Probing Helical PMFs via Circular Polarization of GW in the LISA-TAIJI network

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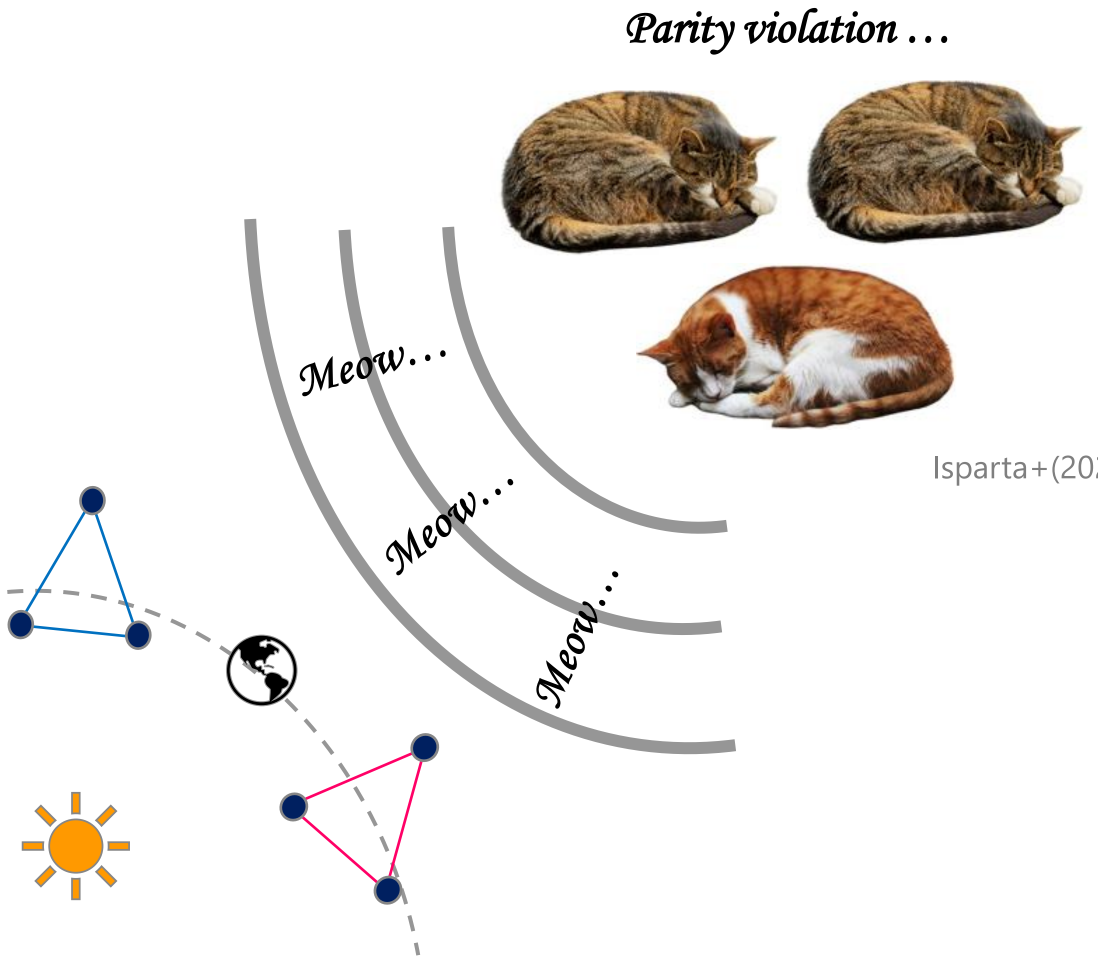
Now Ongoing...



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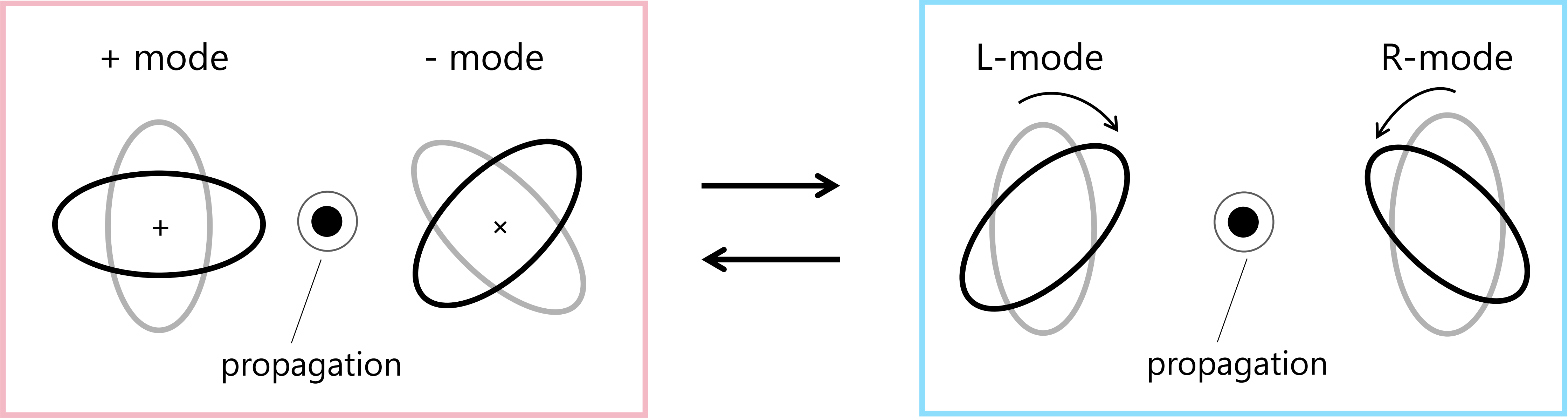
Circular Polarization



$$\text{GW: } h_{ij} = h_+ e_{ij}^+ + h_\times e_{ij}^\times = h_\mathrm{R} e_{ij}^\mathrm{R} + h_\mathrm{L} e_{ij}^\mathrm{L}$$

Basis

$$e_{ij}^\mathrm{R} = \frac{1}{\sqrt{2}}(e_{ij}^+ + i e_{ij}^\times)$$

$$e_{ij}^\mathrm{L} = \frac{1}{\sqrt{2}}(e_{ij}^+ - i e_{ij}^\times)$$

Amp.

$$h_\mathrm{R} = \frac{1}{\sqrt{2}}(h_+ - i h_\times)$$

$$h_\mathrm{L} = \frac{1}{\sqrt{2}}(h_+ + i h_\times)$$

Stochastic GW Background

- **Stochastic GW Background (SGWB)**

$$h_{ij}(t,\mathbf{x}) = \sum_{A=\times,+} \int_{-\infty}^{\infty} df \int d^2\hat{\mathbf{n}} \tilde{h}_A(f,\hat{\mathbf{n}}) e^A_{ij}(\hat{\mathbf{n}}) e^{-2\pi i f (t-\hat{\mathbf{n}}\cdot\mathbf{x}/c)}.$$

- **Cross correlation:**

$$\langle h^*_A(f,\hat{\boldsymbol{n}}) h_{A'}(f,\hat{\boldsymbol{n}}')\rangle = \frac{\delta(f-f')}{2}\frac{\delta^2(\hat{n},\hat{n}')}{4\pi}\begin{pmatrix} I(f,\hat{\boldsymbol{n}})+Q(f,\hat{\boldsymbol{n}}) & U(f,\hat{\boldsymbol{n}})-iV(f,\hat{\boldsymbol{n}}) \\ U(f,\hat{\boldsymbol{n}})+iV(f,\hat{\boldsymbol{n}}) & I(f,\hat{\boldsymbol{n}})-Q(f,\hat{\boldsymbol{n}}) \end{pmatrix}$$

$\boldsymbol{I} = \langle h_{\mathrm{L}}^2 + h_{\mathrm{R}}^2\rangle$: **intensity** $\boldsymbol{V} = \langle |h_{\mathrm{L}}|^2 - |h_{\mathrm{R}}|^2\rangle$: **circular polarization** Q, U: linear polarization

$$I,V(f,\hat{\boldsymbol{n}}) = I,V(f)\sum_{l=0}^{\infty}\sum_{m=-l}^{m=l} c^{I,V}_{lm} Y_{lm}(\hat{\boldsymbol{n}})$$

→ **contribute to Isotropic SGWB!!**

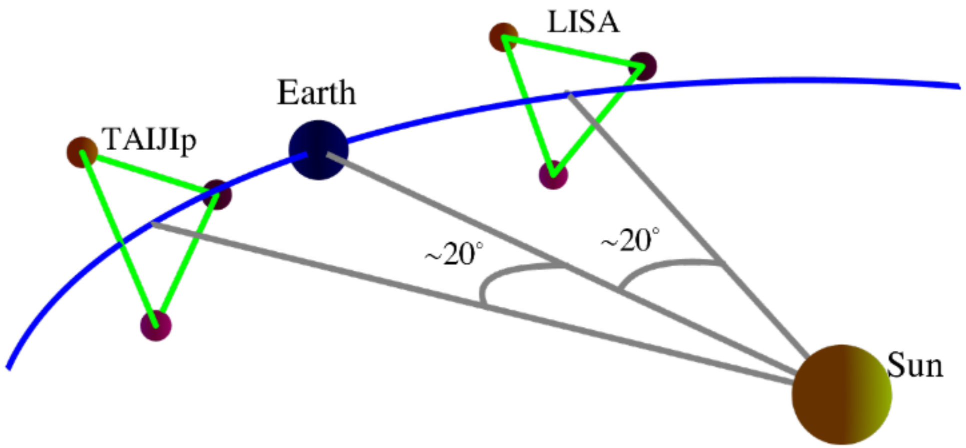
$$(Q\pm iU)(f,\hat{\boldsymbol{n}}) = (Q\pm iU)(f)\sum_{l=4}^{\infty}\sum_{m=-l}^{m=l} c^{\pm}_{lm} {}_{\pm 4}Y_{lm}(\hat{\boldsymbol{n}})$$

→ $\boldsymbol{Q}, \boldsymbol{U} = \boldsymbol{0}$ in monopole…

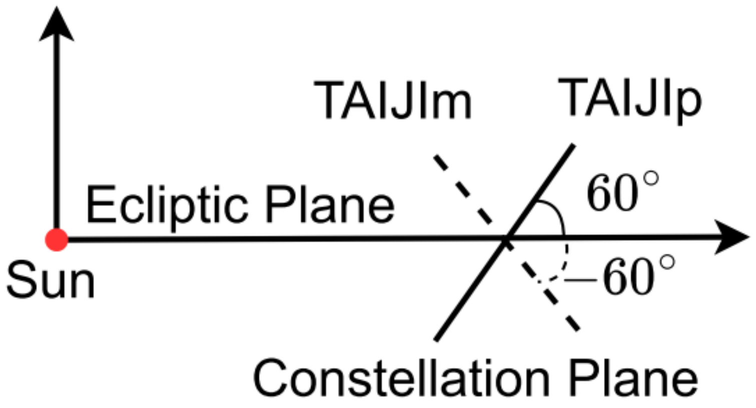
LISA-TAIJI Network!!

Ju Chen+(2024)

Fiducial LISA-TAIJI Network (LISA-TAIJIp)



Alternative LISA-Taiji Network (LISA-TAIJIm)



- Combination of the two space GW detectors LISA & TAIJI ($f_{\rm obs} \in [10^{-5}, 10^{-1}]$)

- Sensitive to the **circular polarization of isotropic GWB**!!

⟶ Possibly probe **parity violation** in generation/propagation of GW

e.g. Chern-Simons coupling, **Helical primordial magnetic field** etc.

Primordial Magnetic Fields (PMFs)

Primordial Magnetic Fields (PMFs)

- generated during phase transition/**inflation**
- can be **seed w/ large correlation length ($\lambda_B \sim 1$ Mpc)**

-> $B \sim 10^{-6}$G by dynamo process

$\approx$ MFs in galaxy & galaxy cluster

-> naturally explain MFs **in voids**

$$B \gtrsim 10^{-16}\,\mathrm{G} \times \begin{cases} 1 & (\lambda_B \leq 1\ \mathrm{Mpc}) \\ \sqrt{1\ \mathrm{Mpc}/\lambda_B} & (\lambda_B \geq 1\ \mathrm{Mpc}) \end{cases}$$

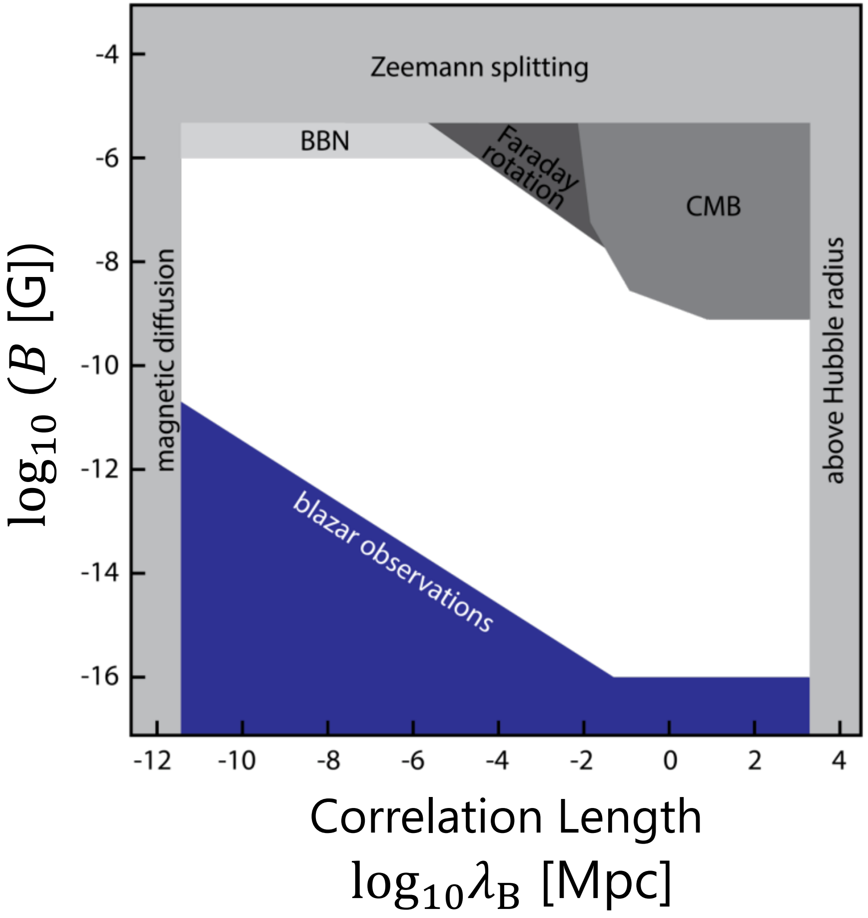


Figure taken from J. schobar

GW can probe PMFs

Our Motivation

- GW obs. can constrain PMFs **at small scale**!!

$(k_{\mathrm{PTA}} \sim 10^6 \mathrm{Mpc}^{-1}, k_{\mathrm{LISA}} \sim 10^{12}\,\mathrm{Mpc}^{-1})$ Saga+(2018)

- **Helical PMFs** can be produced through inflation.

e.g.) $\mathcal{L} \supset -\frac{I(a)}{4}(F_{\mu\nu}F^{\mu\nu} + \gamma F_{\mu\nu}\tilde{F}^{\mu\nu})$

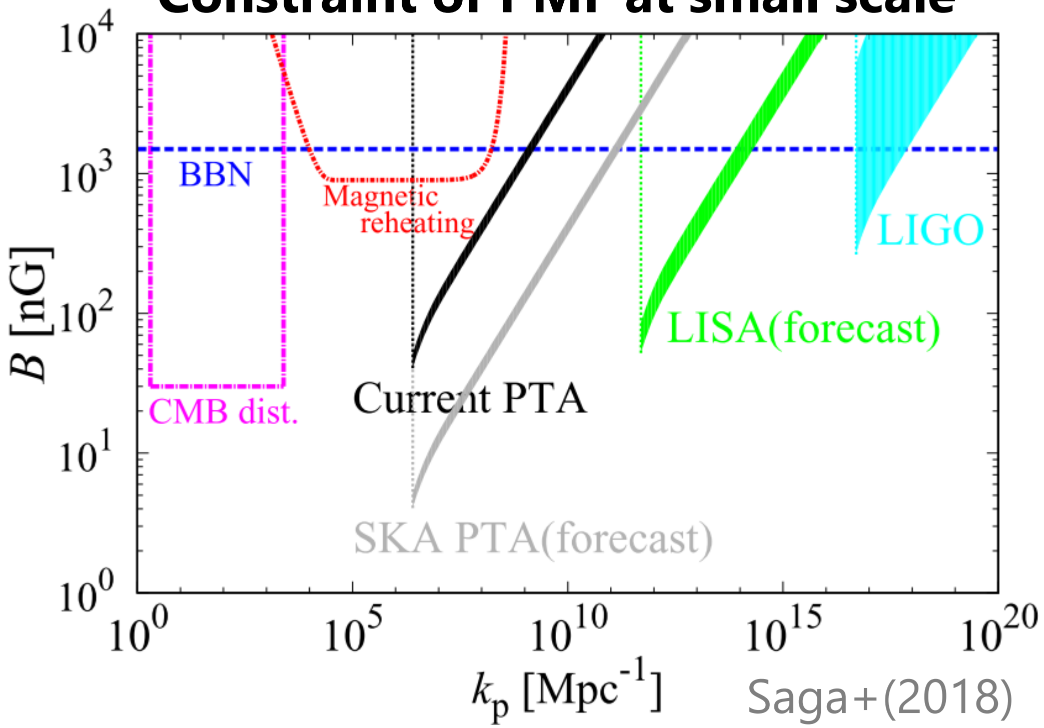
Caprini&Sorbo (2014), Fujita+(2019), Sharma+(2018)

⟶ **Circular polarized SGWB** will be generated!!

▼

Derive the expected constraint on helical PMFs from the LISA-TAIJI network!!

Constraint of PMF at small scale



Saga+(2018)

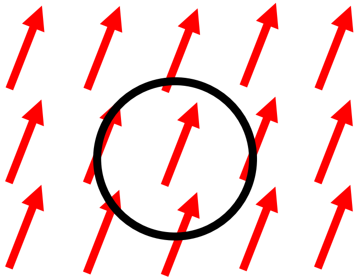
e.g. Spectrum of PMFs

$$P_B(k) = \frac{2\pi^2}{k^3}\mathcal{B}^2\delta_D[\ln(k/k_p)]$$

What we consider is…

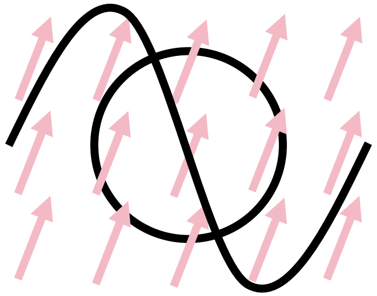
Scenario

PMFs are produced
at the end of inflation

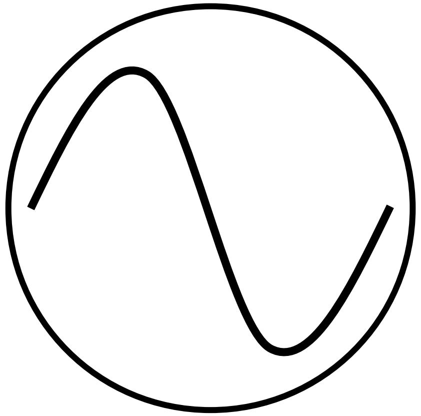


$\eta = \eta_{\rm B}(= 10^{-12}\eta_\nu)$

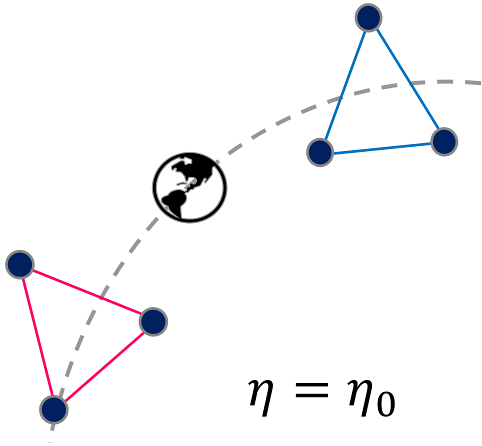
GWB generated
at super-horizon scale



PMFs decayed,
but GWB is still alive



LISA-TAIJI network
observes GWB!!



$\eta = \eta_0$

Calculation

1. Put helical PMF power spectrum
   controlled 2 params. $(\mathcal{B}\ , r_{\rm H})$

$$\langle B_i(\boldsymbol{k}) B_j^*(\boldsymbol{k}')\rangle = \frac{(2\pi)^3}{2}\delta_{\rm D}^{(3)}(\boldsymbol{k}-\boldsymbol{k}')[P_{ij}S(k) + \epsilon_{ijl}\hat{k}_l A(k)]$$

2. Solve GW propagation eq.

$$h''_{ij}(\eta,\boldsymbol{k}) + \frac{2}{\eta}h'_{ij}(\eta,\boldsymbol{k}) + k^2 h_{ij}(\eta,\boldsymbol{k}) = \frac{2}{\eta^2}R_\gamma \pi^B_{ij}(\boldsymbol{k})$$

3. Evaluate detectability
   - Calculate SNR
   - Fisher Forecast

PMF Power Spectrum

Saga+(2018) Caprini+(2003)

2-pt. correlation of helical PMFs :

$$\langle B_i(\boldsymbol{k}) B_j^*(\boldsymbol{k'})\rangle = \frac{(2\pi)^3}{2}\delta_{\rm D}^{(3)}(\boldsymbol{k}-\boldsymbol{k'})[P_{ij}S(k) + \epsilon_{ijl}\hat{k}_l A(k)]$$

Non-helical ($S(k)$) Helical ($A(k)$)

- $P_{ij}(\hat{k})$: Projection operator

$$P_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j$$

- ϵ_{ijk}: anti-sym. tensor
- $\boldsymbol{S(k)} \geq |\boldsymbol{A(k)}|$

PMF power spectrum (params. : $\mathcal{B}$, $|r_{\rm H}| \leq 1$)

- delta-function type:

$$S(k) = \frac{2\pi^2}{k^2}\mathcal{B}^2 \delta_{\rm D}(\ln(k/k_{\rm p}))$$

$$A(k) = r_{\rm H} S(k)$$

$k_{\rm p}$: peak wave number

- scale-invariant type:

$$S(k) = \frac{2\pi^2}{k^3}\mathcal{B}^2 \ln^{-1}\left(\frac{k_{\max}}{k_{\min}}\right)$$

$$A(k) = r_{\rm H} S(k)$$

for $k_{\min} \leq k \leq k_{\max}$

$k_{\min} = \eta_\nu^{-1}$

$k_{\max} = 10^8 \eta_{\rm B}^{-1}$

Let's connect PMF to GW

Saga+(2018)

GW propagation :

$$h''_{ij}(\eta,\boldsymbol{k})+\frac{2}{\eta}h'_{ij}(\eta,\boldsymbol{k})+k^2h_{ij}(\eta,\boldsymbol{k})=\frac{2}{\eta^2}R_\gamma\pi^B_{ij}(\boldsymbol{k})$$

$$h_{ij}(\eta,\boldsymbol{k})=\underline{T_{\rm h}(\eta,\boldsymbol{k})}R_\gamma\pi^{\rm B}_{ij}(\boldsymbol{k})$$

Transfer function

- Anisotropic stress of PMFs:

$$\pi^{\rm B}_{ij}(\boldsymbol{k})=-\frac{3}{4\pi\bar{\rho}_{\gamma,0}}\int\frac{{\rm d}^3k'}{(2\pi)^3}\Lambda_{ij,lm}(\hat{k})B_i(\boldsymbol{k})B_j(\boldsymbol{k}-\boldsymbol{k}')$$

- $R_\gamma\equiv\rho_\gamma/\rho_{\rm r}$

2pt. correlation of GW :

$$\langle h_{ij}(\eta,\boldsymbol{k})h^*_{lm}(\eta,\boldsymbol{k}')\rangle=T^2_{\rm h}(\eta,\boldsymbol{k})R^2_\gamma\langle\pi^{\rm B}_{ij}(\boldsymbol{k})\pi^{\rm B*}_{lm}(\boldsymbol{k}')\rangle$$

⟶ Derive $\langle\pi^{\rm B}_{ij}(\boldsymbol{k})\pi^{\rm B}_{ij}(\boldsymbol{k}')\rangle$ to compute $I(f),V(f)$!!

How does helicity work?

Caprini+(2003)

2-pt. corr. of anisotropic stress:

$$\langle \pi^{\mathrm{B}}_{ij}(\boldsymbol{k})\pi^{\mathrm{B}*}_{lm}(\boldsymbol{k}')\rangle = \frac{(2\pi)^3}{4}\delta^{(3)}_{\mathrm{D}}(\boldsymbol{k}-\boldsymbol{k}')[\mathcal{M}_{ijlm}f(k) + i\mathcal{A}_{ijlm}g(k)]$$

$\propto$ **intensity** ($f(k)$) $\propto$ **circ. pol.** ($g(k)$)

$$\mathcal{M}_{ijlm} \equiv e^{+}_{ij}\otimes e^{+}_{lm} + e^{\times}_{ij}\otimes e^{\times}_{lm}$$

$$\mathcal{A}_{ijlm} \equiv e^{+}_{ij}\otimes e^{\times}_{lm} - e^{\times}_{ij}\otimes e^{+}_{lm}$$

- 2-pt. corr. of helical PMFs :

$$\langle B_i(\boldsymbol{k})B^*_j(\boldsymbol{k}')\rangle = \frac{(2\pi)^3}{2}\delta^{(3)}_{\mathrm{D}}(\boldsymbol{k}-\boldsymbol{k}')[P_{ij}S(k) + \epsilon_{ijl}\hat{k}_l A(k)]$$

Non-helical ($S(k)$) **Helical!!** ($A(k)$)

$$f(k) = \frac{1}{2^2}\frac{3^2}{(4\pi\bar{\rho}_{\gamma,0})^2}\frac{1}{(2\pi)^3}\int \mathrm{d}^3p[(1+\gamma^2)(1+\beta^2)S(p)S(|\boldsymbol{k}-\boldsymbol{p}|) + 4\gamma\beta A(p)A(|\boldsymbol{k}-\boldsymbol{p}|)]$$

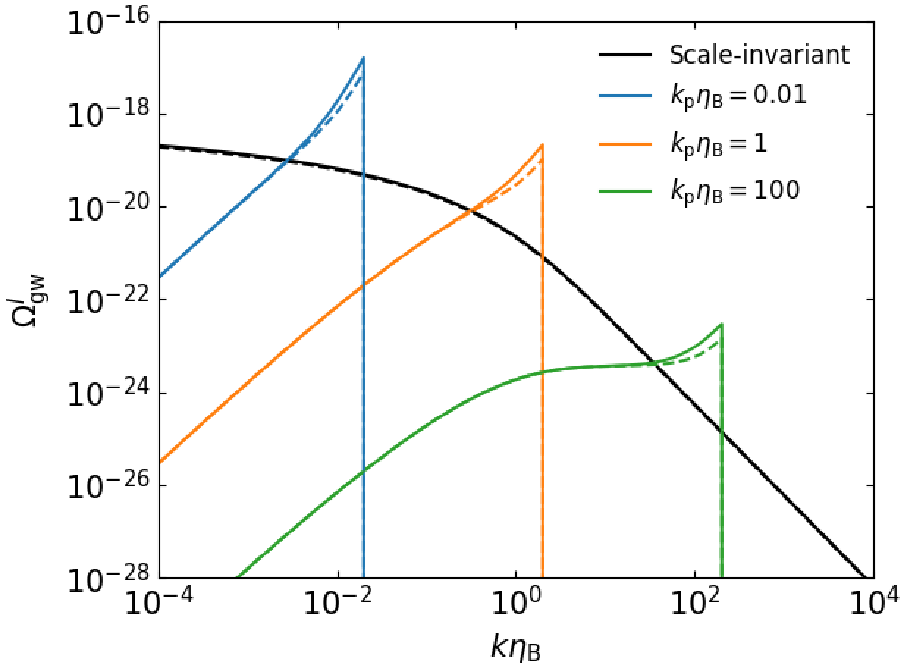
$$g(k) = \frac{1}{2^2}\frac{3^2}{(4\pi\bar{\rho}_{\gamma,0})^2}\frac{1}{(2\pi)^3}2\int \mathrm{d}^3p[\beta(1+\gamma^2)S(p)A(|\boldsymbol{k}-\boldsymbol{p}|) + \gamma(1+\beta^2)A(p)S(|\boldsymbol{k}-\boldsymbol{p}|)]$$

where $\gamma = \hat{k}\cdot\hat{p}, \beta = \hat{k}\cdot\widehat{k-p}$

GW spectrum

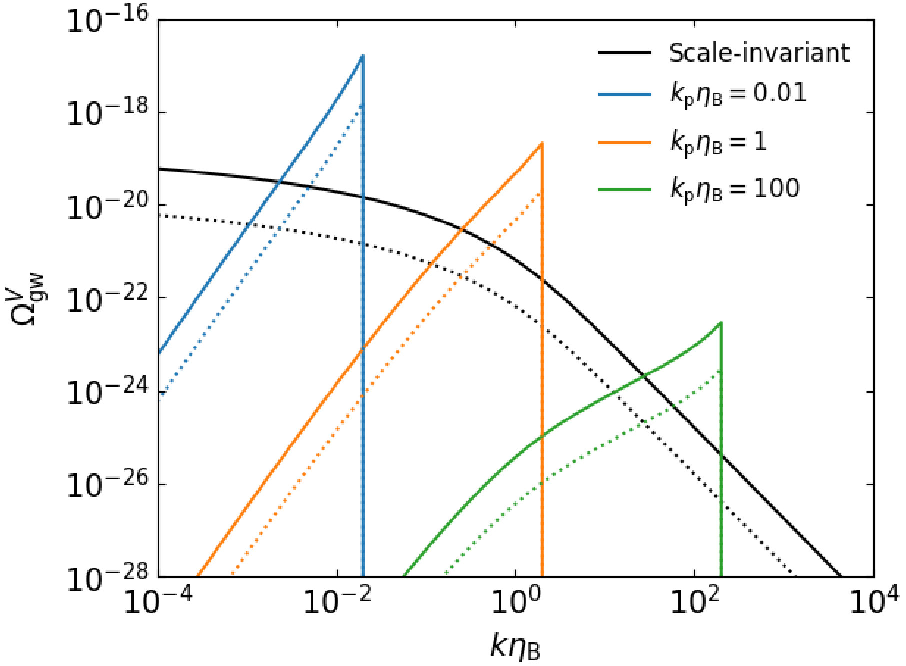
Intensity $\Omega^I_{\rm GW}$ ($\mathcal{B} = 1$nG)

(solid: $r_{\rm H} = 1$/dashed: $r_{\rm H} = 0$)



Circ. pol. $\Omega^V_{\rm GW}$ ($\mathcal{B} = 1$nG)

(solid: $r_{\rm H} = 1$/dotted: $r_{\rm H} = 0.1$)



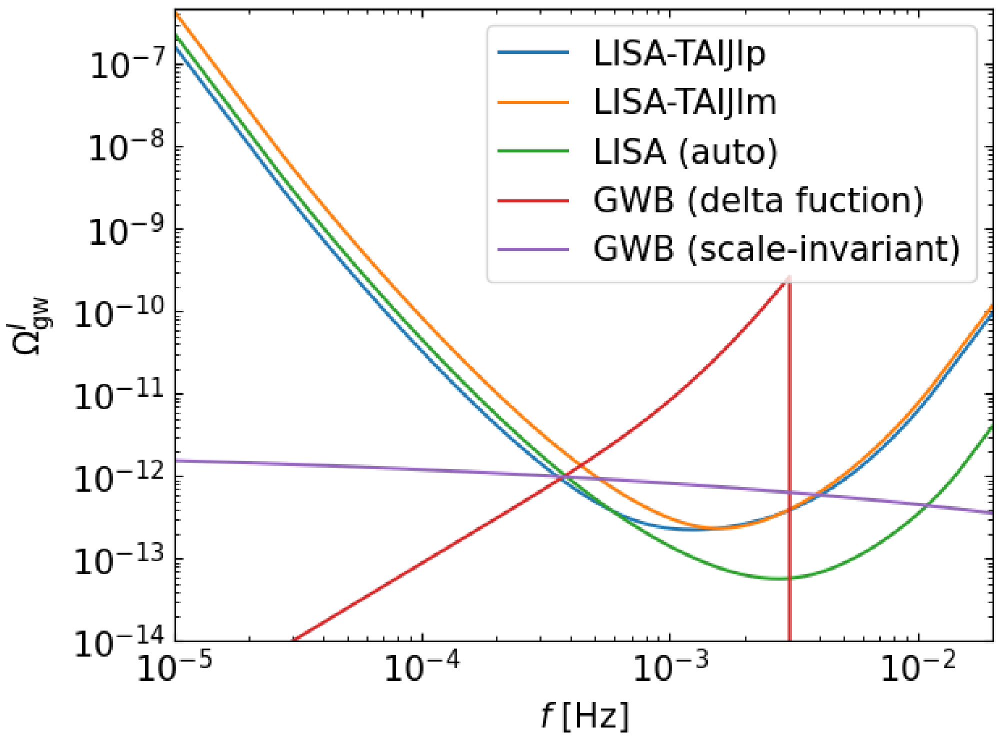
- Helicity does not affect on $\Omega^I_{\rm GW}$ significantly...

- Circ. pol. $\Omega^V_{\rm GW}$ depends on $r_{\rm H}$ linearly

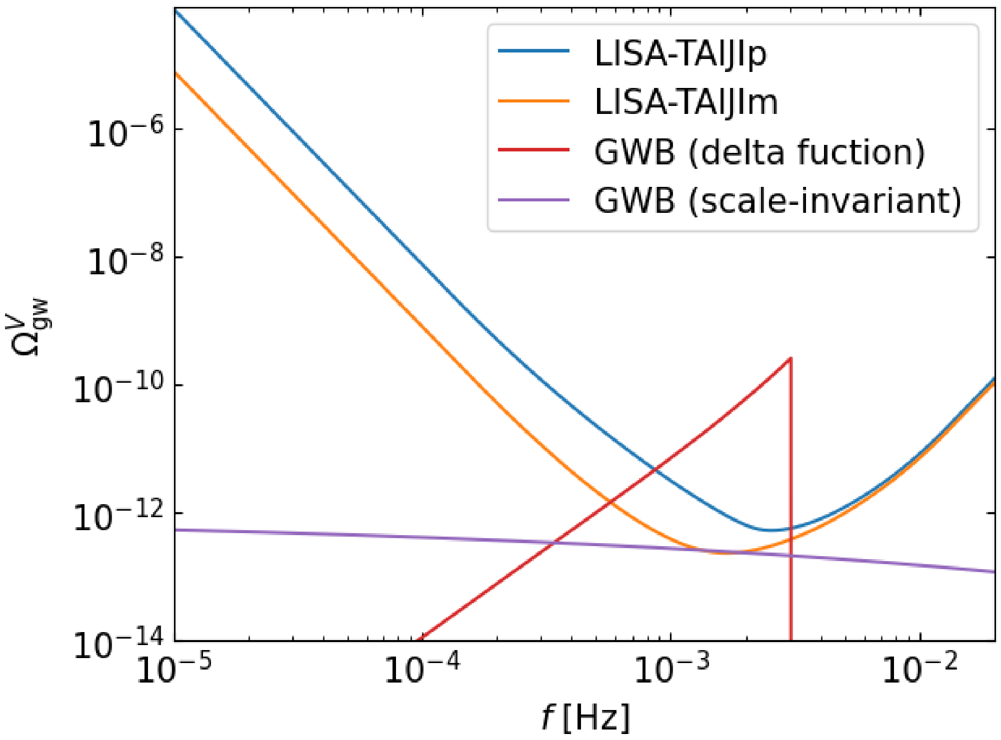
→ **Circ. pol. is sensitive to helicity!!**

Sensitivity

Intensity $\Omega^I_{\rm GW}$ ($\mathcal{B} = 50\text{nG}, r_{\rm H} = 1$)



Circ. Pol. $\Omega^V_{\rm GW}$ ($\mathcal{B} = 50\text{nG}, r_{\rm H} = 1$)



- We need the cross corr. between LISA and TAIJI to observe circular polarization.

- LISA-TAIJIm is the most suitable to see the parity violation

Fisher Forecast

■ Fisher forecast by LISA-TAIJIm

- LISA can only see the intensity...

- Circ. Pol. depends on r_{H}

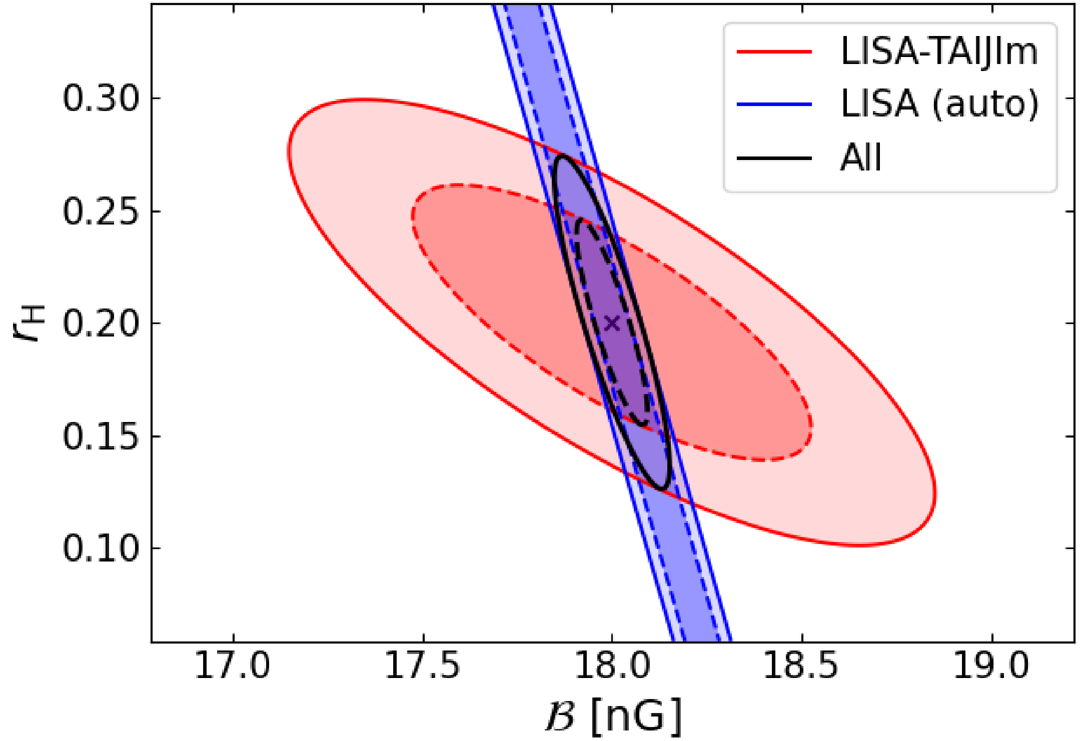
  more strongly than GW intensity

▼

Combining LISA-TAIJI cross corr.

with LISA auto corr., we can identify $\boldsymbol{\mathcal{B}}$ **&** $\boldsymbol{r_{\mathrm{H}}}$!!

Delta-function type ($\mathcal{B} = 18\ \mathrm{nG}, r_{\mathrm{H}} = 0.2$)



Red : LISA-TAIJIm

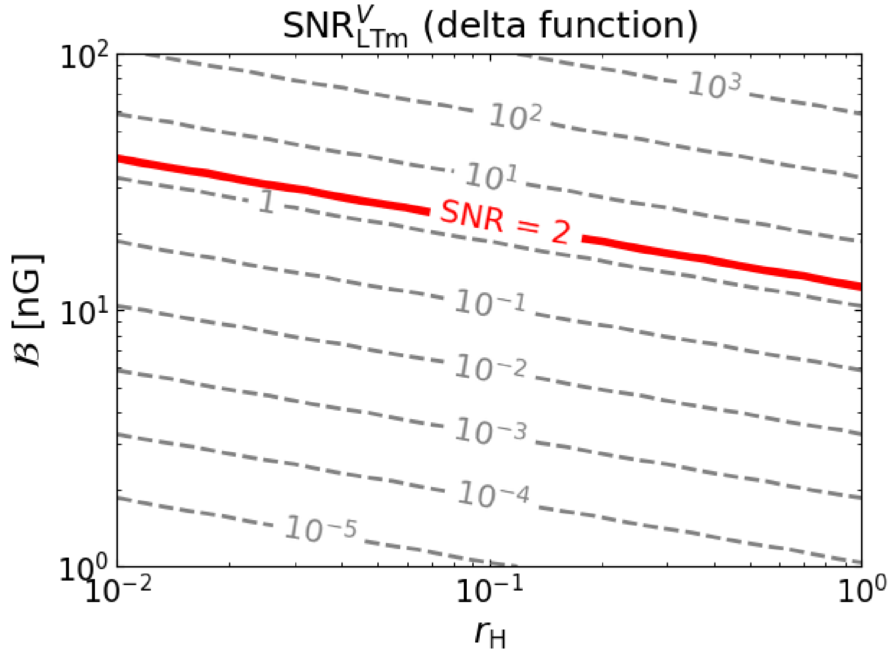
Blue : LISA only

Black : Red+Blue

Signal-to-Noise Ratio

■ SNR for circ. pol. (LISA-TAIJIm)

- delta-function type:

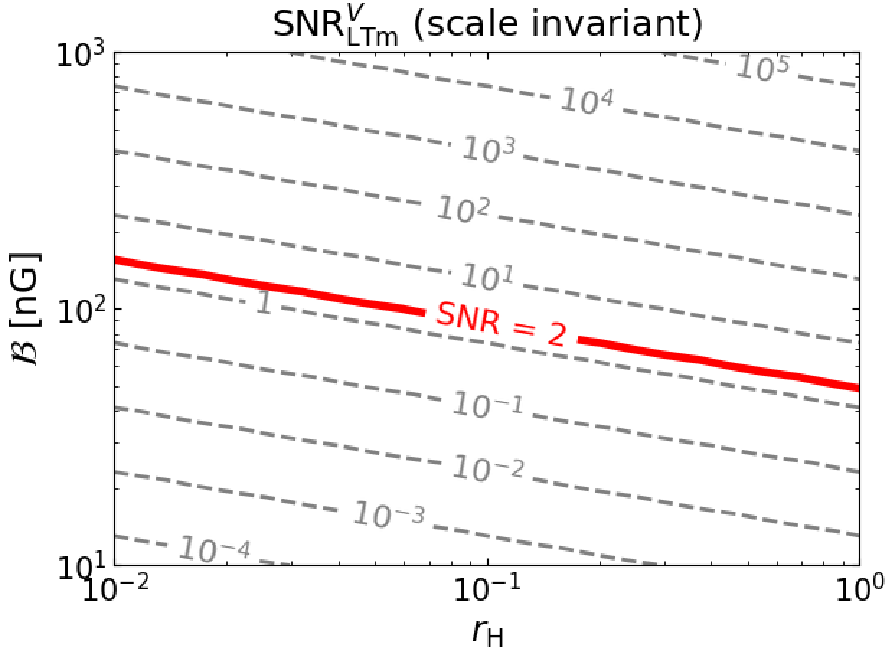


$$\mathrm{SNR}^{V}_{\mathrm{LTm}} \simeq 0.84 \left(\frac{\mathcal{B}}{10\ \mathrm{nG}}\right)^4 r_{\mathrm{H}}$$

▶ If we detect helicity with $\mathrm{SNR}^V > 2$

PMFs needs at least $\mathcal{B} \gtrsim 10$ nG.

- scale-invariant type:



$$\mathrm{SNR}^{V}_{\mathrm{LTm}} \simeq 2.1 \left(\frac{\mathcal{B}}{50\ \mathrm{nG}}\right)^4 r_{\mathrm{H}}$$

▶ If we detect helicity with $\mathrm{SNR}^V > 2$

PMFs needs at least $\mathcal{B} \gtrsim 50$ nG.

Summary

Summary

- **LISA-TAIJI network** has the sensitivity to GW **circular polarization** and it can probe **parity violating signature** in the early universe.
- As an example, we consider **the presence of Helical Primordial magnetic fields (PMFs)** and estimate the GW intensity and circular polarization phenomenologically.

▶ We confirm the LISA-TAIJI network **potentially breaks the degeneracy between $\mathcal{B}$ and r_H.**

▶ To detect helicity, we need PMF amplitude at least
- Delta-function type : $\boldsymbol{\mathcal{B} \gtrsim 10\ \mathrm{nG}}$
- Scale-invariant type : $\boldsymbol{\mathcal{B} \gtrsim 50\ \mathrm{nG}}$

Future work

- Derive the constraint in different frequency range (e.g. PTA, CMB E-B mode)
- Consider the specific scenario of Inflationary magnetogenesis

Stokes Parameter

- **Stokes parameter**

  a set of values that describe the polarization state

$$I = \langle |h_+|^2 + |h_\times|^2 \rangle = \langle |h_\mathrm{L}|^2 + |h_\mathrm{R}|^2 \rangle$$: **Intensity**

$$V = i\langle h_+^* h_\times - h_\times^* h_+ \rangle = \langle |h_\mathrm{L}|^2 - |h_\mathrm{R}|^2 \rangle$$: **Circular polarization**

$$Q = \langle |h_+|^2 - |h_\times|^2 \rangle = \langle h_\mathrm{L}^* h_\mathrm{R} + h_\mathrm{R}^* h_\mathrm{L} \rangle$$

$$U = \langle h_+^* h_\times + h_\times^* h_+ \rangle = i\langle h_\mathrm{L}^* h_\mathrm{R} - h_\mathrm{R}^* h_\mathrm{L} \rangle$$

} : Linear polarization

Considering the ψ rotation around the axis $\hat{n}$,

$$I' = I, V' = V, \quad Q' \pm U' = \mathrm{e}^{\pm 4i\psi}(Q \pm iU)$$

SNR

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Signal to Noise Ratio (SNR)

- Quantity to discuss the detectability of GW signals

$$\mathrm{SNR}_I^2 = \left(\frac{3}{10}\right)^2 T_{\mathrm{obs}} \int \mathrm{d}f [\gamma_{\mathrm{eff}}^I(f)]^2 \frac{I^2(f)}{N^2(f)}$$

$$\mathrm{SNR}_V^2 = \left(\frac{3}{10}\right)^2 T_{\mathrm{obs}} \int \mathrm{d}f [\gamma_{\mathrm{eff}}^V(f)]^2 \frac{V^2(f)}{N^2(f)}$$

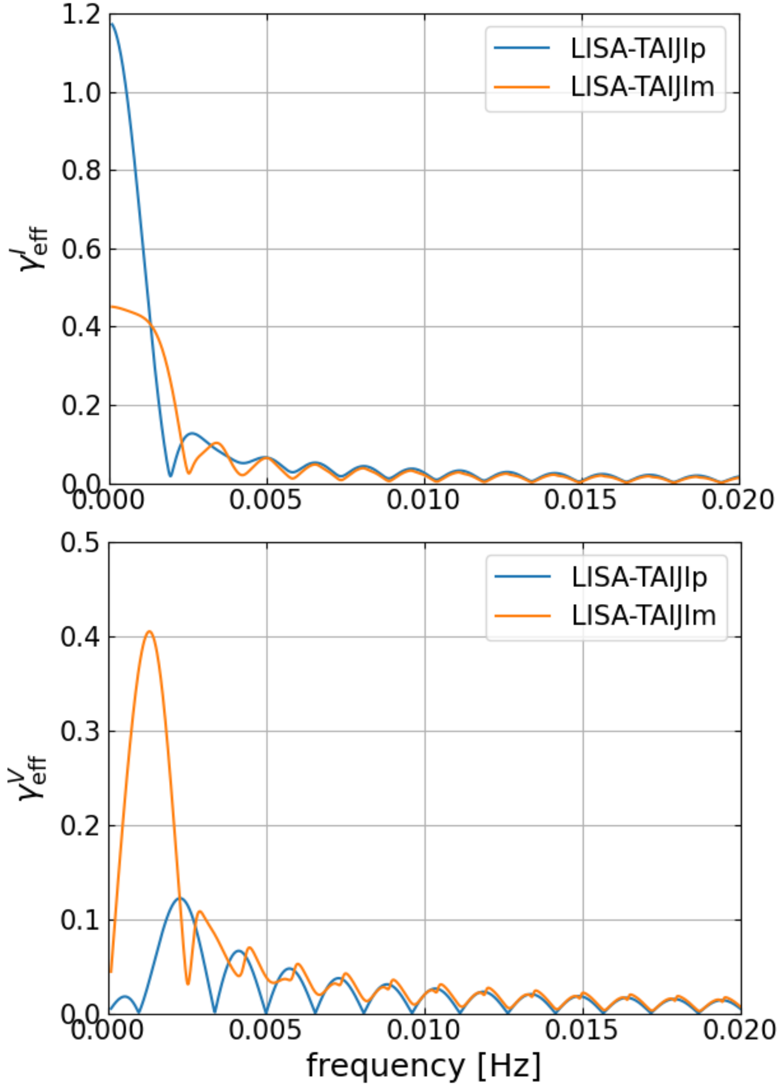
Configulation of Network

- $N(f)$: Noise power spectrum of the network

- $\boldsymbol{\gamma_{\mathrm{eff}}^{I,V}(f)}$: Effective overlap reduction function

→ reflect the **detector's response to GW signals**

Effective overlap reduction function



Overlap reduction function

■ Overlap reduction function (ORF):

$$\gamma^I_{ij}(f) = \frac{5}{2}\int \frac{\mathrm{d}^2\hat{k}}{4\pi}[F_i^+(\hat{k})F_j^+(\hat{k}) + F_i^\times(\hat{k})F_j^+(\hat{k})]e^{-i\hat{k}\cdot\Delta\vec{r}/c},$$

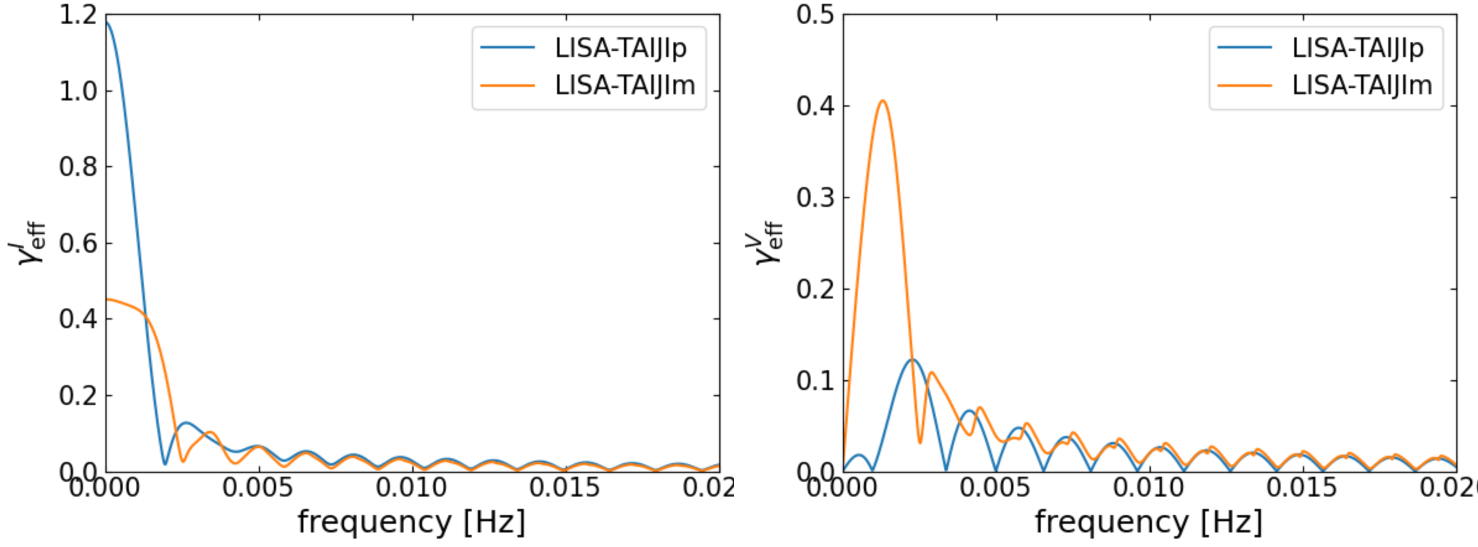
$$\gamma^V_{ij}(f) = \frac{5}{2}\int \frac{\mathrm{d}^2\hat{k}}{4\pi}(-i)[F_i^+(\hat{k})F_j^\times(\hat{k}) - F_i^\times(\hat{k})F_j^+(\hat{k})]e^{-i\hat{k}\cdot\Delta\vec{r}/c}.$$

F_i^A: antenna pattern of i–th detector Δr: separation between two detectors

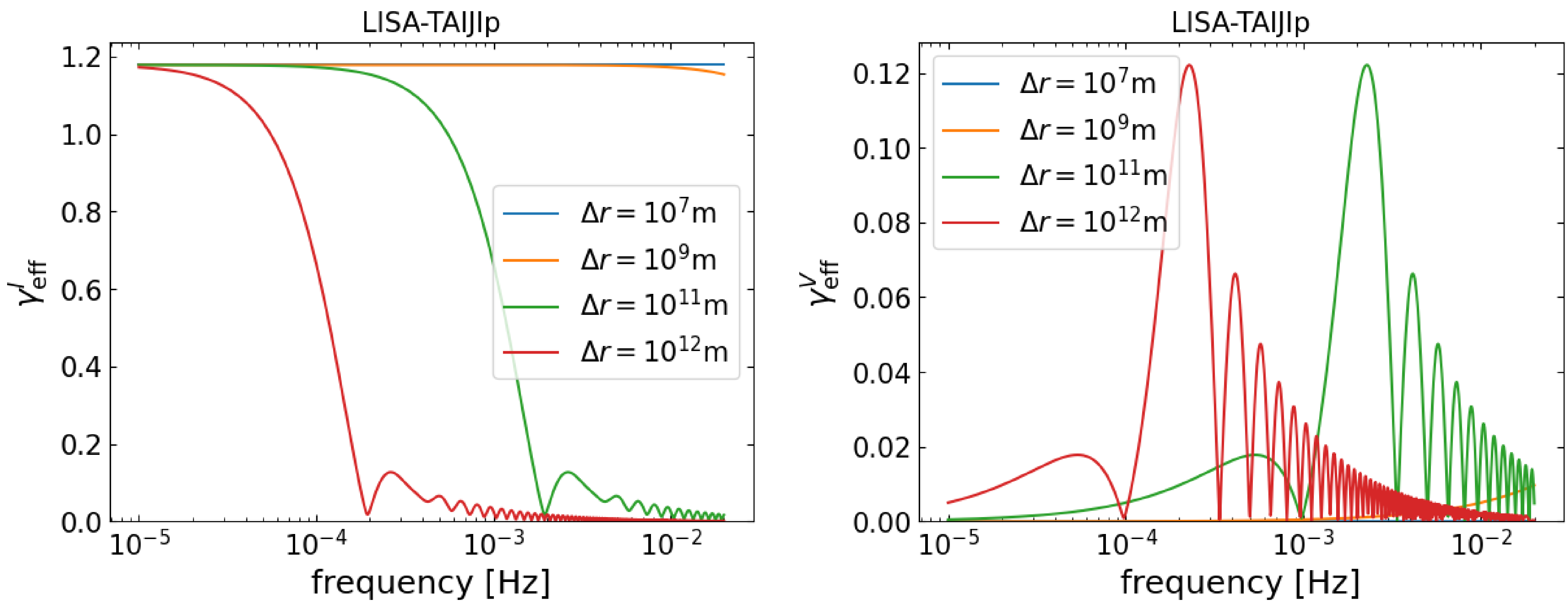
■ Effective ORF:

$$\gamma^I_{\rm eff} = \sqrt{\sum_\kappa(\gamma^I_\kappa)^2 - \frac{(\sum_\kappa \gamma^I_\kappa\gamma^V_\kappa)^2}{\sum_\kappa(\gamma^V_\kappa)^2}},$$

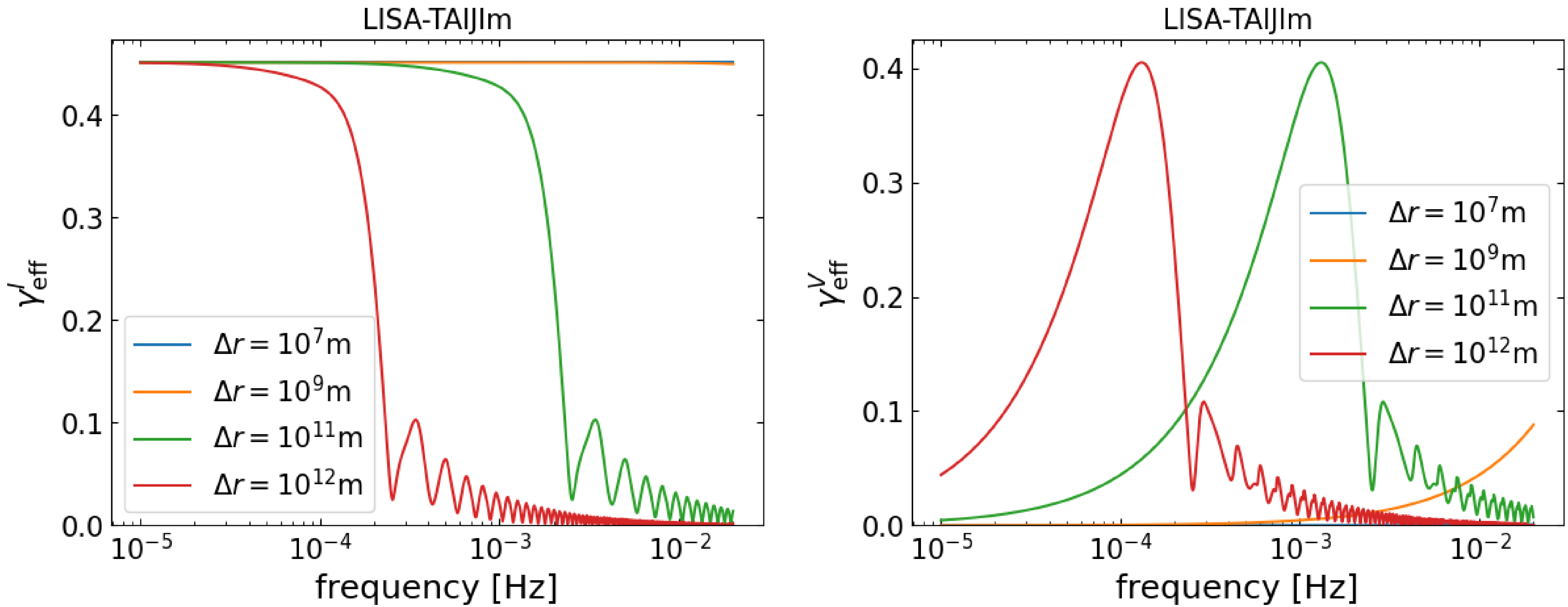
$$\gamma^V_{\rm eff} = \sqrt{\sum_\kappa(\gamma^V_\kappa)^2 - \frac{(\sum_\kappa \gamma^I_\kappa\gamma^V_\kappa)^2}{\sum_\kappa(\gamma^I_\kappa)^2}}.$$



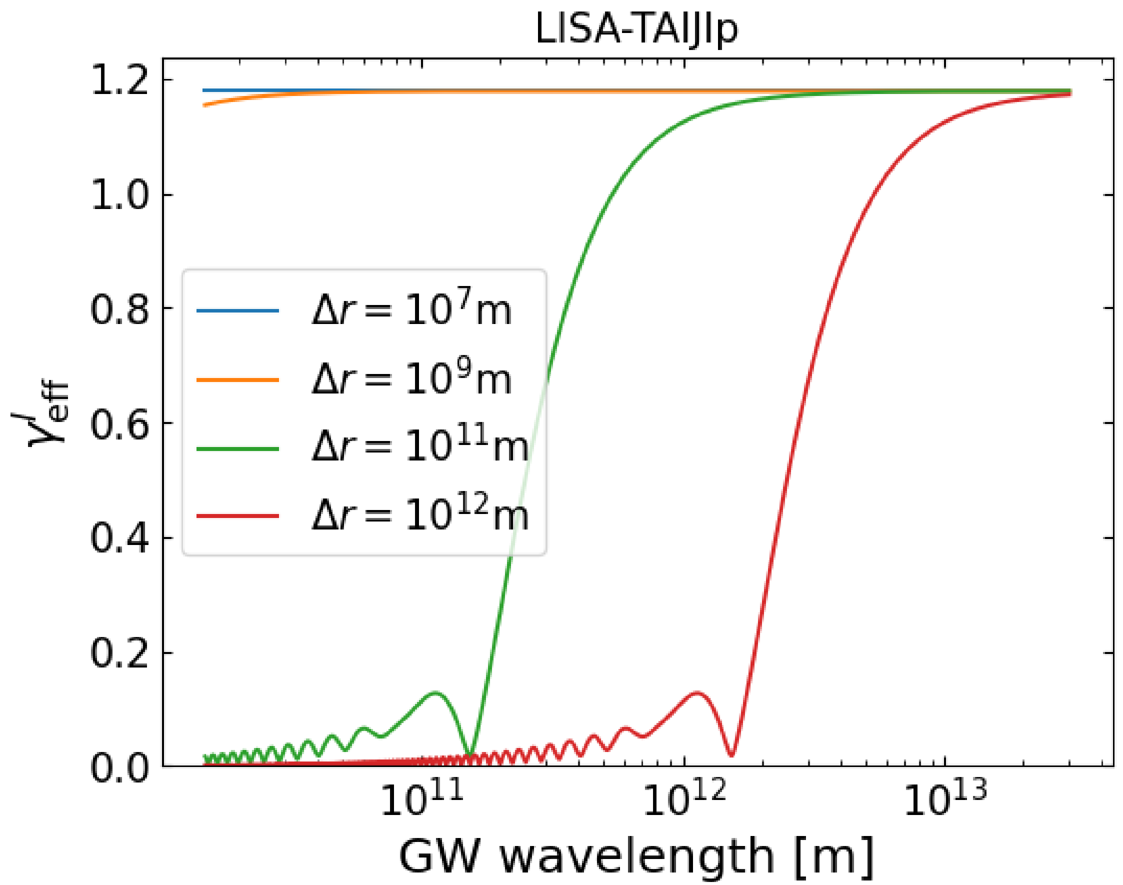
Overlap reduction function

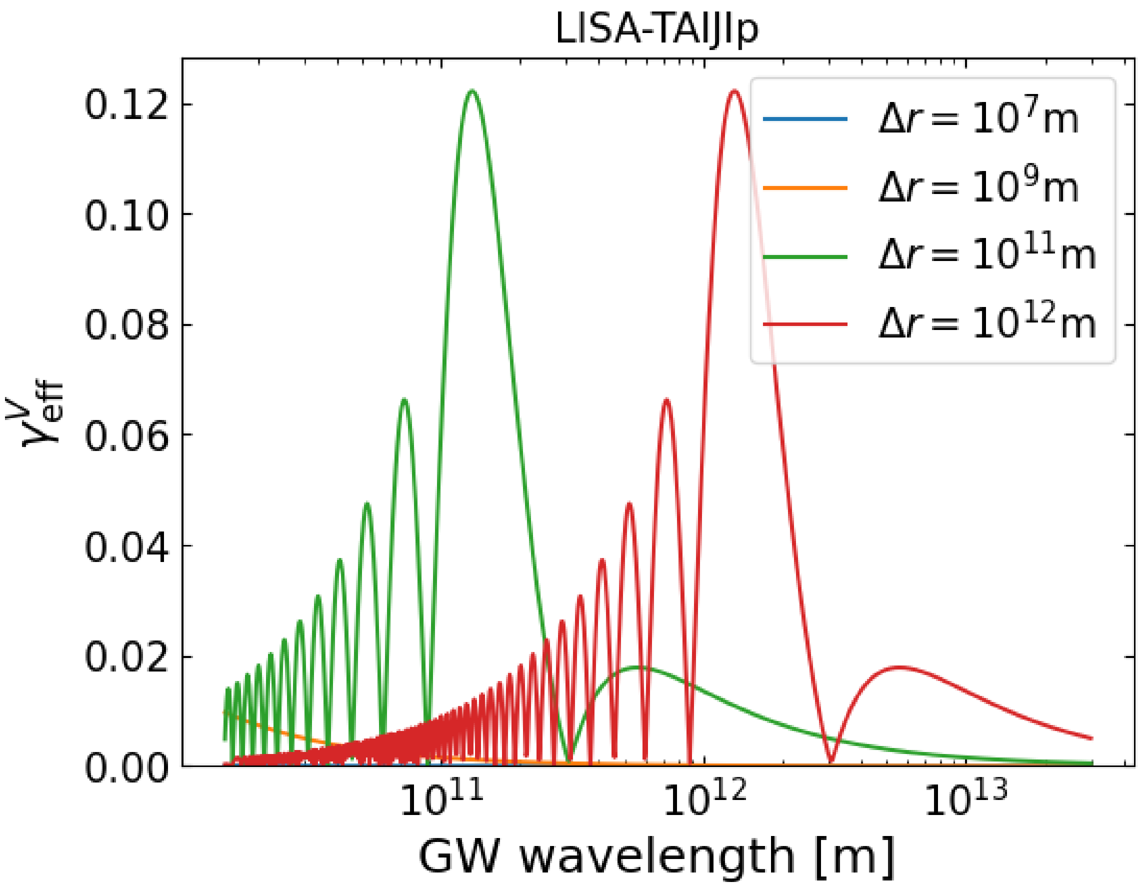


Overlap reduction function



Overlap reduction function



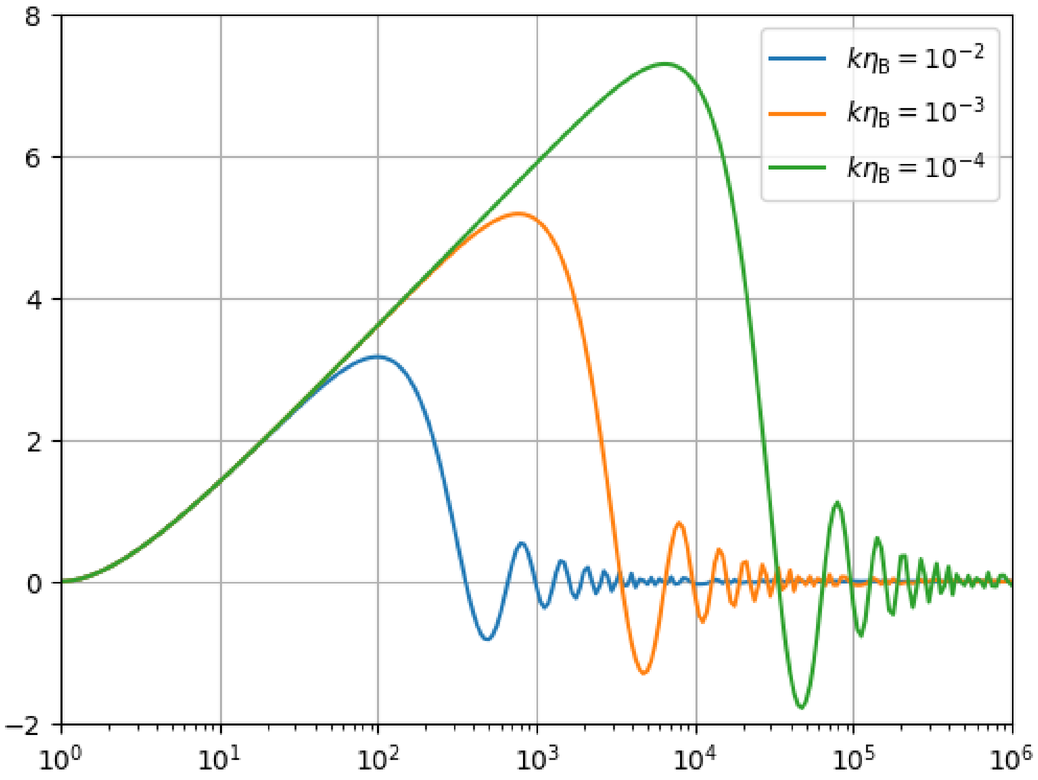


Transfer function

$$h_{ij}(\eta, \boldsymbol{k}) = T_{\rm h}(\eta, \boldsymbol{k}) R_\gamma \pi^{\rm B}_{ij}(\boldsymbol{k})$$

Transfer function

Anisotropic stress



- super-horizon ($k\eta \ll 1$):

$$T_h(\eta, k) \approx \ln\left(\frac{\eta}{\eta_{\rm B}}\right) + \frac{\eta_{\rm B}}{\eta} + 1$$

- sub-horizon ($k\eta \gg 1$):

$$T_h(\eta, k) \approx -\ln(k\eta_{\rm B}) \frac{\sin k\eta}{k\eta}$$

$\propto \eta^{-1} \propto a^{-1}$: adiabatic decay

Previous Work

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Lateralized sleeping positions in domestic cats

Sevim Isparta [1,12] · Sebastian Ocklenburg [2,3,4,12] · Marcello Siniscalchi [1] · … · Nadja Freund [8] · Onur Güntürkün [2,10,13] · Yasemin Salgirli Demirbas [7,11,13] … Show more

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Show Outline

Summary

Both vertebrates and invertebrates show a multitude of left–right asymmetries of brains and behaviors[1]. For example, cats, dogs, and many other species have a preferred paw when handling food[2]. But why should humans and other animals have lateralized brains? Based on a large comparative approach[1], it is likely that asymmetries serve several purposes. First, by specializing on one limb or one side of its sensory system, the contralateral hemisphere goes through life-long cycles of motor and perceptual learning, thereby increasing the speed of processing and motor efficacy, decreasing reaction time, and enhancing discrimination ability. Second, by having two complementary, specialized hemispheres, neural processes are computed in parallel, thereby reducing cognitive redundancy[1]. For example, the right hemisphere excels in processing threat-related stimuli, providing the left visual field an advantage in reacting to a predator approaching from the left[3]. Here, we report that two-thirds of cats prefer a leftward sleeping position, giving their left visual field and thus their right brain half a privileged view of approaching animals without being obstructed by their own body.