gravitational self-interaction of dark matter axions causes quantum squeezing within a surprisingly short period
- Experimental detection seems non-feasible ☹

Phys. Rev. D 106, 043517, 2022

Eg) Gravitational wave from blackhole merger

PHYSICAL REVIEW LETTERS **136,** 061404 (2026)

Coherent State Description of Gravitational Waves from Binary Black Holes

Sugumi Kanno,[1,2,3] Jiro Soda,[4] and Akira Taniguchi[1]

→ Coherent state

$$\exp\left[-i\int dtd^3x\left\{T_{ij}(x^i,t)\hat{h}_{ij}(x^i,t) + \Lambda_{ijkl}(x^i,t)\hat{h}_{ij}(x^i,t)\hat{h}_{kl}(x^i,t) + \cdots\right\}\right]$$

→ squeezed state

1st order time evolution → Displacement operator

$$\hat{U}(t,\bar{\boldsymbol{x}}_N) = \exp\left[-i\frac{2}{M_{\rm p}}\sum_{N=1,2}\frac{\gamma_N^3 m_N}{2}\sum_{P=+,\times}\int^t dt'\int\frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}} \times\left(\frac{e^{-i\omega_k t'}}{\sqrt{2\omega_k}}e_{ij}^{(P)}(\boldsymbol{k})a(\boldsymbol{k}) + \frac{e^{i\omega_k t'}}{\sqrt{2\omega_k}}e_{ij}^{(P)}(-\boldsymbol{k})a^\dagger(-\boldsymbol{k})\right)e^{i\boldsymbol{k}\cdot\bar{\boldsymbol{x}}_N}v_N^i v_N^j\right]$$

2nd order time evolution → Squeezing operator

$$\hat{S}(\beta) = \prod_P \exp\left[\int d^3\boldsymbol{k}\int d^3\boldsymbol{k}'\left(\beta_{kk'}^{(P)}a^{(P)\dagger}(\boldsymbol{k})a^{(P)\dagger}(\boldsymbol{k}') - \beta_{kk'}^{(P)*}a^{(P)}(\boldsymbol{k})a^{(P)}(\boldsymbol{k}')\right)\right]$$

Decoherence is neglected

Squeezing parameter

$$\zeta \simeq \frac{4\pi}{3}(2\Omega)^3|\beta| \simeq \frac{1}{8\pi M_{\rm p}^2}\mu(a\Omega)^4 f$$

$$\zeta \simeq 2\times 10^{-4}\left(\frac{\mu}{16M_\odot}\right)\left(\frac{a\Omega}{0.41}\right)^4\left(\frac{f}{68\ \text{Hz}}\right). \qquad (22)$$

Thus, the quantum state of gravitational waves from GW150914 is characterized by a squeezing parameter of order 10^{-4}. Note that the squeezing parameter in Eq. (22)

(how to detect it???)

Quantum nature of axion DM: thermalization

$$\mathcal{L}[a,\chi] = \frac{1}{2}\partial_\mu a(x)\partial^\mu a(x) - \frac{1}{2}m_a^2 a^2(x) - \boxed{g a(x)\,\mathcal{O}_\chi(x) + \mathcal{L}_\chi}$$

Interaction to environment (SM)

$$a(\vec{x}, t=0) = \frac{1}{\sqrt{V}}\sum_{\vec{k}}\frac{1}{\sqrt{2\omega_k}}\left[b_{\vec{k}}\,e^{i\vec{k}\cdot\vec{x}} + b^\dagger_{\vec{k}}\,e^{-i\vec{k}\cdot\vec{x}}\right]$$

$$n(\omega_q) = \frac{1}{e^{\beta\,\omega_q} - 1}$$

$$\hat{\rho}(t) = e^{-iHt}\hat{\rho}(0)e^{iHt}$$

Time evolution of density matrix

$$\rho(0) = \rho_a(0)\otimes\rho_\chi(0)$$

Initial condition: axion x environment

$$\rho_\chi(0) = \frac{e^{-\beta H_\chi}}{\mathrm{Tr}e^{-\beta H_\chi}}$$

Environment: thermal bath (CMB)

$$\rho_a(0) = |\Delta\rangle\langle\Delta|$$

Initial axion: coherent state from misalignment mechanism

$$|\Delta\rangle = \Pi_{\vec{k}}\, e^{-\frac{1}{2}|\Delta_{\vec{k}}|^2}\, e^{-\Delta_{\vec{k}} b^\dagger_{\vec{k}}}\,|0\rangle$$

<u>Quantum master equation</u>

→ Initial coherent axions become decoherent due to thermalization

$$\mathcal{E}(t) = \frac{1}{V}\sum_{\vec{q}} N_q(t)\omega_q = \frac{1}{2}\left[\overline{\pi}^2 + m_a^2\,\overline{a}^2\right]\boxed{e^{-\gamma_0 t}} + \int\frac{d^3q}{(2\pi)^3}\,\omega_q\, n(\omega_q)\boxed{(1 - e^{-\gamma_q t})}$$

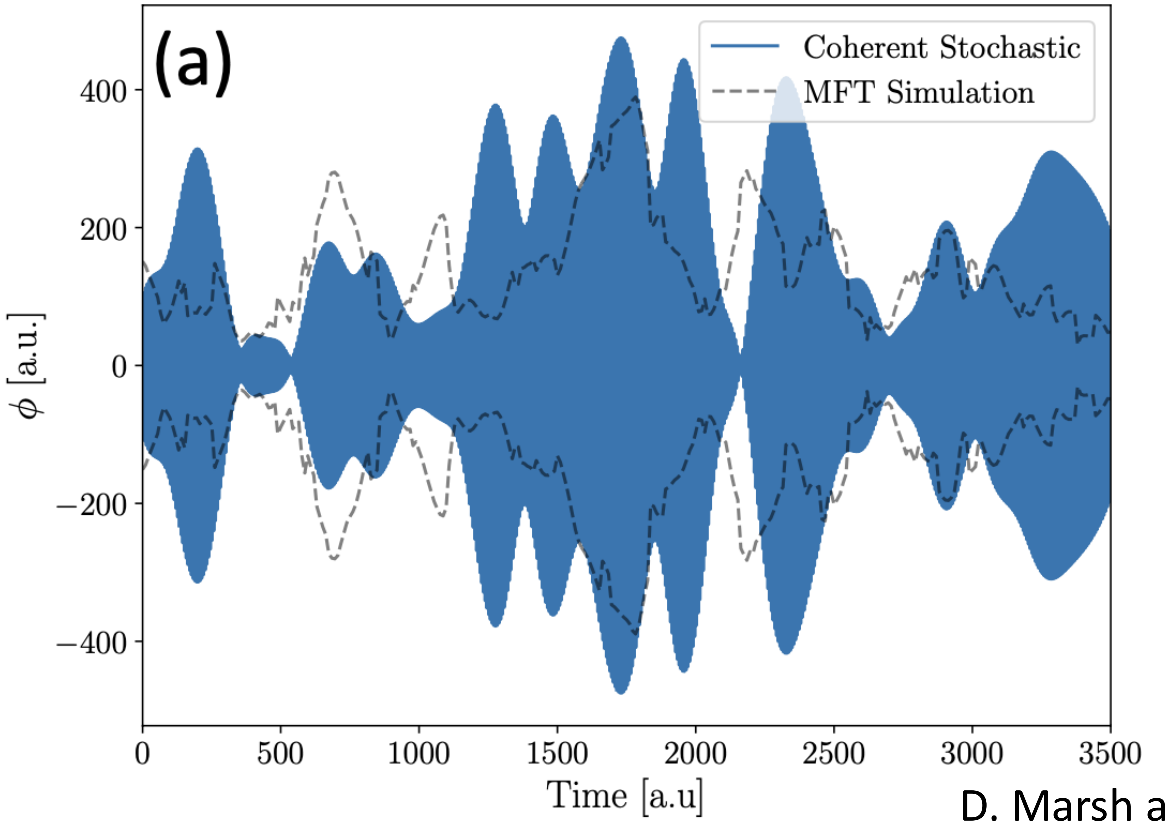
Coherent term decays

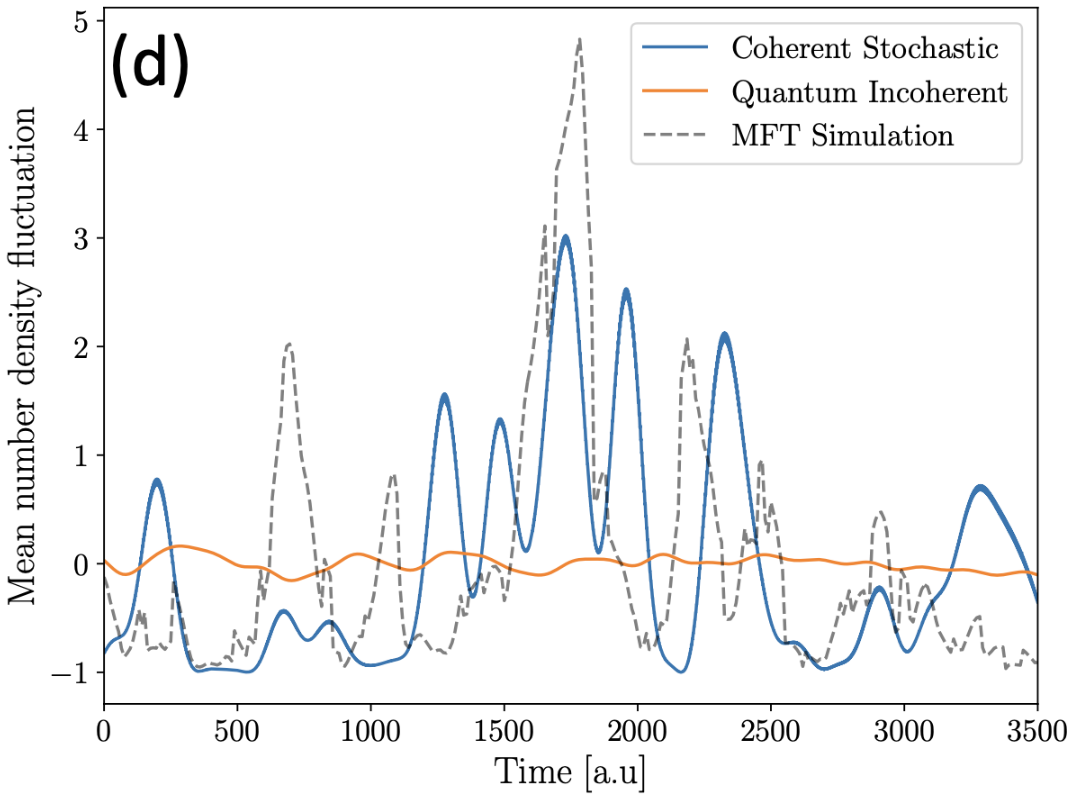
Thermal term grows

Decay rate: $\gamma_q = \Gamma_q(\infty) = \dfrac{g^2}{2\omega_q}\rho(\omega_q, q)$

Phys. Rev. D 107, 063518 (2023)

Temporal coherence of axion DM





D. Marsh arXiv:2211.13602

<u>Coherent stochastic model</u>

$$\phi(t) = \frac{\sqrt{2\rho_{\rm loc}}}{m} \sum_{i=0}^{i_{\rm max}} \alpha_i \cos\left[\left(1 + \frac{v_i^2}{2}\right) mt + \delta_i\right]$$

DM axion is in quantum coherent state ~ truly classical wave

<u>Quantum incoherent model</u>

$$n = \frac{1}{2} m \langle |\psi_k|^2 \rangle_k$$

$$\psi_k = \frac{\sqrt{2\rho_{\rm loc}}}{m} \sum_{i=0}^{i_{\rm max}} \alpha_i \exp\left[i\left(m\frac{v_i^2}{2}t + \delta_i + \varepsilon_{ik}\right)\right]$$

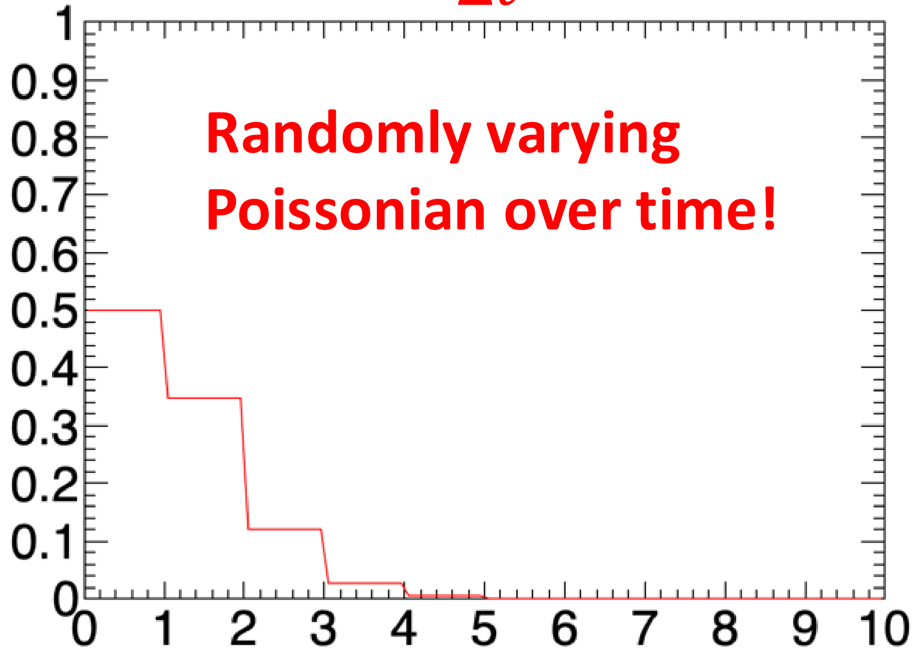
What happens to the quantum detector response

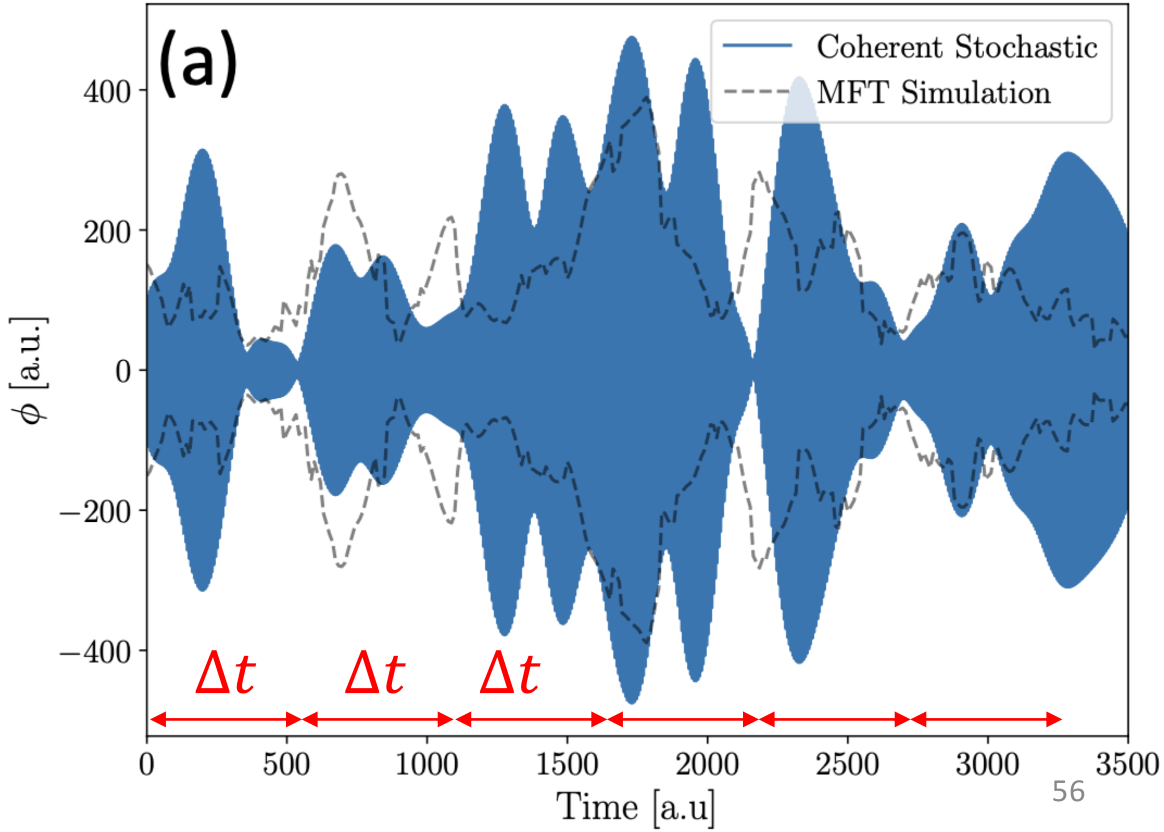
$$P_n = |\langle n|\psi(t)\rangle|^2 = \frac{e^{-|\beta(t)|^2}|\beta(t)|^{2n}}{n!}$$

$$\beta(t) = \exp(-i\omega t)\int_0^t dt'\phi(t')\exp(i\omega t')$$

Δt

- The photon number obeys Poisson distribution of varying mean
- Depends on integral of classical axion amplitude over temporal coherency
- Δt → response time of quantum sensor? Or DAQ?





Part 3: Quantum detection scheme

- Quantum coherent states
- Glauber's theorem
- Thermal noise and Standard Quantum Limit
- Squeezing and photon counting
- Conclusion of part 3

- Global conclusion of the lecture courses

Conclusion of Part 3

- Quantum coherent states are the basis of quantum optics
  - The eigen state of the annihilation operator (non-Hermitian = non-observable)
  - Classical electrodynamics is the expectation value of a field operator of a coherent state
  - The number of photons obey a Poisson distribution
- Glauber's theorem
  - Classical driving force generally produces a quantum coherent state
  - Dark matter axion act as a displacement operator
- Thermal noise and Standard Quantum Limit
  - Thermal noise is a 1D blackbody radiation to be reduced by lower T but
  - Reading out both I/Q quadrants faces fundamental noise limit due to the commutation relation and corresponding Kennard inequality (uncertainty principle)
- Squeezing and photon counting
  - Parametric amplifier can be operated in a phase sensitive way to implement squeezing
  - Photon / power sensing is free from SQL and potentially enhances S/N for axion DM search
  - The quantum sensing is for microwave photons and DM axion itself is still treated classically
  - Quantum nature of axions is a one of the hot topics of theoretical studies yet no clear suggestions for the experimental detection scheme

Part 3: Quantum detection scheme

- Quantum coherent states
- Glauber's theorem
- Thermal noise and Standard Quantum Limit
- Squeezing and photon counting
- Conclusion of part 3

- Global conclusion of the lecture courses

Global conclusion and take-away messages

- Axion dark matter is an ***excellent target in particle physics and cosmology*** in post-LHC era
    - Somewhat similar to SUSY/WIMP before LHC/XENON
    - Phase space is wide open despite lacking immediate feasibility in experiments
    - No anthropic solution of strong CP unlike TeV-scale SUSY
- Axion search forces ***experimentalists*** to do ***advanced theory of detectors***
    - Material science / condensed matter physics
    - Modern classical microwave engineering
    - Quantum optics / circuit QED
- Axion experiments require knowledge, skills, and infrastructure uncommon in conventional ***particle*** physics experiments
    - ***Particle*** physics: scintillator, semi-conductor detector, PMT, NIM, oscilloscope
    - ***Microwave*** physics: Vector Network Analyzer, Spectrum Analyzer, Linear Amplifier, heterodyne mixers, I/Q sampling ADC, dilution refrigerator
    - Surprisingly expensive and fragile!
- Challenging to start axion experiments from scratch → **contact experts**
    - Cosmic Microwave Background (**CMB**) people (40-200 GHz)
    - **Radioastronomy** people (60-300 GHz)
    - **RF particle accelerator** people (FCCee/PIP2-DUNE: 400-800 MHz, ILC: 1.3 GHz, CLIC: 12 GHz)

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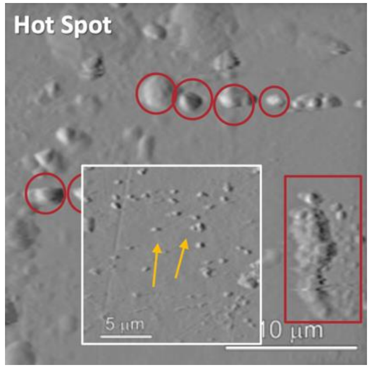
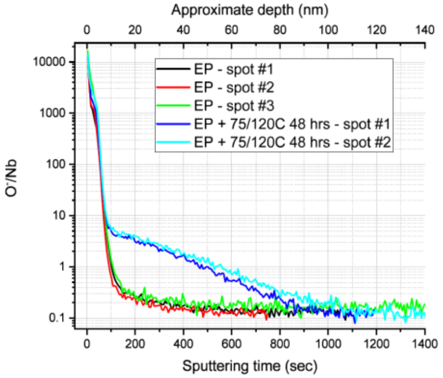
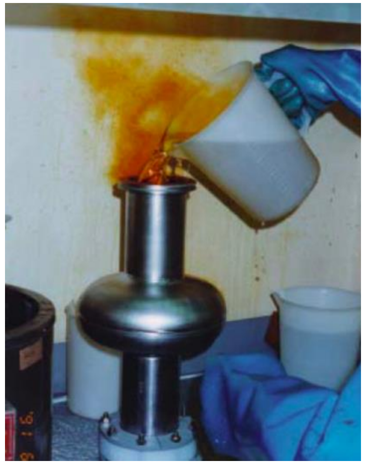
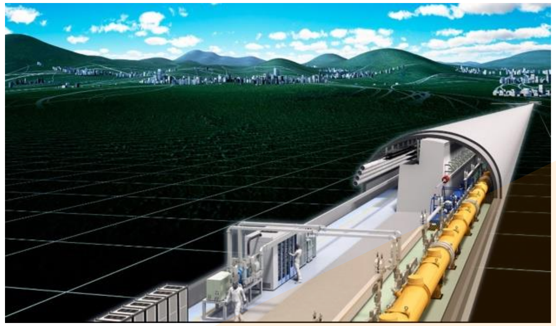
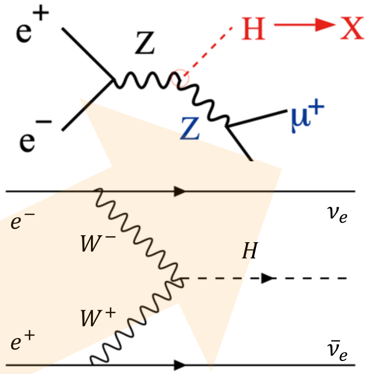
My vision in superconducting RF accelerators

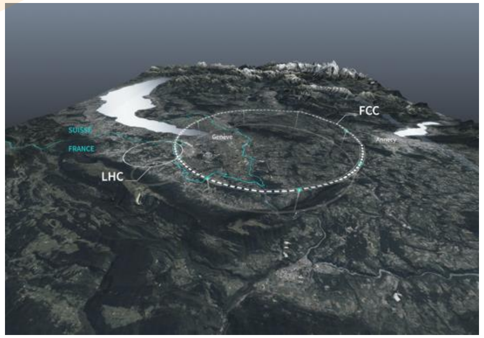


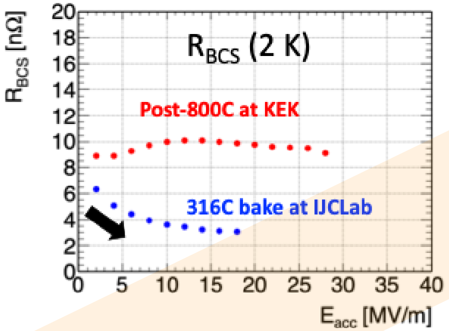


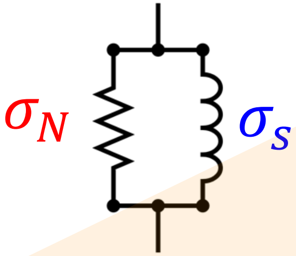


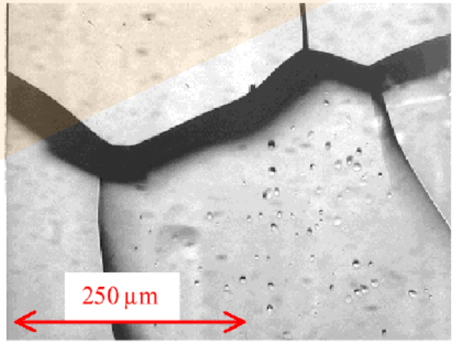


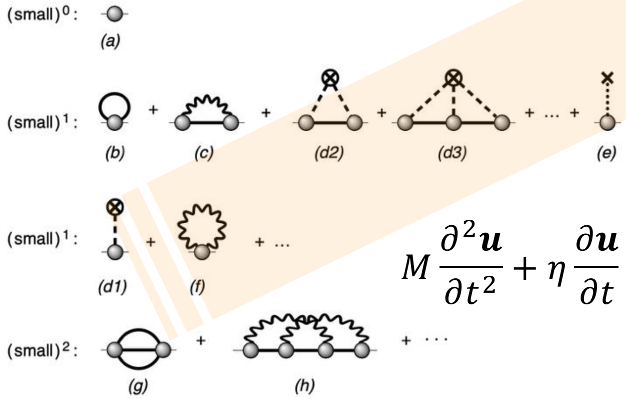






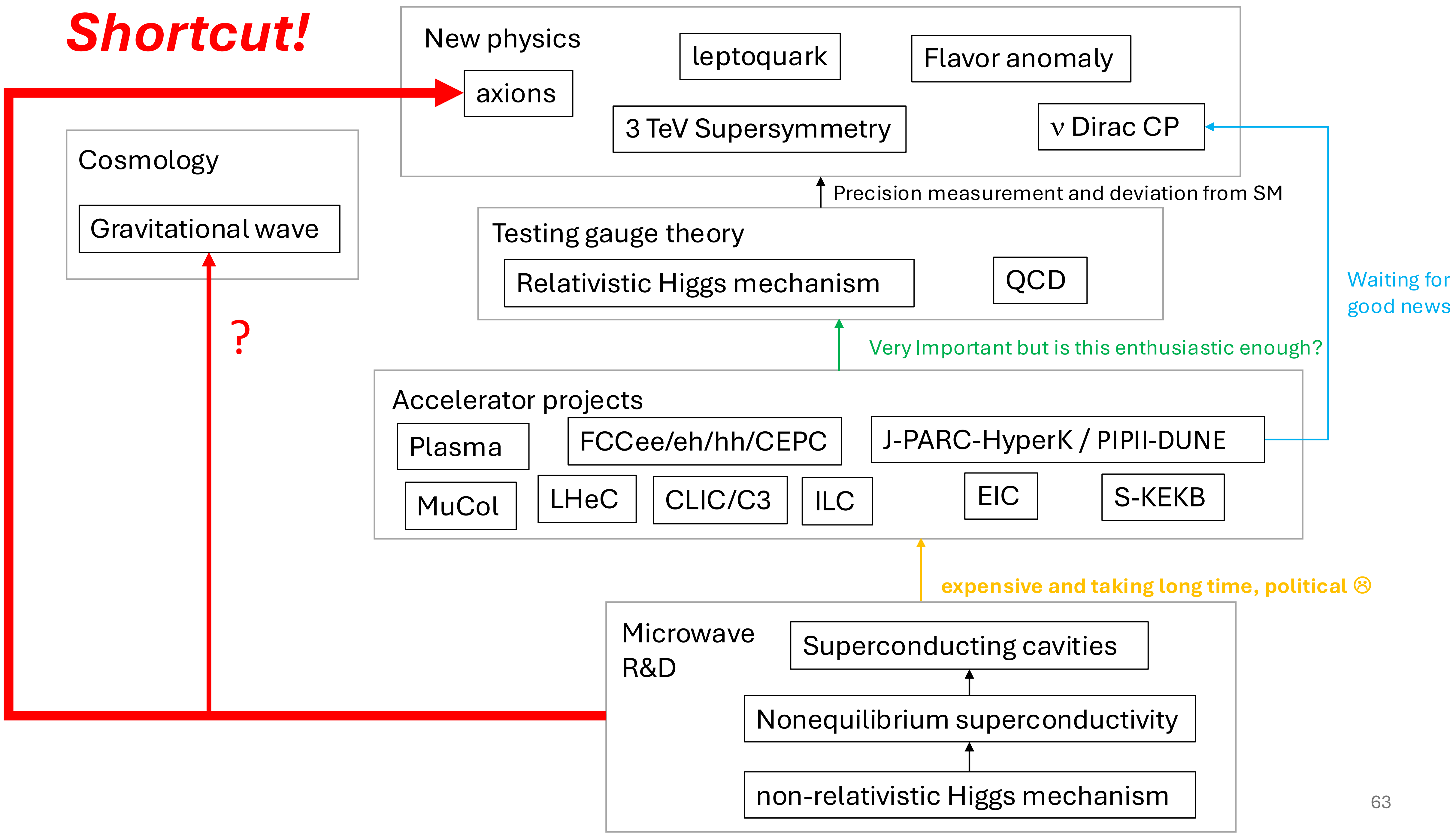






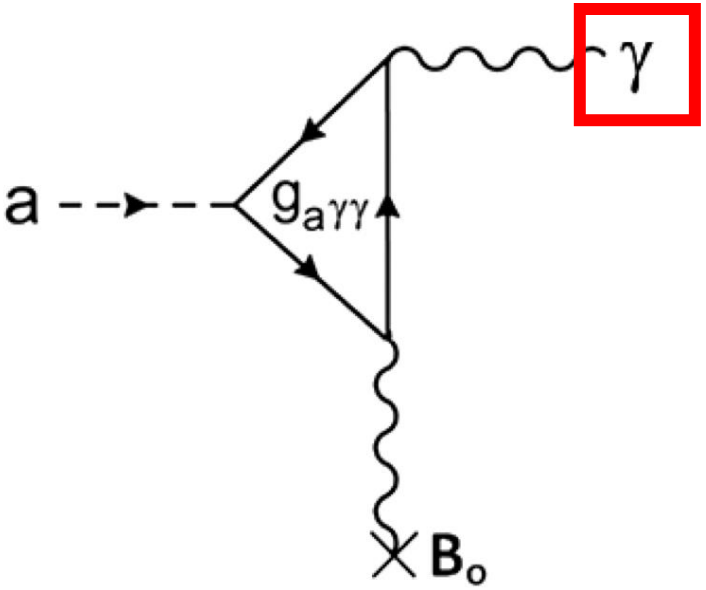
$$M\frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \eta\frac{\partial \boldsymbol{u}}{\partial t} - \epsilon\frac{\partial^2 \boldsymbol{u}}{\partial z^2} + \nabla U(z,u) = \boldsymbol{J}_{RF}(z,u) \times \boldsymbol{B}_{\text{ext}}$$

From superconductivity to particle physics through huge MW & cryogenic infrastructure


Shortcut!
New physics
axions
leptoquark
Flavor anomaly
3 TeV Supersymmetry
ν Dirac CP
Cosmology
Gravitational wave
Precision measurement and deviation from SM
Testing gauge theory
Relativistic Higgs mechanism
QCD
?
Very Important but is this enthusiastic enough?
Waiting for
good news
Accelerator projects
Plasma
FCCee/eh/hh/CEPC
J-PARC-HyperK / PIPII-DUNE
MuCol
LHeC
CLIC/C3
ILC
EIC
S-KEKB
expensive and taking long time, political ☹
Microwave
R&D
Superconducting cavities
Nonequilibrium superconductivity
non-relativistic Higgs mechanism

Microwave photons may address fundamental physics

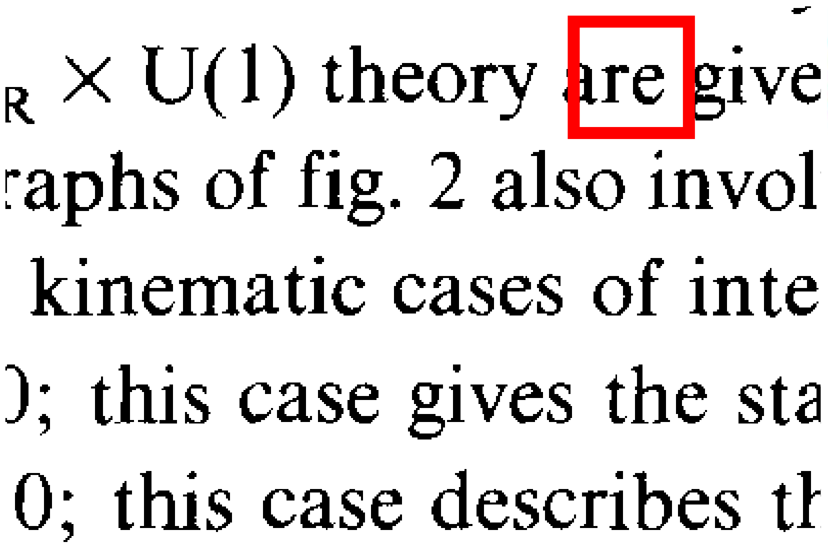
Axions



Inverse Primakoff effect

Minimal extension of SM

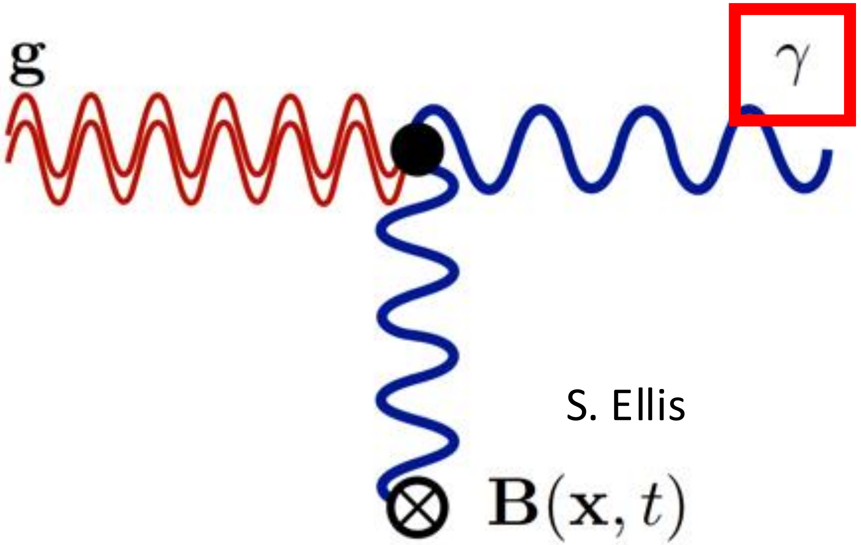
Neutrinos



Cosmic neutrino background

Extension of SM and/or SM

Gravitational waves



Inverse Gertsenshtein effect

Solution of general relativity

Welcome to future ~~particle~~ fundamental physics via microwaves

backup

Cf) Heisenberg uncertainty ≠ Kennard

Robertson-Kennard inequality is fundamental property of Hilbert space

The relation between the standard deviation (=statistical error) of two canonical operators

Commutation relation Schwarz inequality

$$[p, q] = \frac{i\hbar}{2} \rightarrow \langle \Delta p^2 \rangle \langle \Delta q^2 \rangle \geq |\langle [p, q] \rangle|^2 \rightarrow \boxed{\sigma_p \sigma_q \geq \frac{\hbar}{2}}$$

Fundamental uncertainty principle of quantum physics (I do not call this Heisenberg)

Original idea: Heisenberg's indeterminacy principle is measurement back-action

Heisenberg "The more accurately one property is measured, the less accurately the other property can be known"

Less systematic error ε
→ more disturbance η

Original Heisenberg: $\varepsilon_p \eta_q \geq \frac{\hbar}{2}$ This was overcome by Ozawa $\varepsilon_p \eta_q + \varepsilon_p \sigma_q + \sigma_p \eta_q \geq \frac{\hbar}{2}$

→ More general argument by Lee & Tsutsui

Measurement theory in modern quantum information theory