Cosmological phase transitions as sources of gravitational waves
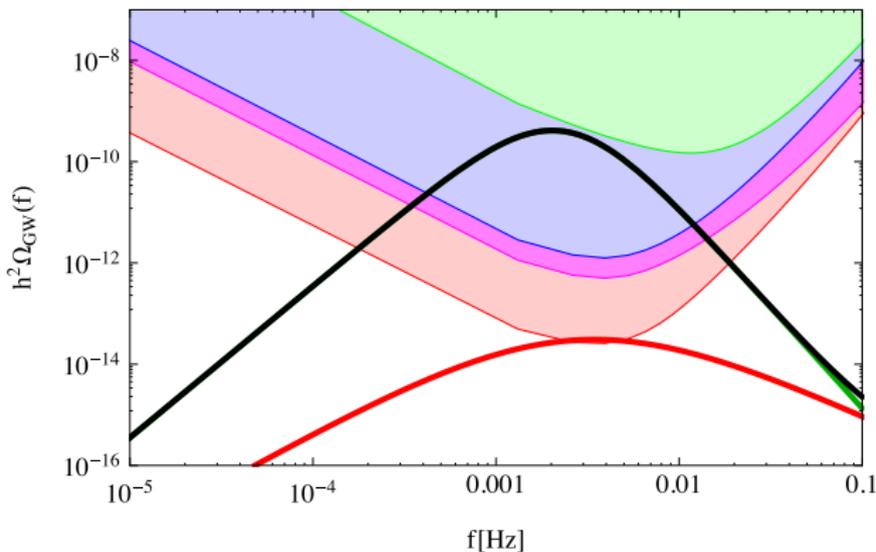
Philipp Schicho

philipp.schicho@unige.ch
pschicho.github.io

Département de Physique Théorique, Université de Genève

Gravitational Waves and the Early Universe:
Accelerated Expansion, Dynamical Inhomogeneity, and Beyond
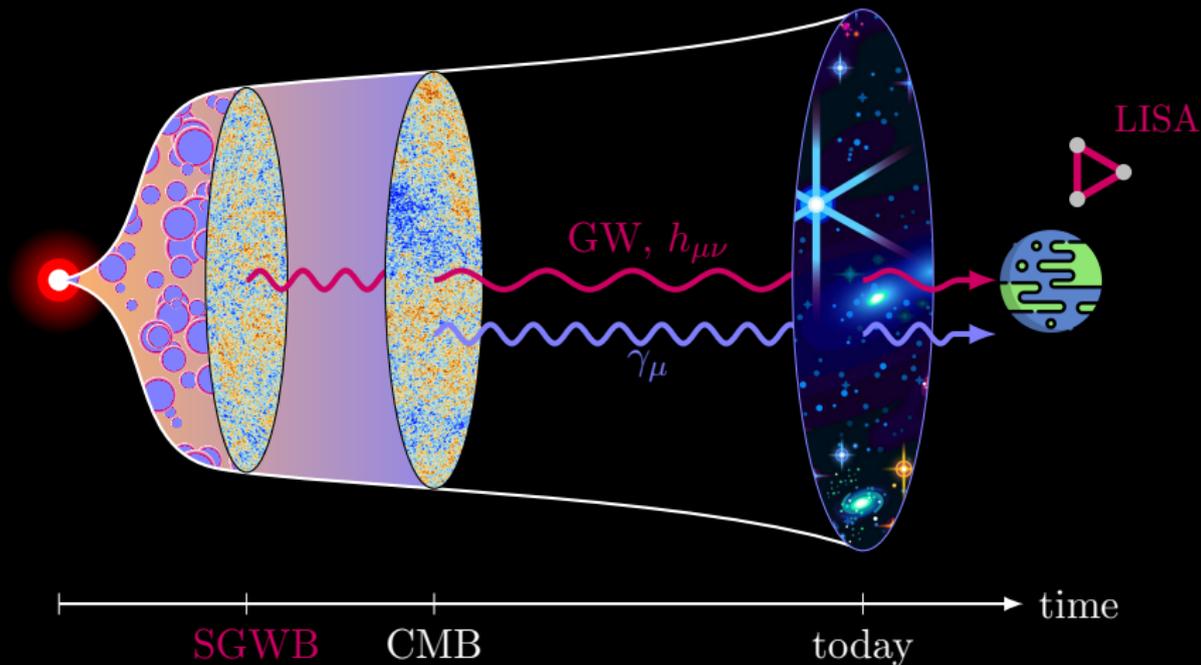素粒子宇宙起源研究所 , KMI, Nagoya, 03/2026

predictions for gravitational waves from cosmological phase transitions in LISA Cosmology Working Group.[1]

Gravitational Waves (GWs) can be

sourced by early-universe phase transitions caused by new physics.
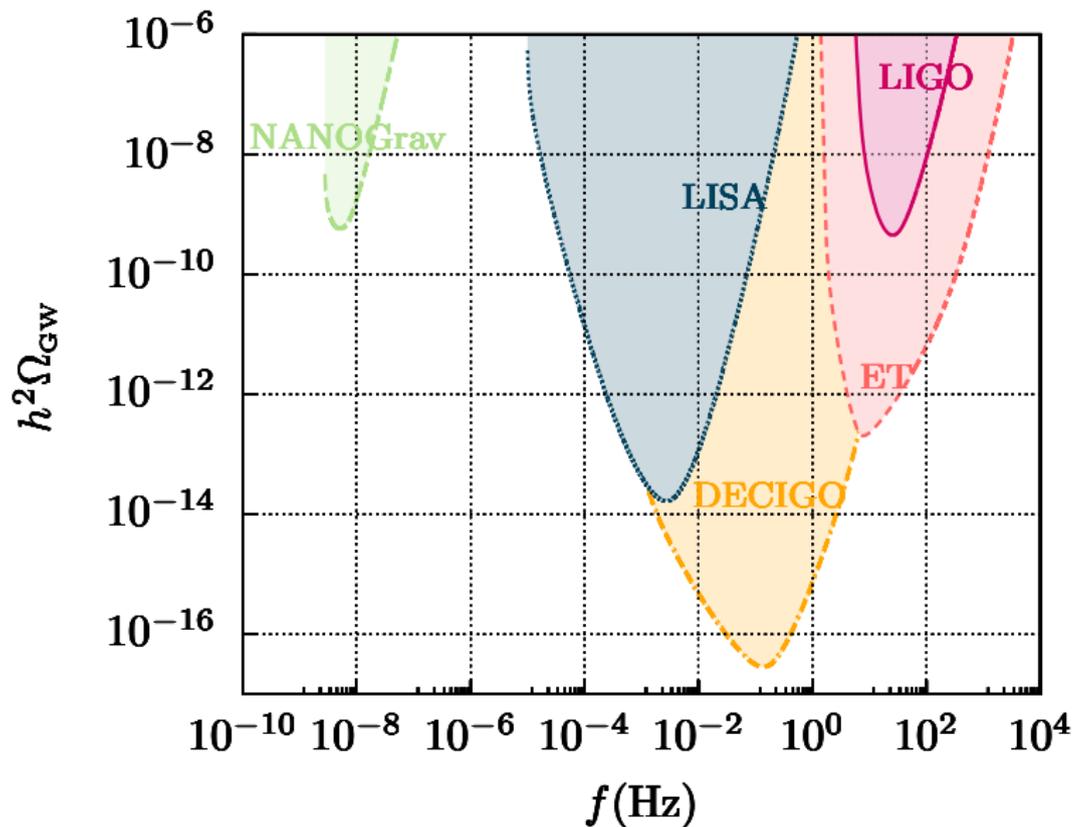
LISA

GW, $h_{\mu\nu}$

γ_μ

time

SGWB   CMB   today

Cosmic Microwave Background (CMB)
Stochastic GW Background (SGWB)
Laser Interferometer Space Antenna (LISA)
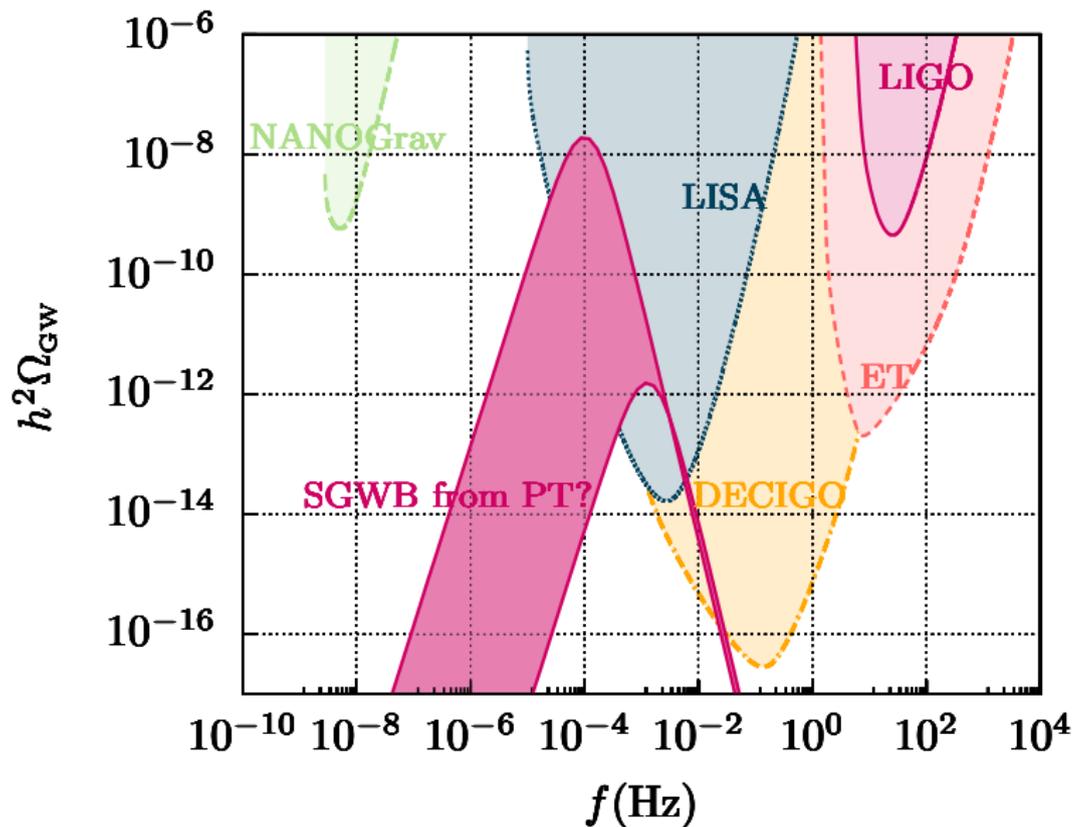
Lessons from the early universe

Does the universe contain a network of cosmic strings, remnant of breaking of a discrete symmetry?[2]
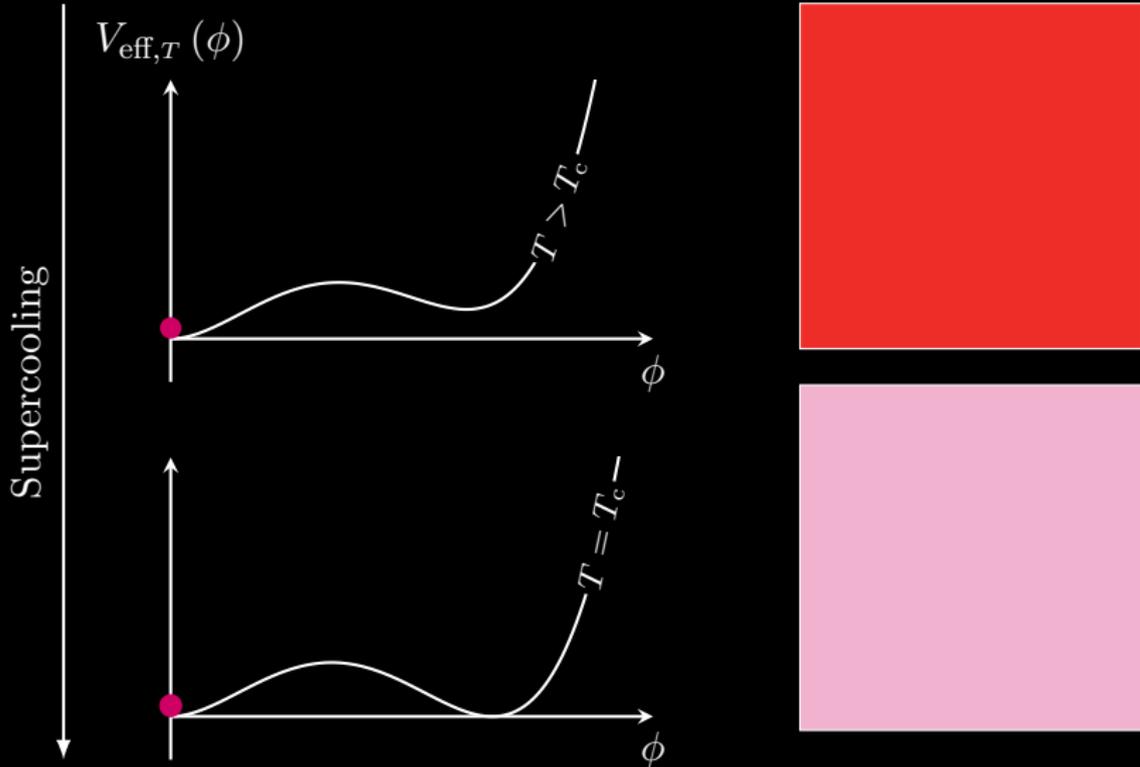
Did the early universe undergo a first order phase transition?

If the matter-antimatter asymmetry was sourced in electroweak baryogenesis, can we observe traces of the generation in GWs?
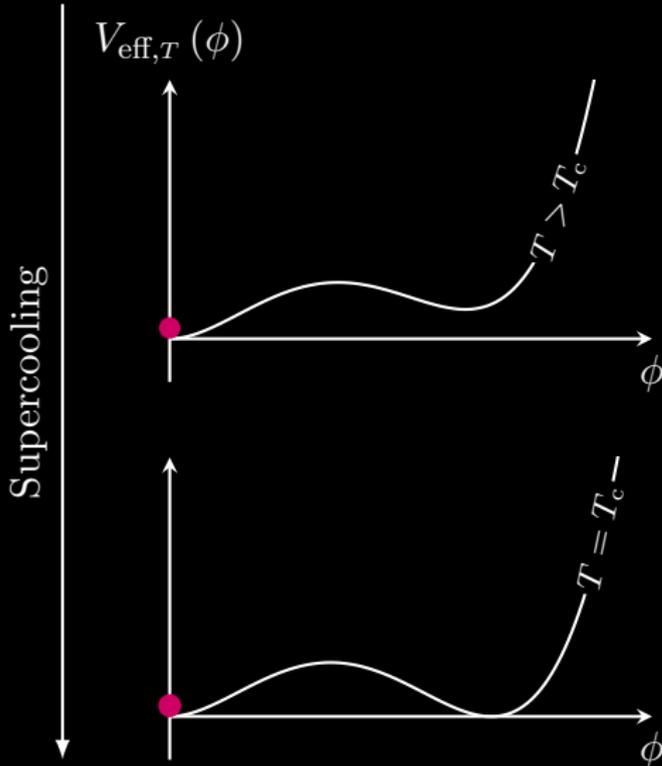
---

[2]See talk by **K. Schmitz** on **Sat 14:50**

Cosmological phase transitions

Cosmological first order phase transition

Cosmological first order phase transition
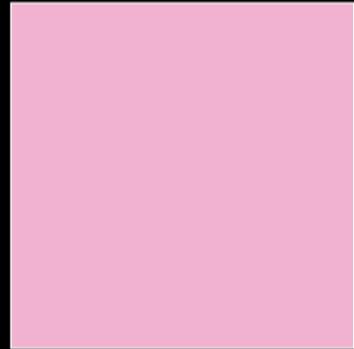

UV [thermal] corrections modify the zero-T potential and restore symmetry.

$$V_{\text{eff},T} \supset \text{⊗} \text{⊗} \text{⊗} \text{⊗} \ldots$$

Cosmological first order phase transition
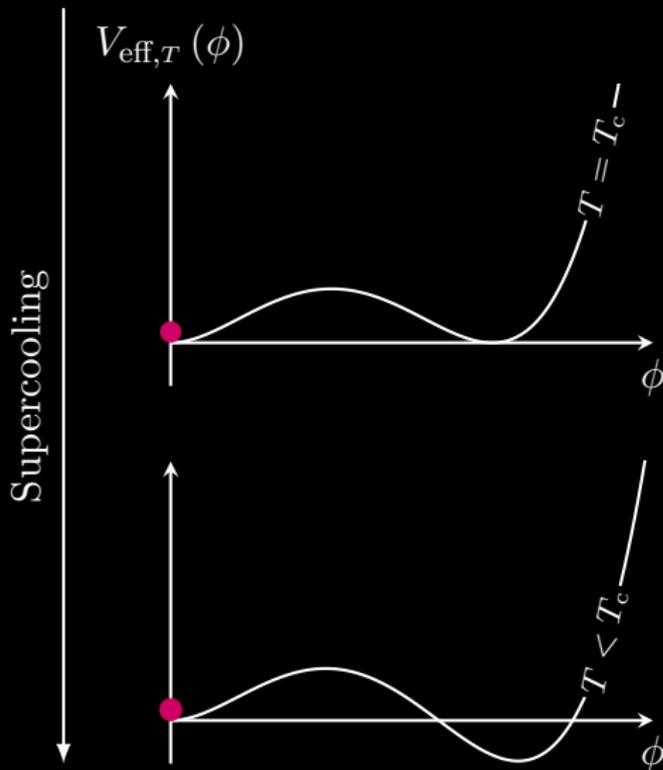
When the universe cools down, the field transitions.

If a barrier separates the phases, the transition is first order.

Supercooling

$V_{\mathrm{eff},T}(\phi)$

$T = T_c$

$T < T_c$

ϕ
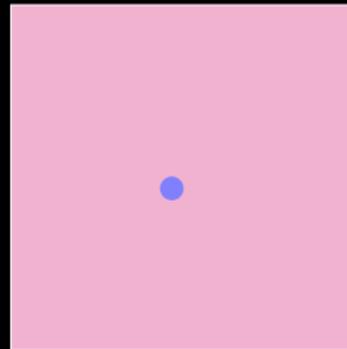
Cosmological first order phase transition
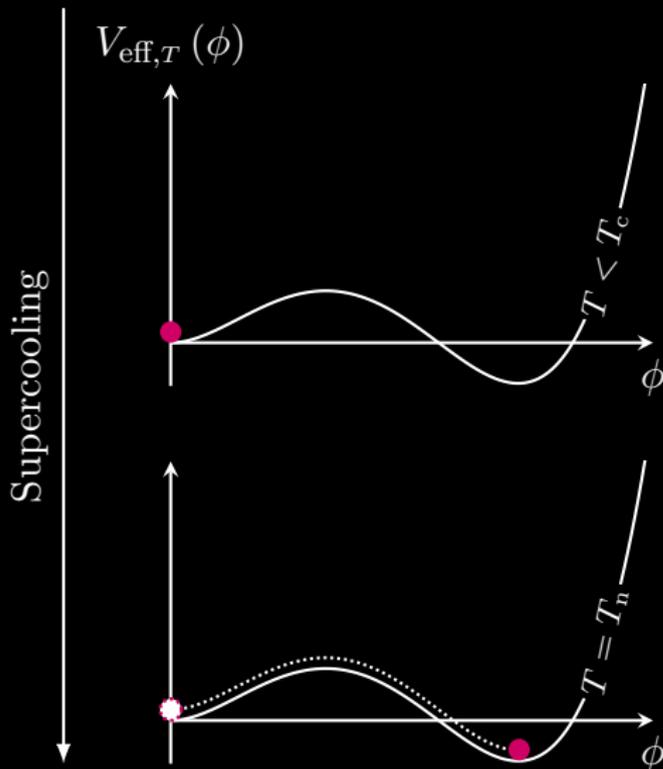
The phase transition proceeds via nucleation of bubbles.

Part of the released energy can be converted into gravitational waves: $\Box h_{ij} = 4\pi\, G\, T_{ij}$
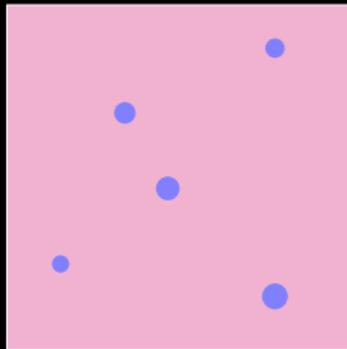
and the sum of independent events (interactions between the bubbles).

The spectral parameters (amplitude, peak frequency, powers in the IR/UV) contain information about the source.

GWs from a cosmological phase transition: New physics?

In Standard Model, EWSB occurs via a smooth crossover[3] but possible that it is first-order in Beyond the Standard Model (BSM) extensions.

[3] Kajantie, Laine, Rummukainen, Shaposhnikov, *Is there a hot electroweak phase transition at $m_H \gtrsim m_W$?*, Phys. Rev. Lett. **77** (1996) 2887 [hep-ph/9605288], D'Onofrio, Rummukainen, *Standard model cross-over on the lattice*, Phys. Rev. D **93** (2016) 025003 [1508.07161]

GWs from a cosmological phase transition: New physics?

In Standard Model, EWSB occurs via a smooth crossover[3] but possible that it is first-order in Beyond the Standard Model (BSM) extensions.

Study **BSM physics** near EW scale in context of phase transitions:

▷ Light fields strongly coupled to Higgs

▷ Collider targets. BSM testing pipeline: Collider phenomenology
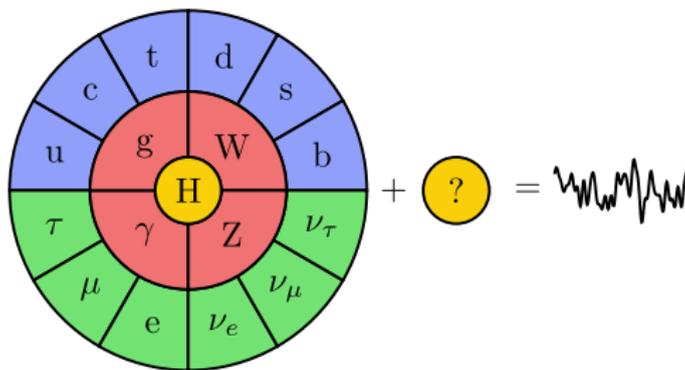
[3] Kajantie, Laine, Rummukainen, Shaposhnikov, *Is there a hot electroweak phase transition at $m_H \gtrsim m_W$?*, Phys. Rev. Lett. **77** (1996) 2887 [hep-ph/9605288], D'Onofrio, Rummukainen, *Standard model cross-over on the lattice*, Phys. Rev. D **93** (2016) 025003 [1508.07161]

Predicting the gravitational wave signal: From micro- to macrophysics

Large dynamic range to resolve Ω_{GW} at both low and high frequency. Reliable GW predictions require control at each scale.
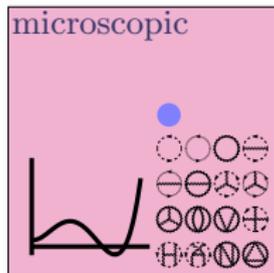
Bubble nucleation
thermodynamics
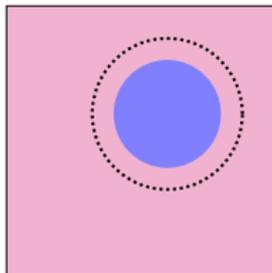
growth
wall-fluid system
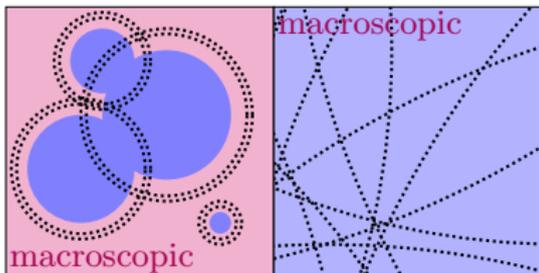
collisions

sound waves
shocks
turbulence

Connecting micro- and macrophysics

Large dynamic range to resolve Ω_{GW} at both low and high frequency. Reliable GW predictions require control at each scale.
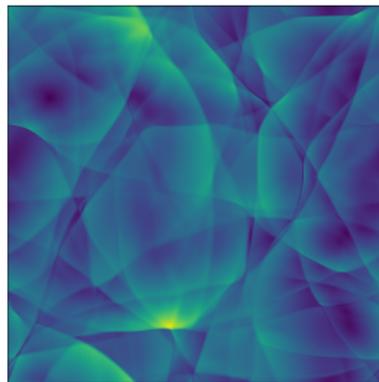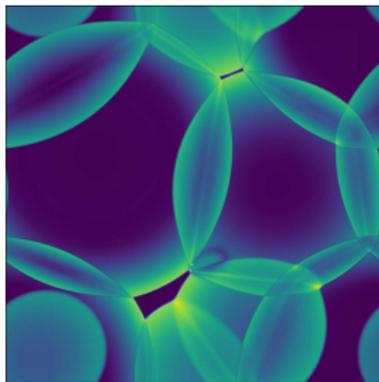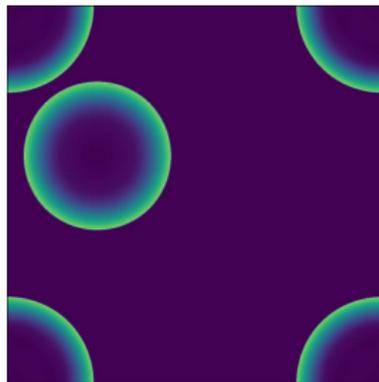
figures by Cutting, Hindmarsh, Weir, *Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions*, Phys. Rev. Lett. **125** (2020) 021302 [1906.00480]

Connecting micro- and macrophysics

Large dynamic range to resolve Ω_{GW} at both low and high frequency.
Reliable GW predictions require control at each scale.

Bubble nucleation
thermodynamics

growth
wall-fluid system

collisions

sound waves
shocks
turbulence
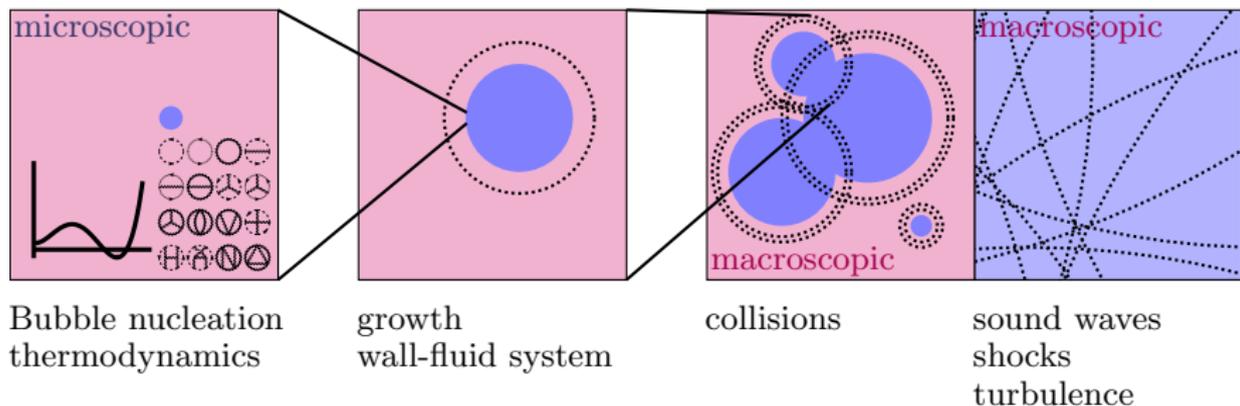
Strategy:
Effective field theory (EFT) to connect micro- and macrophysics.

Microscopic scales
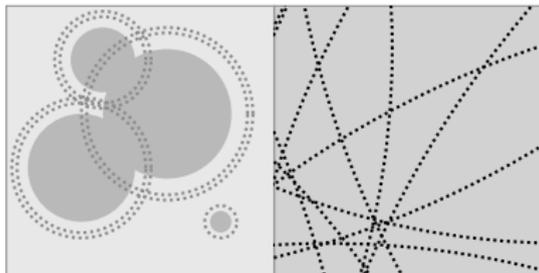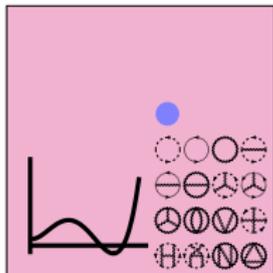
using a few equilibrium parameters:

$T_\star$ reference temperature of the transition ($T_\star = T_{\mathrm{nucl}}, T_{\mathrm{p}}$),

α phase transition strength,

β/H inverse duration of the transition (also $r_\star$),

c_s speed of sound in the plasma

How robust are theoretical predictions?

$\mathcal{O}(10^4)$ uncertainty even for purely perturbative regimes[4] as Ω_{GW} depends strongly on the transition temperature, $T_\star$, in simulation fits:

$$\Omega_{\mathrm{GW}} \propto \frac{(\Delta V_\star)^2}{T_\star^8} .$$

Vary RG scale $\bar{\mu}$ in SM extensions:

▷ SMEFT:[5] SM + $\frac{1}{M^2}(\phi^\dagger\phi)^3$

▷ xSM:[6] SM + singlet

▷ BSM models in general

---
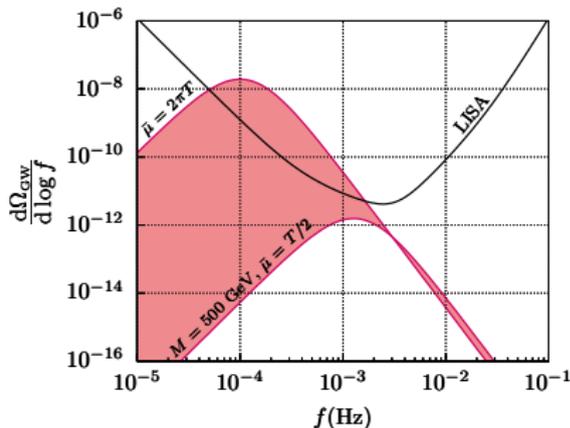
[4] Croon, Gould, Schicho, Tenkanen, White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080]

[5] Chala, Fiore, Gil, *Hot news on the phase-structure of the SMEFT,* [2507.16905]

[6] Gould, Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

How robust are theoretical predictions?

$\mathcal{O}(10^4)$ uncertainty even for purely perturbative regimes[4] as Ω_{GW} depends strongly on the transition temperature, $T_\star$, in simulation fits:

$$\Omega_{\mathrm{GW}} \propto \frac{(\Delta V_\star)^2}{T_\star^8} \, .$$

Vary RG scale $\bar{\mu}$ in SM extensions:

▷ SMEFT:[5]   SM $+ \frac{1}{M^2}(\phi^\dagger \phi)^3$

▷ xSM:[6]   SM + singlet

▷ BSM models in general
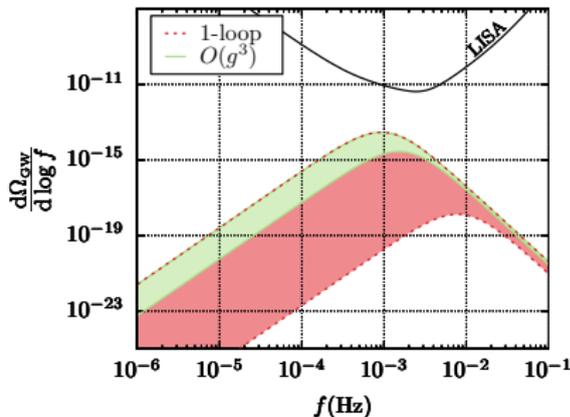
[4] Croon, Gould, Schicho, Tenkanen, White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080]

[5] Chala, Fiore, Gil, *Hot news on the phase-structure of the SMEFT,* [2507.16905]

[6] Gould, Tenkanen, *On the perturbative expansion at high temperature and implications for cosmological phase transitions*, JHEP **06** (2021) 069 [2104.04399]

Perturbative phase transitions need scale hierarchies

for quantum effects ΔV_{fluct} to influence the tree-level potential

$$V_{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}} \ .$$

Relevant operators ($\sigma > 0$) in the IR get large UV contributions[7] and

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 \left[\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}}\right]^{\sigma} \overset{!}{\sim} 1 \quad \Rightarrow \begin{cases} \text{strong coupling} & g^2 \gtrsim 1 \\ \text{scale hierarchy} & \frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \sim \left[\frac{1}{g^2}\right]^{\frac{1}{\sigma}} \gg 1 \end{cases}$$

How to reorganize perturbation theory for large ΔV_{fluct}?

---

[7] Consider e.g. hierarchy: $M_\chi \gg m_\Phi$, $(\Delta m_\Phi^2)\Phi^\dagger\Phi = \underbrace{} \sim g^2 M_\chi^2 \Phi^\dagger\Phi$, $(\Delta m_\Phi^2)/m_\Phi^2 = g^2 \left[M_\chi/m_\Phi\right]^2$.

Thermodynamics $\mathcal{Z} = \text{Tr}\, e^{-\beta \mathcal{H}}$ formulated in $\mathbb{R}^3 \times S^1_\beta$.

$$\underbrace{\int_{\mathbb{R}^3 \times S^1_\beta} \mathcal{L}}_{\text{bosons + fermions}} \rightarrow \underbrace{\int_{\mathbb{R}^3} \mathcal{L}_{\text{eff}}}_{\text{bosons only}} \ .$$
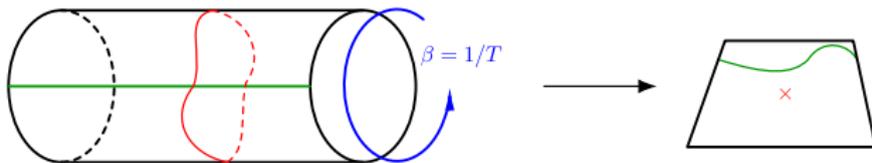
In 3d EFT, modified canonical mass dimensions render marginal operators relevant, $[A_0^a] = [g_{3,abc}] = 1/2$: **superrenormalizable** theory.

$$\int_{\mathbb{R}^3} \mathcal{L}_{\text{eff}} \supset \left[\frac{1}{2} \partial_i A_0^a \partial_i A_0^b - g_{3,abc} A_i^a A_0^b \partial_i A_0^c + \frac{m_{3,ab}^2}{2} A_0^a A_0^b \right] .$$

[8] Braaten, Nieto, *Effective field theory approach to high temperature thermodynamics,* Phys. Rev. D **51** (1995) 6990 [hep-ph/9501375],

Ekstedt, Schicho, Tenkanen, DRalgo: *A package for effective field theory approach for thermal phase transitions,* Comput. Phys. Commun. **288** (2023) 108725 [2205.08815]

Powercounting the SM-like $3d$ EFT (SU(2)+Higgs)

Describes the thermodynamics[9] of several parent $4d$ theories:

$$\mathcal{L}_{3d}^{\text{softer}} = \frac{1}{4}F_{ij}^a F_{ij}^a + (D_i\Phi)^\dagger(D_i\Phi) + V(\Phi) ,$$

$$V(\Phi) = m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 .$$

If $m_{A_i} \sim g_3\phi \gg m_3$, integrating out vector boson introduces LO barrier

$$V_{\text{LO}}(\Phi) = \quad \bullet \quad + \quad \big\{\;\big\} = m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 - \frac{g_3^3}{2\pi}\Big(\frac{\Phi^\dagger\Phi}{2}\Big)^{3/2} .$$

---

[9] Kajantie, Laine, Rummukainen, Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

[10] Gould, Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions,* JHEP **01** (2024) 048 [2309.01672], cf. talk by **L. Gil** on **Thu 11:10**

Powercounting the SM-like $3d$ EFT ($SU(2)$+Higgs)

Describes the thermodynamics[9] of several parent $4d$ theories:

$$\mathcal{L}_{3d}^{\text{softer}} = \frac{1}{4}F_{ij}^a F_{ij}^a + (D_i\Phi)^\dagger(D_i\Phi) + V(\Phi) \ ,$$

$$V(\Phi) = m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 \ .$$

If $m_{A_i} \sim g_3\phi \gg m_3$, integrating out vector boson introduces LO barrier

$$V_{\text{LO}}(\Phi) \to y\Phi^\dagger\Phi + x(\Phi^\dagger\Phi)^2 - \frac{1}{2\pi}\left(\frac{\Phi^\dagger\Phi}{2}\right)^{3/2} \ .$$

Since $x \sim \frac{m_3^2}{m_{A_i}^2} \ll 1$, and at the phase transition $y \sim 1/x$, we strictly[10] x-pand the perturbative series using 3d EFT dimensionless couplings

$$x \equiv \frac{\lambda_3}{g_3^2} \ , \qquad\qquad y \equiv \frac{m_3^2}{g_3^4} \ .$$

---

[9] Kajantie, Laine, Rummukainen, Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

[10] Gould, Tenkanen, *Perturbative effective field theory expansions for cosmological phase transitions,* JHEP **01** (2024) 048 [2309.01672], cf. talk by **L. Gil** on Thu 11:10

The electroweak phase diagram

One-loop

$$V_{\text{eff},T} = V_0 + $$

$$= \frac{1}{2} m_3^2(T)\phi^2 + \frac{1}{4}\lambda_3(T)\phi^4$$

The electroweak phase diagram

One-loop

$$V_{\text{eff},T} = V_0 + \cdots$$
$$= \frac{1}{2} m_3^2(T)\phi^2 + \frac{1}{4}\lambda_3(T)\phi^4$$

Leading-order resummations

$$V_{\text{eff},T} = V_0 + \cdots + \cdots$$
$$= \frac{1}{2}m_3^2\phi^2 + \frac{1}{4}\lambda_3\phi^4 - \frac{g_3^3}{16\pi}\phi^3$$

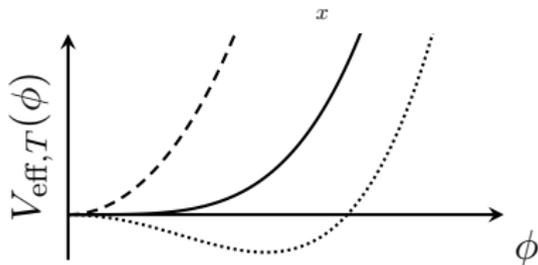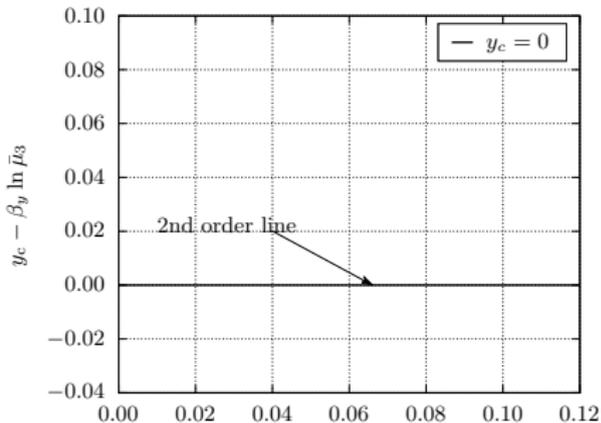An EFT x-pansion[11] in

$$x = \lambda_3/g_3^2.$$

---
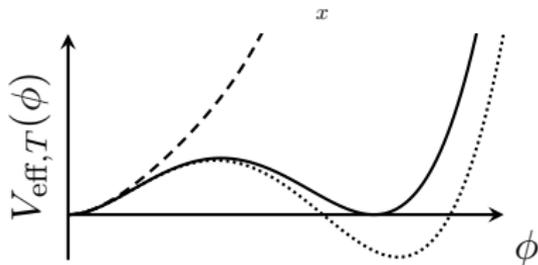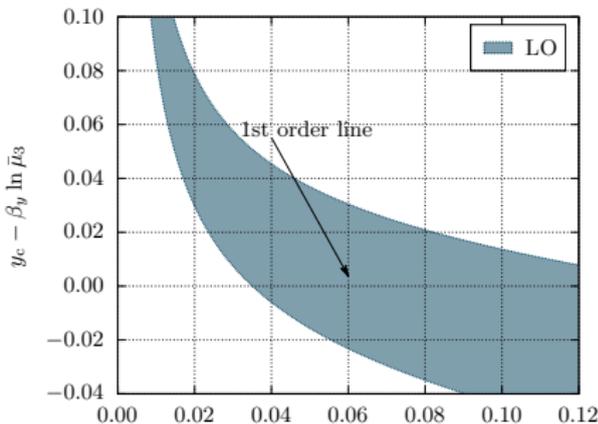
[11] Kajantie, Laine, Rummukainen, Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

The electroweak phase diagram

Infrared breakdown:

$$V_{\text{eff}} = ?$$

All higher loops $\subset$ N^{5}LO. Resolve with lattice Monte-Carlo.[12]
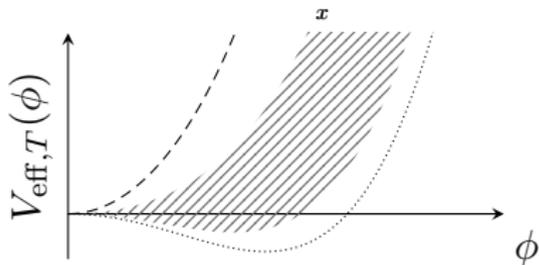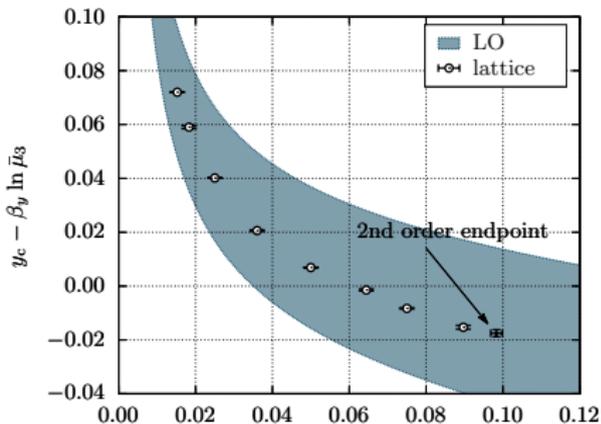
[12] Farakos, Kajantie, Rummukainen, Shaposhnikov, *3-d physics and the electroweak phase transition: A Framework for lattice Monte Carlo analysis,* Nucl. Phys. B **442** (1995) 317 [hep-lat/9412091], Kajantie, Laine, Rummukainen, Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], Gould, Güyer, Rummukainen, *First-order electroweak phase transitions: A nonperturbative update,* Phys. Rev. D **106** (2022) 114507 [2205.07238]

The electroweak phase diagram

Infrared breakdown:

$$V_{\text{eff}} = ?$$

All higher loops $\subset$ N^{5}LO. Resolve with lattice Monte-Carlo.[12]

Resummations at

  ▷ NLO[13]

  ▷ N^{2}LO[14]

  ▷ N^{4}LO[15]
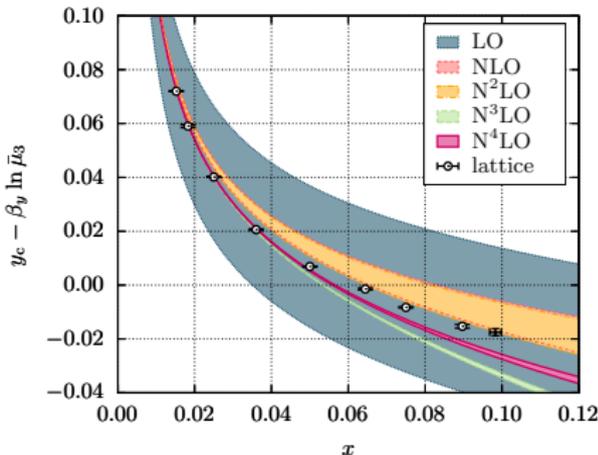
[13] Arnold, Espinosa, *Effective potential and first-order phase transitions: Beyond leading order,* Phys. Rev. D **47** (1993) 3546 [9212235]

[14] Ekstedt, Gould, Löfgren, *Radiative first-order phase transitions to next-to-next-to-leading order,* Phys. Rev. D **106** (2022) 036012 [2205.07241]

[15] Ekstedt, Schicho, Tenkanen, *Cosmological phase transitions at three loops: The final verdict on perturbation theory,* Phys. Rev. D **110** (2024) 096006 [2405.18349]

Order of the electroweak phase transition

Leading order (LO):[16]

1st order, $\quad V_{\text{eff},T} = V_0 +$

Next-to-leading order (NLO):[17]

2nd order, $\quad V_{\text{eff},T} = V_0 +$

---
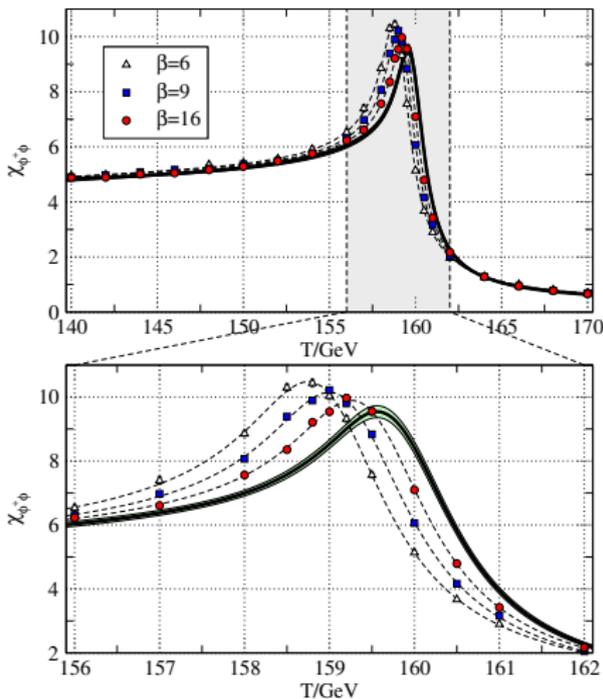
[16] Dolan, Jackiw, *Symmetry behavior at finite temperature,* Phys. Rev. D **9** (1974) 3320,

[17] Arnold, Espinosa, *Effective potential and first-order phase transitions: Beyond leading order,* Phys. Rev. D **47** (1993) 3546 [9212235]

Order of the electroweak phase transition

Higher-order infrared problems:[18]

     (?) order

EFT + lattice resolves all issues:[19]

   crossover transition

Accurate SM thermodynamics[20]

---
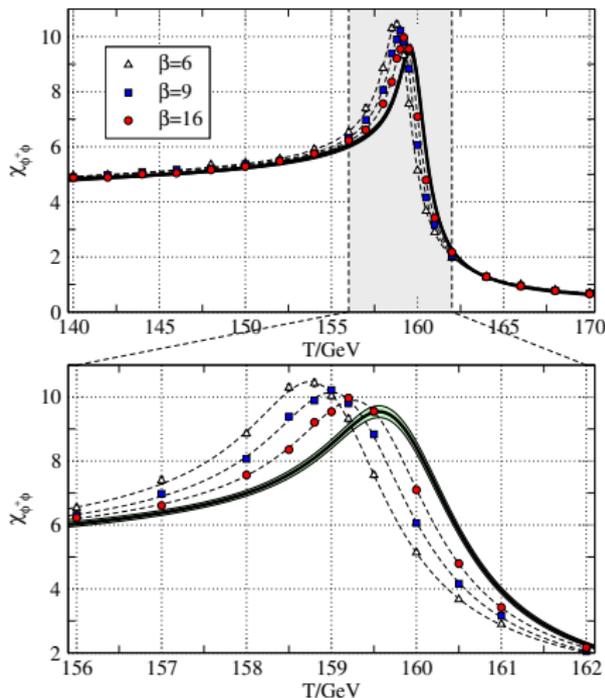
[18] Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas,* Phys. Lett. B **96** (1980) 289

[19] Kajantie, Laine, Rummukainen, Shaposhnikov, *Is there a hot electroweak phase transition at $m_H \gtrsim m_W$?,* Phys. Rev. Lett. **77** (1996) 2887 [hep-ph/9605288]

[20] D'Onofrio, Rummukainen, *Standard model cross-over on the lattice,* Phys. Rev. D **93** (2016) 025003 [1508.07161]

Perturbation theory and lattice agree remarkably

at the last perturbative thermodynamic order (for SU(2) + Higgs).
By using $F \sim V_{\text{eff}}(\phi_{\min})$, determine the *critical mass* y_c (or T_c)

$$\Delta F(y_c(x), x) = [F_{\text{bro}} - F_{\text{sym}}](y_c(x), x) = 0\,,$$

and scalar *condensates*[21] $\Delta\langle\Phi^\dagger\Phi\rangle \equiv \partial_y \Delta F$ and $\Delta\langle(\Phi^\dagger\Phi)^2\rangle \equiv \partial_x \Delta F$.

[21]Lattice data: Kajantie, Laine, Rummukainen, Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], Gould, Güyer, Rummukainen, *First-order electroweak phase transitions: A nonperturbative update,* Phys. Rev. D **106** (2022) 114507 [2205.07238]

Perturbation theory and lattice agree remarkably

at the last perturbative thermodynamic order (for SU(2) + Higgs).
By using $F \sim V_{\text{eff}}(\phi_{\min})$, determine the *critical mass* y_c (or T_c)

$$\Delta F(y_c(x), x) = [F_{\text{bro}} - F_{\text{sym}}](y_c(x), x) = 0,$$

and scalar *condensates*[21] $\Delta\langle\Phi^\dagger\Phi\rangle \equiv \partial_y\Delta F$ and $\Delta\langle(\Phi^\dagger\Phi)^2\rangle \equiv \partial_x\Delta F$.
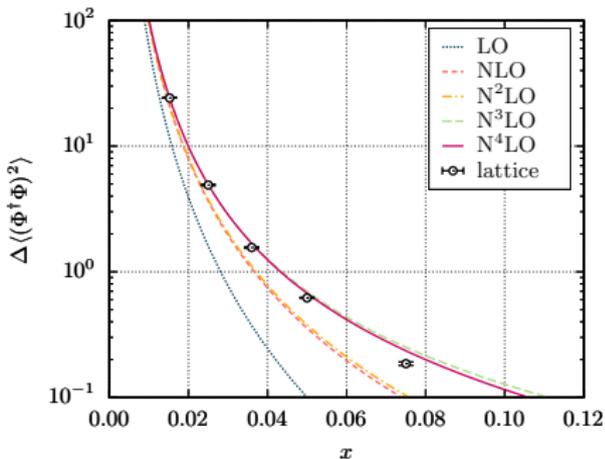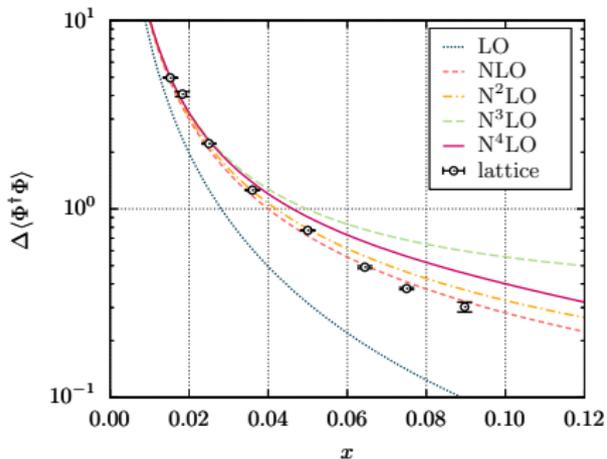
[21]Lattice data: Kajantie, Laine, Rummukainen, Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], Gould, Güyer, Rummukainen, *First-order electroweak phase transitions: A nonperturbative update,* Phys. Rev. D **106** (2022) 114507 [2205.07238]

Perturbation theory and lattice agree remarkably

at the last perturbative thermodynamic order (for U(1) + Higgs). By using $F \sim V_{\text{eff}}(\phi_{\min})$, determine the *critical mass* y_c (or T_c)

$$\Delta F(y_c(x), x) = [F_{\text{bro}} - F_{\text{sym}}](y_c(x), x) = 0\,,$$

and scalar *condensates*[21] $\Delta\langle\Phi^\dagger\Phi\rangle \equiv \partial_y\Delta F$ and $\Delta\langle(\Phi^\dagger\Phi)^2\rangle \equiv \partial_x\Delta F$.
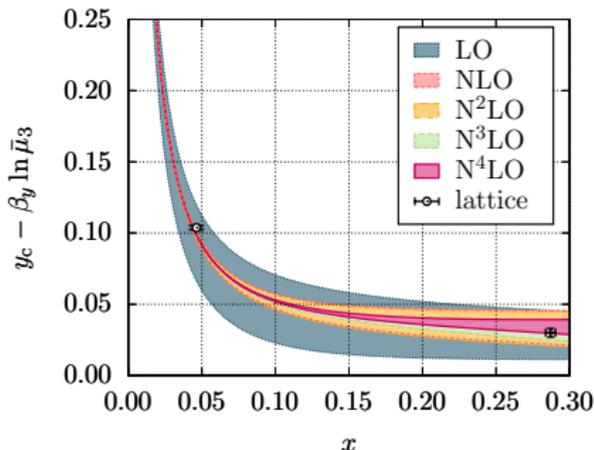
[21] Lattice data: Kajantie, Karjalainen, Laine, Peisa, *Three-dimensional U(1) gauge + Higgs theory as an effective theory for finite temperature phase transitions,* Nucl. Phys. B **520** (1998) 345 [hep-lat/9711048], Mo, Hove, Sudbo, *The Order of the metal to superconductor transition,* Phys. Rev. B **65** (2002) 104501 [cond-mat/0109260]

Beyond the SM and evading the rhombus of misery

$\alpha/(\beta/H)$ rhombus of SM-like EFT with no prospect for large SNR.[22]
Better access interesting LISA SNR by increasing loop order:

BM-A xSM with weakly portal-coupled singlet (decoupled)
BM-B xSM with strongly portal-coupled singlet
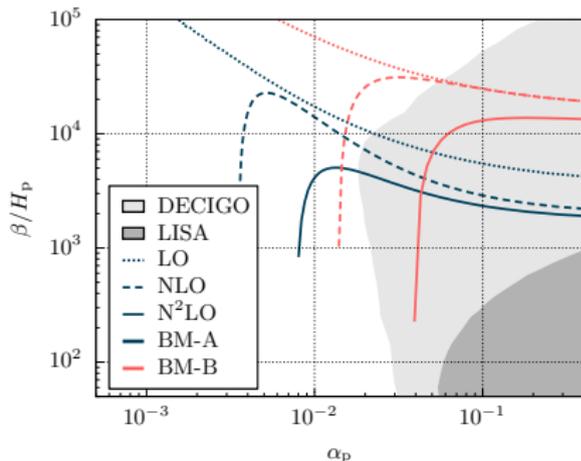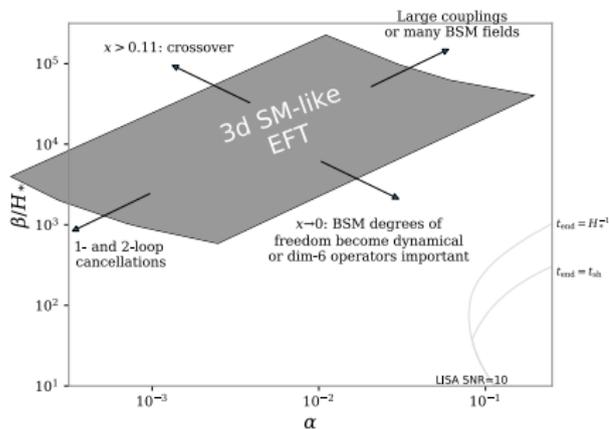
[22] Gould, Kozaczuk, Niemi, Ramsey-Musolf, Tenkanen, Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition*, Phys. Rev. D **100** (2019) 115024 [1903.11604]

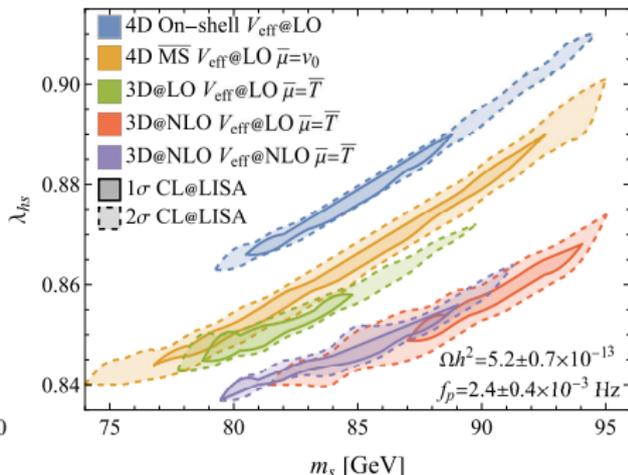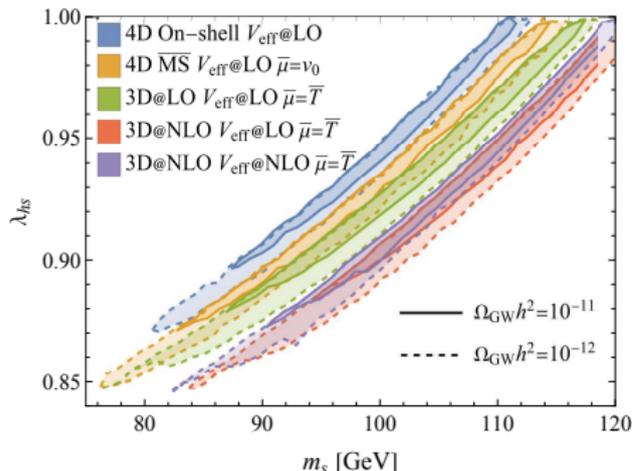# Computational diligence at $\mathcal{O}(g^4)$

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \to x + s$. 2-loop corrections move "Bananas": significant effect on GW signal.[23]

Parameters	m_s	λ_{hs}	$\lambda_s = 1$	$\alpha > 1$	$\mathrm{SNR_{LISA}} > 10$

---

[23] Lewicki, Merchand, Sagunski, Schicho, Schmitt, *Impact of theoretical uncertainties on model parameter reconstruction from GW signals sourced by cosmological phase transitions,* Phys. Rev. D **110** (2024) 023538 [2403.03769], Niemi, Schicho, Tenkanen, *Singlet-assisted electroweak phase transition at two loops,* Phys. Rev. D **103** (2021) 115035 [2103.07467], talk by **Van Que Tran** on **Fri 15:00**

Does the **banana** move?

Monitor Higgs (v) and **real singlet** (x) VEV after shift $s \to x + s$.
2-loop corrections move "Bananas": significant effect on GW signal.[23]

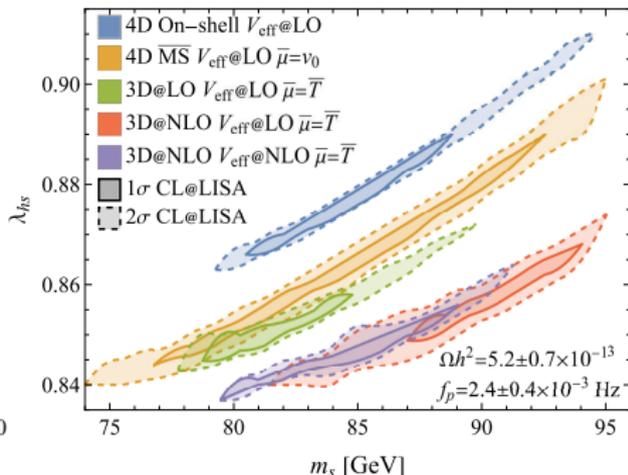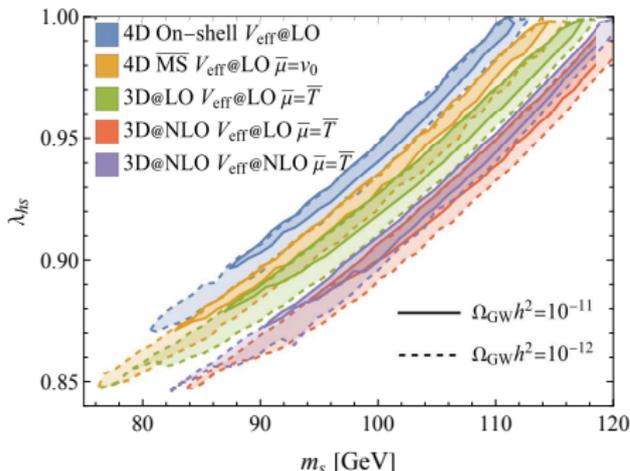| Parameters	m_s	λ_{hs}	$\lambda_s = 1$	$\alpha > 1$	$SNR_{LISA} > 10$

[23] Lewicki, Merchand, Sagunski, Schicho, Schmitt, *Impact of theoretical uncertainties on model parameter reconstruction from GW signals sourced by cosmological phase transitions,* Phys. Rev. D **110** (2024) 023538 [2403.03769], Niemi, Schicho, Tenkanen, *Singlet-assisted electroweak phase transition at two loops,* Phys. Rev. D **103** (2021) 115035 [2103.07467], talk by **Van Que Tran** on **Fri 15:00**

Dimension-six operators in U(1) + Higgs[24]

WHAT IF WE TRIED
MORE LOOPS ?

So far truncated operators at high T at dimension 4:

$$S_{\text{soft}}^{\text{3d}} = \frac{1}{T} \int_{\mathbf{x}} \left\{ \mathcal{L}_{\text{soft}}^{\text{3d}} + \sum_{n \geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\},$$

$$S_{\text{softer}}^{\text{3d}} = \frac{1}{T} \int_{\mathbf{x}} \left\{ \mathcal{L}_{\text{softer}}^{\text{3d}} + \sum_{n \geq 5} \frac{\mathcal{O}_n}{(m_{\text{D}})^n} \right\}.$$

---

[24] Bernardo, Klose, Schicho, Tenkanen, *Higher-dimensional operators at finite temperature affect gravitational-wave predictions*, JHEP **08** (2025) 109 [2503.18904], Bernardo, Chala, Gil, Schicho, *Hard thermal contributions to phase transition observables at NNLO*, [2602.06962]

Dimension-six operators in U(1) + Higgs[24]

So far truncated operators at high T at dimension 4:

$$S_{\text{soft}}^{\text{3d}} = \frac{1}{T} \int_{\mathbf{x}} \left\{ \mathcal{L}_{\text{soft}}^{\text{3d}} + \sum_{n \geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\},$$

$$S_{\text{softer}}^{\text{3d}} = \frac{1}{T} \int_{\mathbf{x}} \left\{ \mathcal{L}_{\text{softer}}^{\text{3d}} + \sum_{n \geq 5} \frac{\mathcal{O}_n}{(m_{\text{D}})^n} \right\}.$$

[24] Bernardo, Klose, Schicho, Tenkanen, *Higher-dimensional operators at finite temperature affect gravitational-wave predictions*, JHEP **08** (2025) 109 [2503.18904], Bernardo, Chala, Gil, Schicho, *Hard thermal contributions to phase transition observables at NNLO*, [2602.06962]

Gravitational wave prospects

Soft scale enhances phase-transition strength, $\alpha(T)$.
High-T expansion is compromised for regime relevant for LISA:
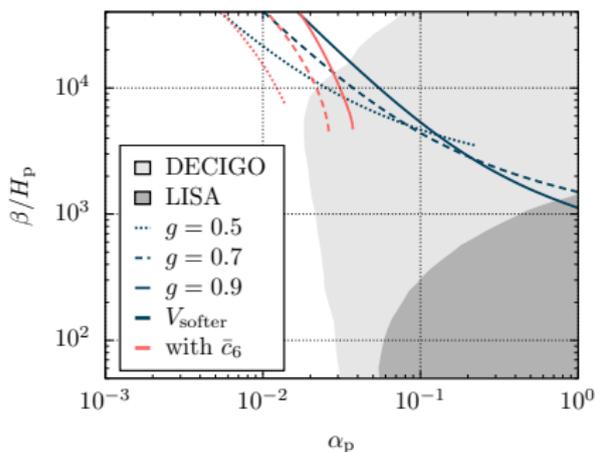
Limitation: conventional lattice results of (dim-4) super-renormalizable 3d EFT do not describe hard/soft-scale driven transitions.[25]

Need 4d full theory lattice simulations for a reliable description?

[25] Laine, *Comparison of 4-D and 3-D lattice results for the electroweak phase transition*, Phys. Lett. B **385** (1996) 249 [hep-lat/9604011]

Gravitational wave prospects

Soft scale enhances phase-transition strength, $\alpha(T)$.
High-T expansion is compromised for regime relevant for LISA:

Limitation: conventional lattice results of (dim-4) super-renormalizable 3d EFT do not describe hard/soft-scale driven transitions.[25]
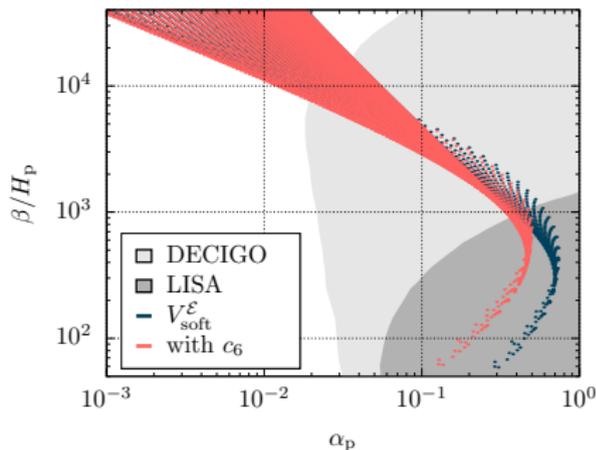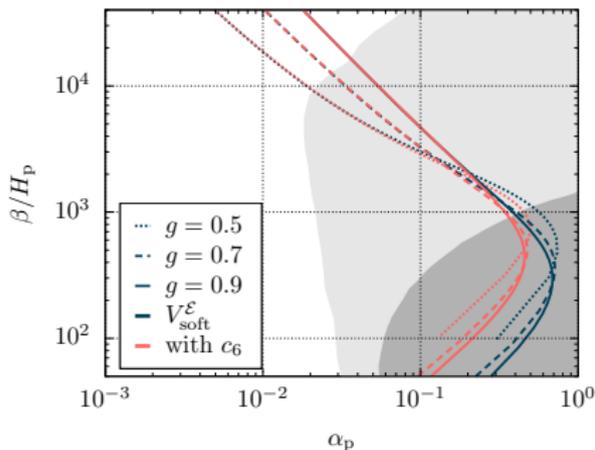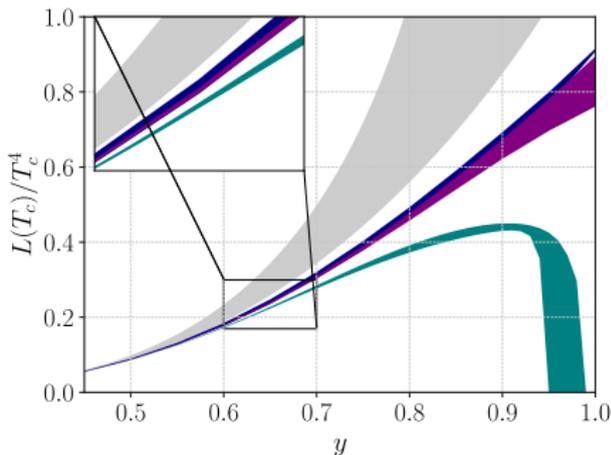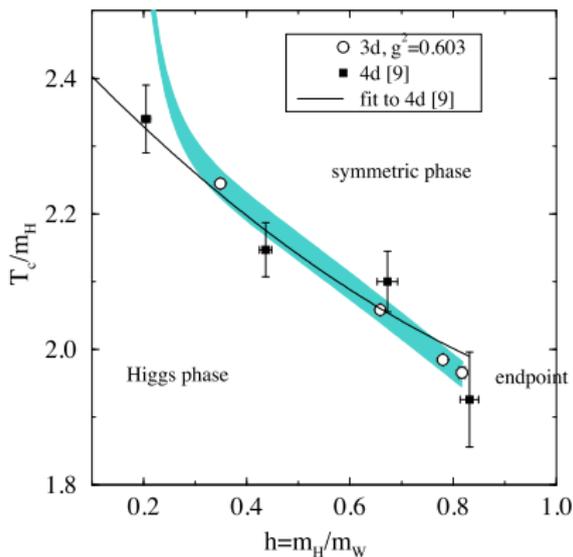
Need 4d full theory lattice simulations for a reliable description?

[25] Laine, *Comparison of 4-D and 3-D lattice results for the electroweak phase transition*, Phys. Lett. B **385** (1996) 249 [hep-lat/9604011]

No surprise since already tests in the 90s showed this breakdown.[26]

New broad-temperature frameworks.[27]

[26] Laine, *The Renormalized gauge coupling and nonperturbative tests of dimensional reduction,* JHEP **06** (1999) 020 [hep-ph/9903513], Csikor, Fodor, Heitger, *Endpoint of the hot electroweak phase transition,* Phys. Rev. Lett. **82** (1999) 21 [hep-ph/9809291]

[27] Navarrete, Paatelainen, Seppänen, Tenkanen, *Cosmological phase transitions without high-temperature expansions,* [2507.07014]

Bubble nucleation rate determines the nucleation temperature and duration of the phase transition[28]

$$\Gamma = \Gamma_{\mathrm{dyn}} \times \Gamma_{\mathrm{stat}} \sim A\, e^{-B}\,,$$

B *Internal energy* of critical bubble. Solve one non-linear ODE

$$\frac{\mathrm{d}^2 \phi_b}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\phi_b}{\mathrm{d}r} - V'(\phi_b) = 0$$

Smoothed 5 times

---

[28] Langer, Turski, *Hydrodynamic model of the condensation of a vapor near its critical point,* Physical Review A **8** (1973) 3230; figure for critical bubble in SU(2)-Higgs model by Gould, Güyer, Rummukainen, *First-order electroweak phase transitions: A nonperturbative update,* Phys. Rev. D **106** (2022) 114507 [2205.07238]
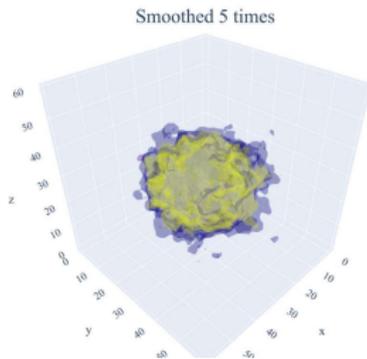
Beyond equilibrium: nucleation rate and temperature

Bubble nucleation rate determines the nucleation temperature and duration of the phase transition[28]

$$\Gamma = \Gamma_{\mathrm{dyn}} \times \Gamma_{\mathrm{stat}} \sim A \, e^{-B} \, ,$$

B *Internal energy* of critical bubble. Solve one non-linear ODE

$$\frac{\mathrm{d}^2\phi_b}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\phi_b}{\mathrm{d}r} - V'(\phi_b) = 0$$

A log *entopy* of fluctuations about bubble. Infinite number of linear ODEs

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{l(l+1)}{r^2} + V''(\phi_b)\right]\psi_l^b(r) = 0$$

Need both A and B for **reliable** rate computation. EFT for Γ_{stat}[29]
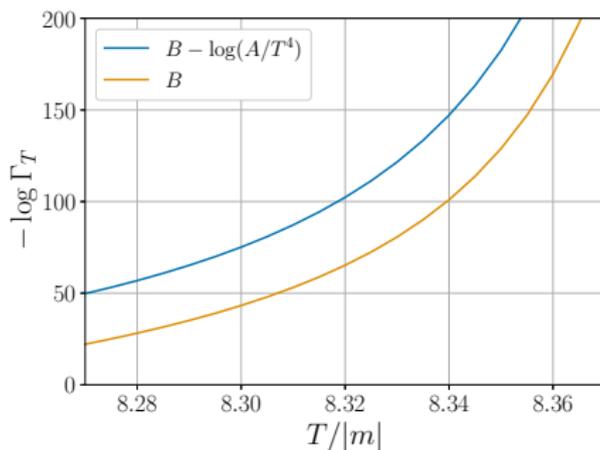
$$\Gamma_{\mathrm{stat}} \sim \left[m_{\mathrm{eff}}^4 e^S\right] \times e^{-E_b/T} \sim m_{\mathrm{eff}}^4 e^{-(E_b - ST)/T} \, .$$

[28] Langer, Turski, *Hydrodynamic model of the condensation of a vapor near its critical point*, Physical Review A **8** (1973) 3230

[29] Gould, Hirvonen, *Effective field theory approach to thermal bubble nucleation*, Phys. Rev. D **104** (2021) 096015 [2108.04377]

Determining the Bubble determinant

`BubbleDet`[30] computes term A. Here: Yukawa model.

Nucleation not just energy and entropy $\rightarrow$ out-of-equilibrium effects[31]

$$\Gamma \sim \frac{\kappa_{\mathrm{dyn}}}{2\pi} m^3 e^{-(E_b - ST)/T} .$$

[30] Ekstedt, Gould, Hirvonen, *BubbleDet: a Python package to compute functional determinants for bubble nucleation*, JHEP **12** (2023) 056 [2308.15652]

[31] Langer, *Statistical theory of the decay of metastable states*, Ann. Phys. (N. Y). **54** (1969) 258, Hirvonen, *Real-time nucleation and off-equilibrium effects in high-temperature quantum field theories*, Phys. Rev. D **111** (2025) 116020 [2403.07987], Gould, Kormu, Weir, *Nonperturbative test of nucleation calculations for strong phase transitions*, Phys. Rev. D **111** (2025) L051901 [2404.01876]
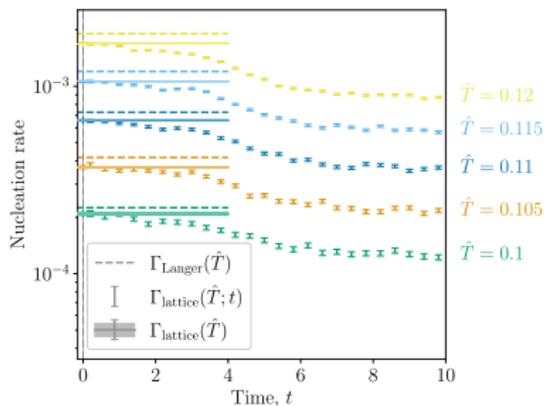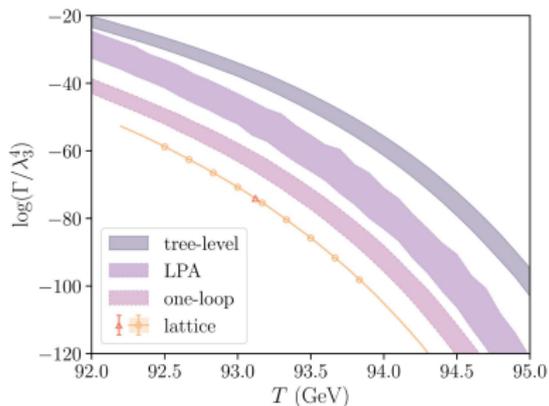
Agreement between perturbation theory and lattice?

Disagreement[32] because initial state is not thermalized on the lattice?

Recent $1 + 1D$ lattice simulations better match perturbation theory.[33]

Agreement up to two-loop order for a thermalized initial state.[34]
The rate is not constant, but **decays over time.**

[32] Gould, Kormu, Weir, *Nonperturbative test of nucleation calculations for strong phase transitions,* Phys. Rev. D **111** (2025) L051901 [2404.01876]

[33] Pîrvu, Shkerin, Sibiryakov, *Thermal false vacuum decay in (1+1) dimensions: Evidence for nonequilibrium dynamics,* Int. J. Mod. Phys. A **39** (2024) 2445007 [2408.06411]

[34] Hirvonen, Gould, *Langer's Nucleation Rate Reproduced on the Lattice,* Phys. Rev. Lett. **136** (2026) 081601 [2505.22732]

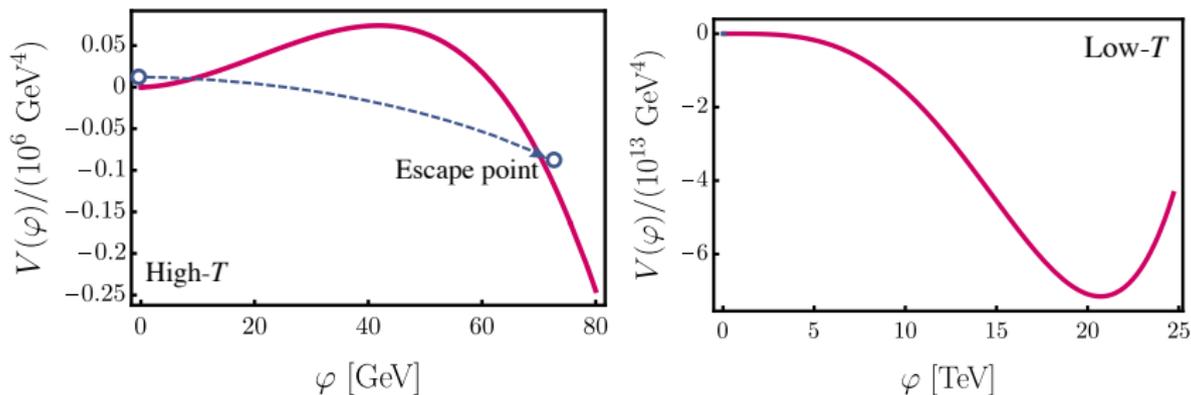# Supercooling in EFT: Classical scale-invariant models

exhibit strong supercooling and phase transition. Barrier until low-T

$$m_\varphi^2(T) = \left[\mu_0^2\right] + m_T^2 .$$

Trapped field φ in false vacuum φ_F until $T_p \ll T_c$. Split computation:[35]

High-T: Small field regime $M(\varphi) < T$ use 3D EFT

**Low-T:**   Large field regime $M(\varphi) > T$ use vacuum potential

[35] Kierkla, Schicho, Swiezewska, Tenkanen, van de Vis, *Finite-temperature bubble nucleation with shifting scale hierarchies*, JHEP **07** (2025) 153 [2503.13597], Kierkla, Swiezewska, Tenkanen, van de Vis, *Gravitational waves from supercooled phase transitions: dimensional transmutation meets dimensional reduction*, JHEP **02** (2024) 234 [2312.12413]

How else can we study bubble nucleation?

Clear disagreement for A-B transition in superfluid ^{3}He.[36]

Good theoretical agreement for rate in $1+1d$ ferromagnetic superfluid, taking entropy (A) as a fit parameter.[37]

^{3}He bulk phase diagram, $B = 0$
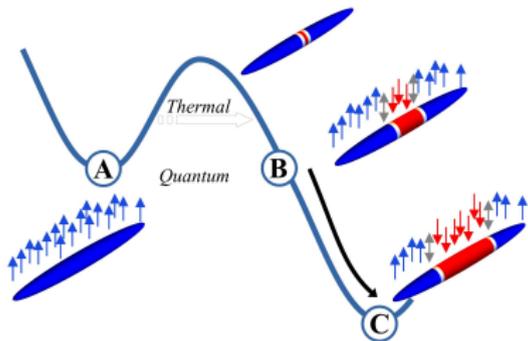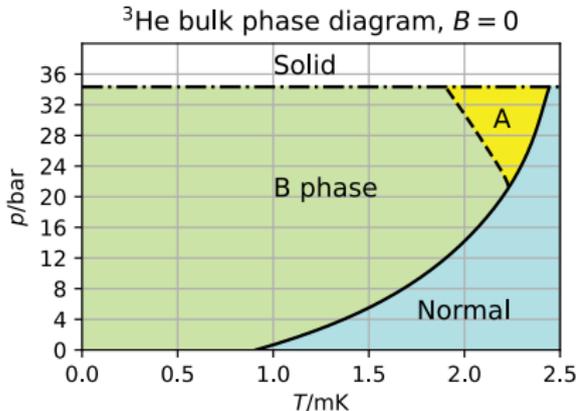
[36] **QUEST-DMC** Collaboration, Hindmarsh *et al.*, *A-B Transition in Superfluid ^{3}He and Cosmological Phase Transitions*, J. Low Temp. Phys. **215** (2024) 495 [2401.07878]

[37] Zenesini, Berti, Cominotti *et al.*, *False vacuum decay via bubble formation in ferromagnetic superfluids*, Nature Phys. **20** (2024) 558 [2305.05225], Cominotti, Baroni, Rogora *et al.*, *Observation of Temperature Effects on False Vacuum Decay in Atomic Quantum Gases*, Phys. Rev. Lett. **135** (2025) 183401 [2504.03528]

In between scales
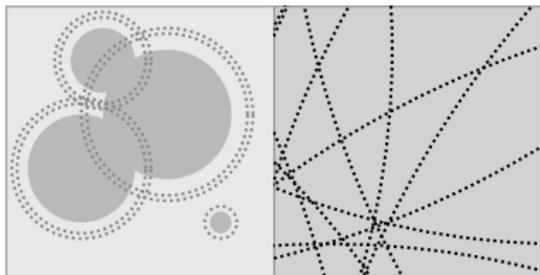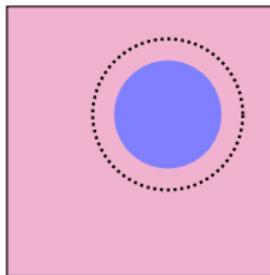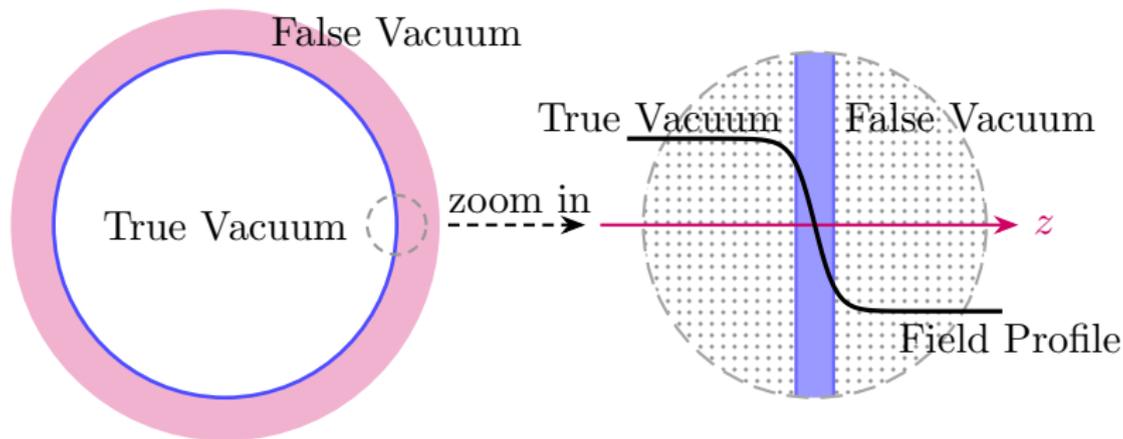
Fingerprinting the GW spectrum via bubble growth

Velocity of expanding bubbles determines the shape and strength of GW signal.[38]
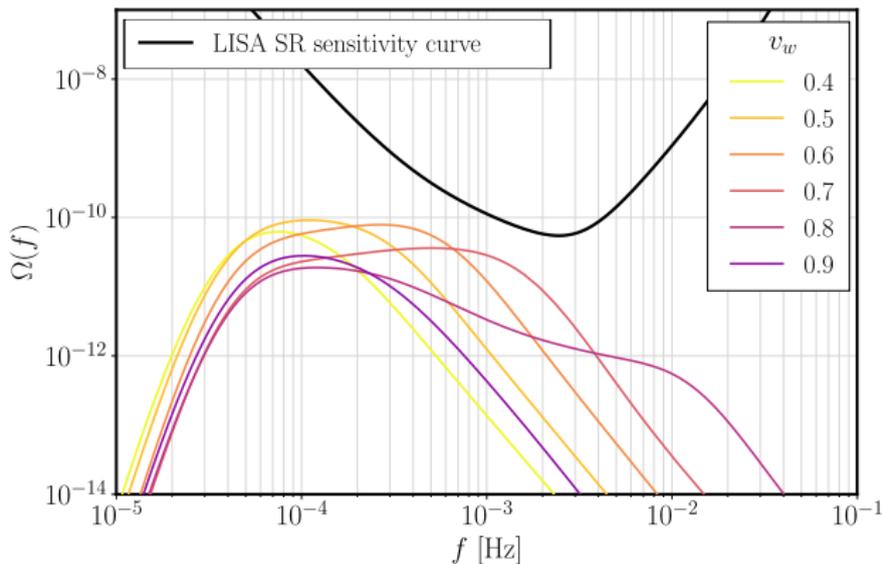
[38] At fixed $\alpha = 0.2$, $r_\star = 0.1$, $T_n = 100$ GeV from Gowling, Hindmarsh, *Observational prospects for phase transitions at LISA: Fisher matrix analysis*, JCAP **10** (2021) 039 [2106.05984]

Microscopic description of the bubble wall

The bubble wall is the region where the field(s) interpolate between the high- and low-temperature vacuum.

figure inspired by Ai, Laurent, van de Vis, *Bounds on the bubble wall velocity*, JHEP **02** (2025) 119 [2411.13641]

Bubbles typically reach terminal velocity before colliding if

$$|P_\text{out}| = |P_\text{in}| \,.$$

ΔP_out Vacuum energy release
(accelerating wall)

ΔP_in Particle friction on the wall
(decelerating wall)

ΔP_in Hydrodynamic backreaction
(decelerating wall)
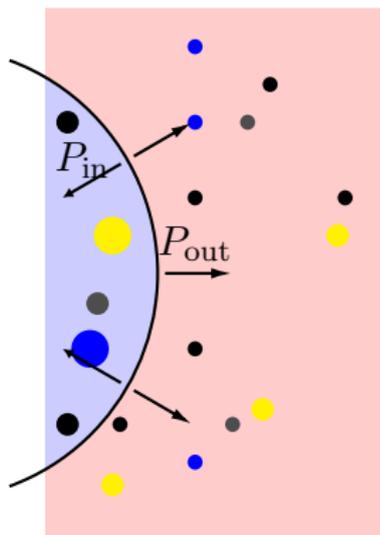
Iteratively solve coupled fluid, scalar field, Boltzmann equations.[39]

Public software for v_w:[40] WallGo.

[39] Moore, Prokopec, *How fast can the wall move? A Study of the electroweak phase transition dynamics*, Phys. Rev. D **52** (1995) 7182 [hep-ph/9506475], Laurent, Cline, *First principles determination of bubble wall velocity*, Phys. Rev. D **106** (2022) 023501 [2204.13120],

[40] Schicho, Ekstedt, Gould, Hirvonen *et al.*, *How fast does the WallGo? A package for computing wall velocities in first-order phase transitions*, JHEP **04** (2025) 101 [2411.04970].
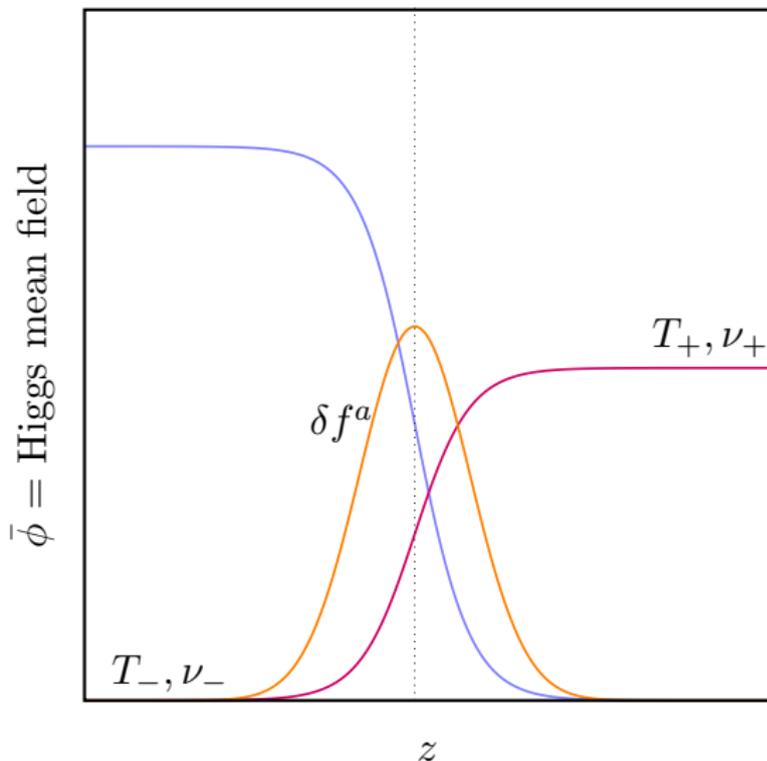
The relevant length scales during a phase transition
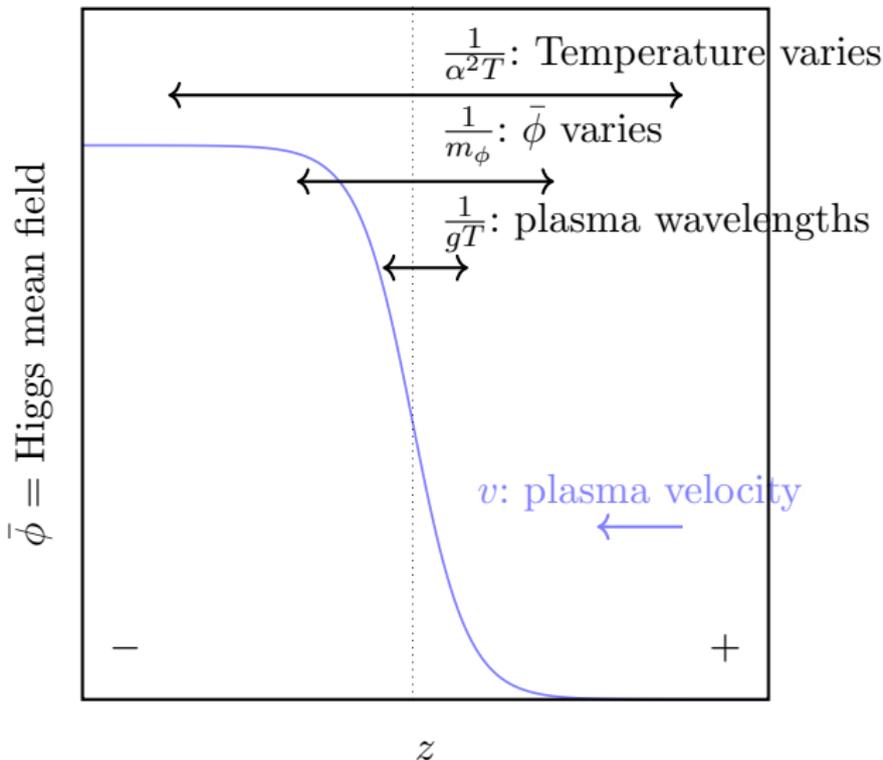
in the wall frame [plasma moves to the left; $\alpha = g^2/(4\pi)$].

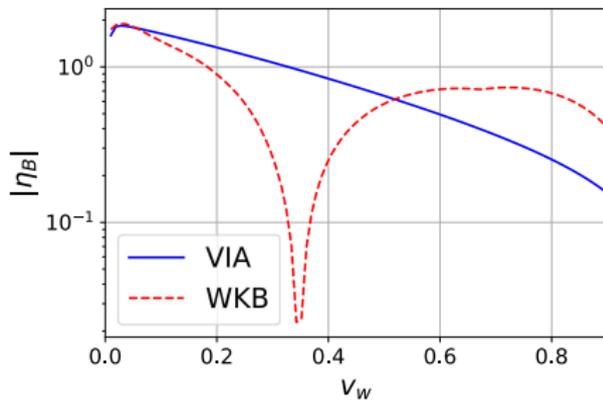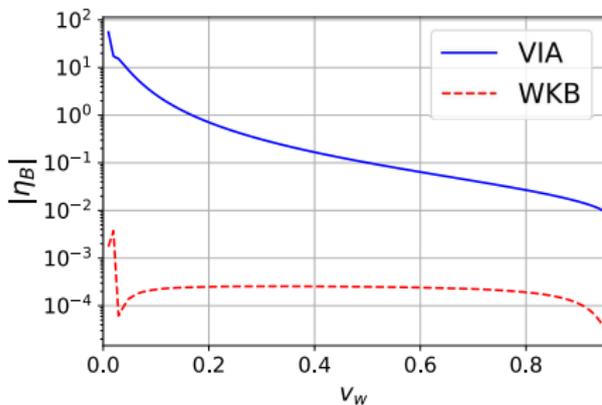## The relevant length scales during a phase transition

in the wall frame [plasma moves to the left; $\alpha = g^2/(4\pi)$].

Baryogenesis from CP violation via lepton higher-dimensional Yukawa.

Baryogenesis by c-t or b-s flavor mixing.

[41] Cline, Laurent, *Electroweak baryogenesis from light fermion sources: A critical study*, Phys. Rev. D **104** (2021) 083507 [2108.04249]

Pinching v_w with LTE and ballistic limit

Upper bound. **Large collision** term ($\mathcal{C}_a \to \infty$ and $\delta f^a \to 0$):
Local thermal equilibrium (LTE).[42,43]

Lower bound. **Small collision** term ($\mathcal{C}_a \to 0$):
Ballistic limit.[44] Can solve Boltzmann equation analytically.
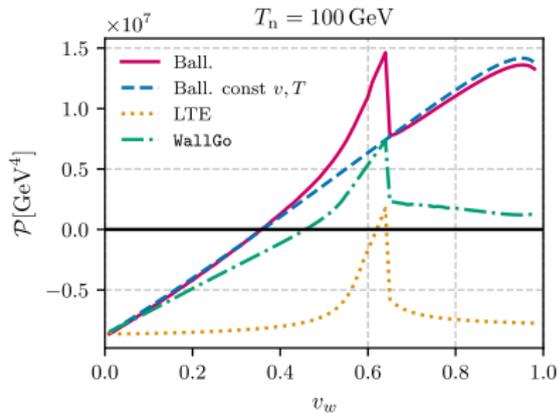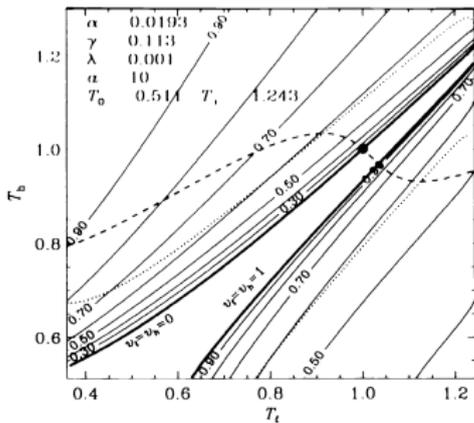
[42] Enqvist, Ignatius, Kajantie, Rummukainen, *Nucleation and bubble growth in a first order cosmological electroweak phase transition,* Phys. Rev. D **45** (1992) 3415

[43] Backreaction from hot fluid, and entropy production ($\Delta_{su}z \geq 0$) can still stop the wall c.f. Eriksson, Laine, *Entropy production at electroweak bubble walls from scalar field fluctuations,* JCAP **09** (2025) 027 [2507.07755]

[44] Ai, Laurent, van de Vis, *Bounds on the bubble wall velocity,* JHEP **02** (2025) 119 [2411.13641] at $m_s = 120$ GeV and $\lambda_{hs} = \lambda_s = 1$

from both the driving pressure ΔP and the wall velocity Δv_w,

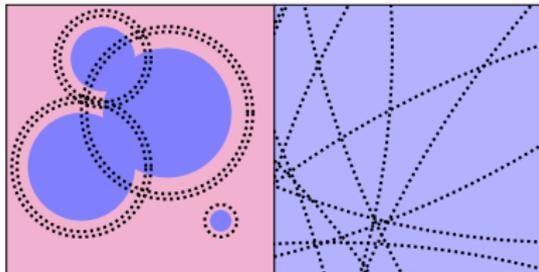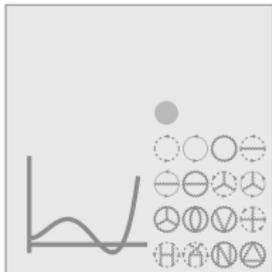$$\Delta v_w \approx \Delta P \Big(\frac{\mathrm{d}P}{\mathrm{d}v_w}\Big)^{-1}.$$

Suggested largest uncertainties:

1) nucleation,
2) leading-log and power-enhancement in collisions,
3) out of equilibrium particles.

Source of uncertainty		$\Delta v_w/v_w$		
		xSM	IDM	$\mathcal{O}(\Delta P/P)$
Boltzmann	Linearization	0.1%	0.01%	$\mathcal{O}(\delta f_2/f_{\mathrm{eq}})$
	Power enhanced	90%	300%	$\mathcal{O}(\delta f_1/f_{\mathrm{eq}})$
	NLL collisions	—	20%	$\mathcal{O}(\delta f_1/(f_{\mathrm{eq}}\ln(1/g)))$
	Out-of-eq. particles	30%	400%	$O(\delta f_1/f_{\mathrm{eq}})$
Scalar EOM	T_{n}	100%	—	$\mathcal{O}(g)$
	V_{eff}	—	20%	$\mathcal{O}(g)$

Macroscopic scales

Predicting the GW signal of colliding bubbles

Simulate numerically the scalar-fluid system consisting of

▷ Scalar field (gradient energy),
▷ Fluid motion.

For weak/intermediate phase transitions, the fluid motion is the dominant source of GWs.[46]

Simulated using phenomenological fluid-scalar interaction

$$\partial_\mu T_\phi^{\mu\nu} = -\eta_T u^\mu \partial_\mu \phi \partial^\nu \phi \,.$$

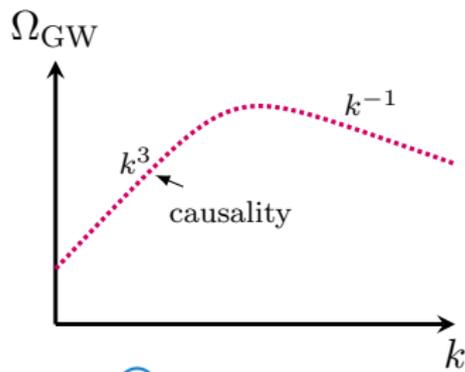Recent computations of $\eta_T(\phi)$ from first principles.[47]

---

[46] Hindmarsh, Huber, Rummukainen, Weir, *Gravitational waves from the sound of a first order phase transition*, Phys. Rev. Lett. **112** (2014) 041301 [1304.2433]

[47] Ekstedt, Konstandin, van de Vis, *Scalar damping in cosmological phase transitions*, [2512.16663]

Gradient energy of scalar field:

  ▷ Envelope approximation[48]

[48] Kosowsky, Turner, *Gravitational radiation from colliding vacuum bubbles: envelope approximation to many bubble collisions,* Phys. Rev. D **47** (1993) 4372 [astro-ph/9211004], Weir, *Revisiting the envelope approximation: gravitational waves from bubble collisions,* Phys. Rev. D **93** (2016) 124037 [1604.08429]

Gradient energy of scalar field:

▷ Envelope approximation[48]

▷ Contributions of collided walls
  can change UV power law[49]
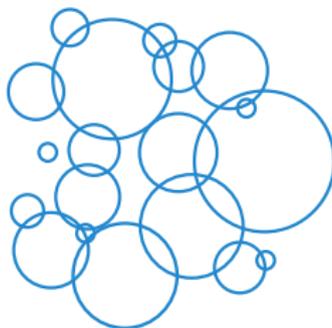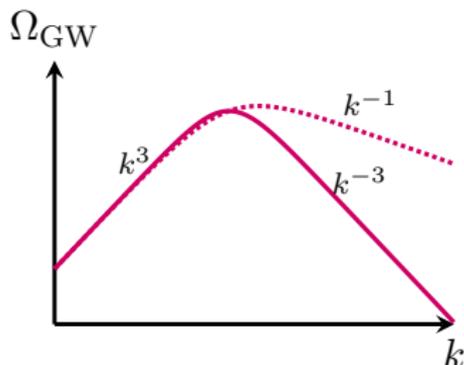
[49] Cutting, Hindmarsh, Weir, *Gravitational waves from vacuum first-order phase transitions: from the envelope to the lattice*, Phys. Rev. D **97** (2018) 123513 [1802.05712], Konstandin, *Gravitational radiation from a bulk flow model*, JCAP **03** (2018) 047 [1712.06869], Jinno, Takimoto, *Gravitational waves from bubble dynamics: Beyond the Envelope*, JCAP **01** (2019) 060 [1707.03111], Lewicki, Vaskonen, *Gravitational waves from colliding vacuum bubbles in gauge theories*, Eur. Phys. J. C **81** (2021) 437 [2012.07826], Gould, Sukuvaara, Weir, *Vacuum bubble collisions: From microphysics to gravitational waves*, Phys. Rev. D **104** (2021) 075039 [2107.05657]
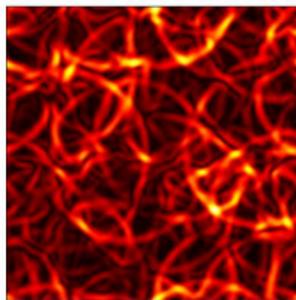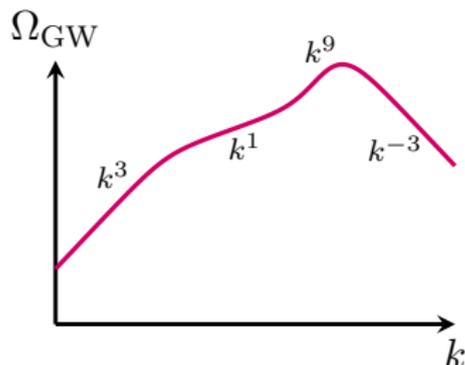
A conspiracy of bubbles: From single bubbles to a GW signal

Gradient energy of scalar field:

▷ Envelope approximation[48]

▷ Contributions of collided walls can change UV power law[49]

Fluid motion:
Sound waves in fluid plasma[50]

[50] Hindmarsh, Huber, Rummukainen, Weir, *Gravitational waves from the sound of a first order phase transition*, Phys. Rev. Lett. **112** (2014) 041301 [1304.2433], Hindmarsh, *Sound shell model for acoustic gravitational wave production at a first-order phase transition in the early Universe*, Phys. Rev. Lett. **120** (2018) 071301 [1608.04735], Sharma, Dahl, Brandenburg, Hindmarsh, *Shallow relic gravitational wave spectrum with acoustic peak*, [2308.12916], Jinno, Konstandin, Rubira, Stomberg, *Higgsless simulations of cosmological phase transitions and gravitational waves*, JCAP **02** (2023) 011 [2209.04369]
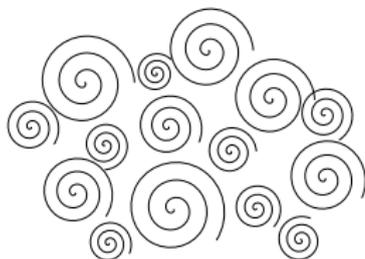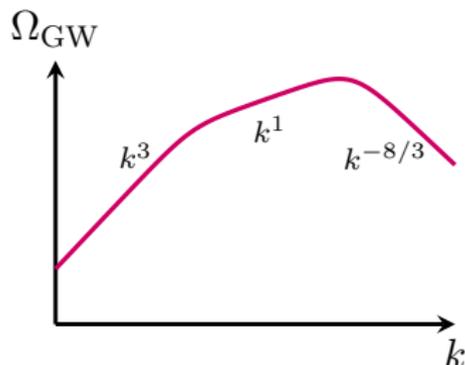
Gradient energy of scalar field:

- ▷ Envelope approximation[48]
- ▷ Contributions of collided walls can change UV power law[49]

Fluid motion:
Sound waves in fluid plasma[50]

Turbulence (vortical and acoustic)[51]

[51] Caprini, Durrer, Servant, *The stochastic gravitational wave background from turbulence and magnetic fields generated by a first-order phase transition,* JCAP **12** (2009) 024 [0909.0622], Roper Pol, Mandal, Brandenburg, Kahniashvili, Kosowsky, *Numerical simulations of gravitational waves from early-universe turbulence,* Phys. Rev. D **102** (2020) 083512 [1903.08585], Auclair, Caprini, Cutting *et al.*, *Generation of gravitational waves from freely decaying turbulence,* JCAP **09** (2022) 029 [2205.02588], Dahl, Hindmarsh, Rummukainen, Weir, *Primordial acoustic turbulence: Three-dimensional simulations and gravitational wave predictions,* Phys. Rev. D **110** (2024) 103512 [2407.05826]

A conspiracy of bubbles: From single bubbles to a GW signal

Gradient energy of scalar field:
- ▷ Envelope approximation[48]
- ▷ Contributions of collided walls can change UV power law[49]
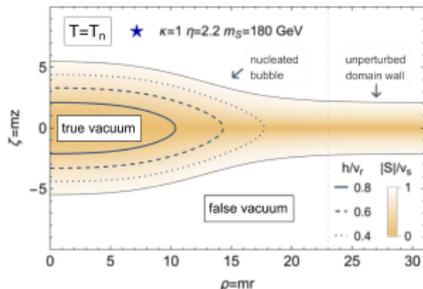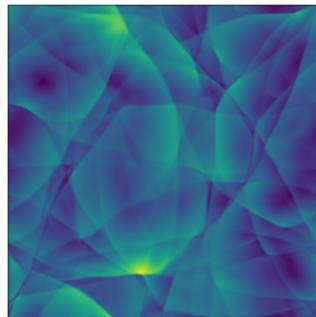
Fluid motion:
Sound waves in fluid plasma[50]

Turbulence (vortical and acoustic)[51]

Fluid shocks[52]
Feebly interacting particles[53]
Domain walls[54]

[52] Dahl, Hindmarsh, Rummukainen, Weir, *Decay of acoustic turbulence in two dimensions and implications for cosmological gravitational waves*, Phys. Rev. D **106** (2022) 063511 [2112.12013]

[53] Jinno, Shakya, van de Vis, *Gravitational Waves from Feebly Interacting Particles in a First Order Phase Transition*, [2211.06405]

[54] Blasi, Mariotti, *Domain Walls Seeding the Electroweak Phase Transition*, Phys. Rev. Lett. **129** (2022) 261303 [2203.16450], Blasi, Jinno, Konstandin, Rubira, Stomberg, *Gravitational waves from defect-driven phase transitions: domain walls*, JCAP **10** (2023) 051 [2302.06952], Agrawal, Blasi, Mariotti, Nee, *Electroweak phase transition with a double well done doubly well*, JHEP **06** (2024) 089 [2312.06749], cf. talk by **A. Vikman** on **Fri 13:30**
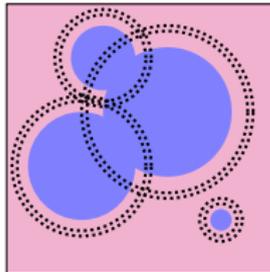
Conclusions: Progress and future challenges
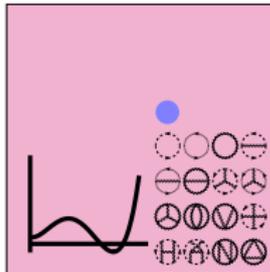
Precision thermodynamics of BSM theories:
- ▷ practical approach: **Effective Theories**
- ▷ Nucleation is hard. Discrepancy with lattice is becoming better.
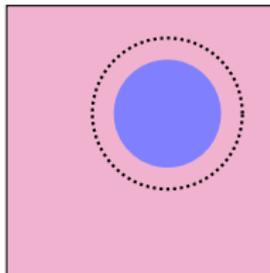
Wall velocity and hydrodynamics of single bubbles:
- ▷ Can be predictive for the gravitational wave signal of colliding bubbles.

Collective effects:
- ▷ Fluid motion dominant source of GW signal,
- ▷ Simulations: decay of source and slow-down of bubbles, call for better modelling.