High Frequency Spectrum of Primordial Gravitational Waves

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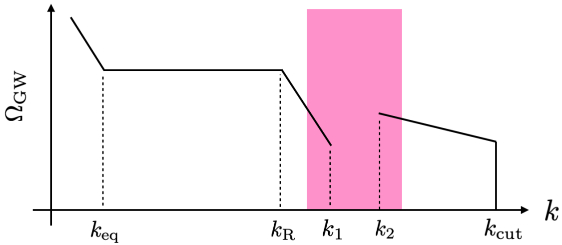
13. 3. 2026

Introduction

- Primordial GWs are created during inflation
- Perturbations freeze at horizon exit
- High frequency modes never exit the horizon, but are affected by reheating dynamics
- Resulting spectrum encodes information about both the inflaton potential and subsequent thermal history
- Low and high frequency limits of the spectrum have been analytically solved
- We present numerical solution for the intermediate region



Background equations

- Einstein-Hilbert action + inflaton

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

- Perturbed FLRW metric

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij}(t,\vec{x}))dx^i dx^j$$

- Friedmann equation

$$3H^2M_{\rm Pl}^2 = \frac{\dot\phi^2}{2} + V$$

- Inflaton EOM

$$\ddot\phi + 3H\dot\phi + \frac{\partial V}{\partial\phi} = 0$$

Tensor perturbation

- Tensor perturbation EOM

$$\ddot{h}_{k,\lambda} + 3H\dot{h}_{k,\lambda} + \left(\frac{k}{a}\right)^2 h_{k,\lambda} = 0$$

- Mukhanov variable $\widetilde{h}_{k,\lambda} \equiv a h_{k,\lambda}$ and conformal time $\tau = \int \frac{dt}{a}$

$$\widetilde{h}''_{k,\lambda} + \omega_k^2 \widetilde{h}_{k,\lambda} = 0, \qquad \omega_k^2 = k^2 - \frac{a''}{a},$$

- Quantization

$$h_{ij}(t,\vec{x}) = \sum_{\lambda=+,\times} \int \frac{d^3k}{(2\pi)^3} \left[a_{k,\lambda} h_{k,\lambda}(t) + a^\dagger_{-k,\lambda} h^*_{k,\lambda}(t) \right] e^{i\vec{k}\cdot\vec{x}} e^\lambda_{ij}$$

$$[a_{k,\lambda}, a^\dagger_{k',\lambda'}] = (2\pi)^3 \delta(\vec{k} - \vec{k}') \delta_{\lambda\lambda'}$$

- Bunch-Davies vacuum

$$\lim_{\tau\to-\infty} \widetilde{h}_{k,\lambda}(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}$$

Low frequency limit

- Modes that exit the horizon during inflation freeze at $\frac{k^3 h_k^2}{2\pi^2} \sim \left(\frac{H_e}{2\pi}\right)^2$ (almost constant for slow-roll inflation)
- Taking into account the evolution during reheating

$$\Omega_{GW}(f) = \Omega_r \times \frac{g_{*R}}{g_{*0}} \left(\frac{g_{*s0}}{g_{*sR}}\right)^{\frac{4}{3}} \times \frac{H_e^2}{12\pi^2 M_{Pl}^2} \left(\frac{a_R H_R}{k}\right)^{\frac{2(1-3w)}{1+3w}}$$

- Modes that reenter the horizon during radiation domination ($w = 1/3$) show flat spectrum
- Modes that reenter during matter domination ($w = 0$) have spectrum scaling as k^{-2}

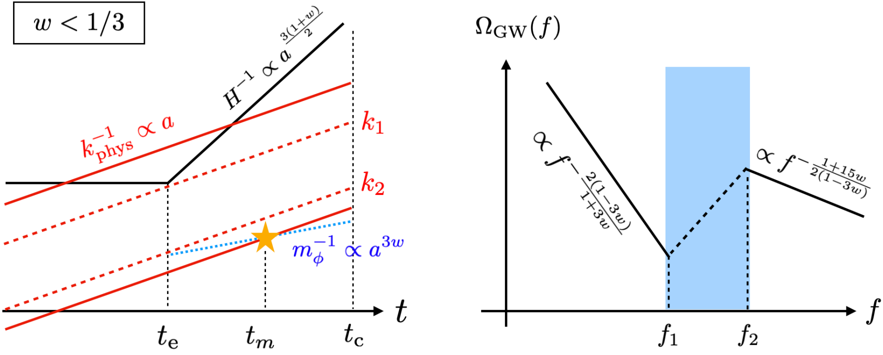
High frequency limit

- Particle production due to oscillating background
- Enhancement at $k/a \simeq m_\phi$
- Inflaton pair to graviton pair annihilation

$$\Gamma_{\varphi\varphi\to hh} \sim \mathcal{C}\frac{\langle \varphi^2 \rangle m_\phi^3}{M_{\mathrm{Pl}}^4}, \quad m_\phi \propto \varphi^{(n-2)/2},\ \mathcal{C} \approx 10^{-2}$$

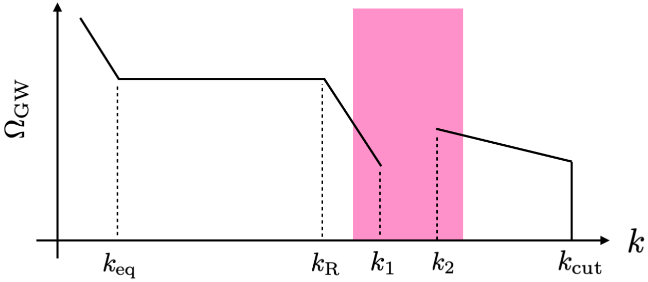
- Taking into account the evolution during reheating

$$\Omega_{\mathrm{GW}}(f) = \Omega_r \times \frac{g_{*\mathrm{R}}}{g_{*0}} \left(\frac{g_{*s0}}{g_{*s\mathrm{R}}}\right)^{\frac{4}{3}} \times \frac{\mathcal{C} m_\phi(t_{\mathrm{e}}) H_{\mathrm{e}}}{3M_{\mathrm{Pl}}^2} \left(\frac{H_{\mathrm{R}}}{H_{\mathrm{e}}}\right)^{\frac{2(1-3w)}{3(1+w)}} \left(\frac{a_{\mathrm{e}} m_\phi(t_{\mathrm{e}})}{k}\right)^{\frac{1+15w}{2(1-3w)}}$$



$w < 1/3$
$H^{-1} \propto a^{\frac{3(1+w)}{2}}$
$k_{\rm phys}^{-1} \propto a$
k_1
k_2
$m_\phi^{-1} \propto a^{3w}$
t
$t_{\rm e}$
t_m
$t_{\rm c}$
$\Omega_{\rm GW}(f)$
$\propto f^{-\frac{2(1-3w)}{1+3w}}$
$\propto f^{-\frac{1+15w}{2(1-3w)}}$
f
f_1
f_2

Intermediate region



$$\frac{\Omega_{\rm GW}(f_2)}{\Omega_{\rm GW}(f_1)} \sim \frac{m_\phi(t_{\rm e})}{H_{\rm e}} \text{ for } w < 1/3$$

- No analytic approximation known

Semiclassical method

- Solve the EOM in terms of $h_{k,\lambda}$ (or $\widetilde{h}_{k,\lambda}$)
- Initial conditions = Bunch-Davies vacuum

$$\widetilde{h}_{k,\lambda}(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}$$

- Graviton energy density is given by

$$\rho_h(k,\tau) = \frac{k^3}{a^4(\tau) 2\pi^2} \left[\left| \widetilde{h}'_{k,\lambda}(\tau) \right|^2 + k^2 \left| \widetilde{h}_{k,\lambda}(\tau) \right|^2 - k \right]$$

- Divergent zero point energy has to be subtracted!
- Not efficient for high k modes

Bogoliubov coefficient method

- Define $\Omega_k(\tau) = \int \omega_k(\tau) d\tau$ and decompose $\widetilde{h}_{k,\lambda}(\tau)$ as

$$\widetilde{h}_{k,\lambda}(\tau) = \alpha_k(\tau) v_k(\tau) + \beta_k(\tau) v_k^*(\tau), \qquad v_k(\tau) \equiv \frac{1}{\sqrt{2k}} e^{-i\Omega_k}$$

- EOM becomes

$$\alpha_k'(\tau) = \frac{\omega_k'}{2\omega_k} \beta_k e^{2i\Omega_k}, \qquad \beta_k'(\tau) = \frac{\omega_k'}{2\omega_k} \alpha_k e^{-2i\Omega_k}$$

- Initial conditions

$$\alpha_k(\tau) = 1, \quad \beta_k(\tau) = 0$$

Bogoliubov coefficient method

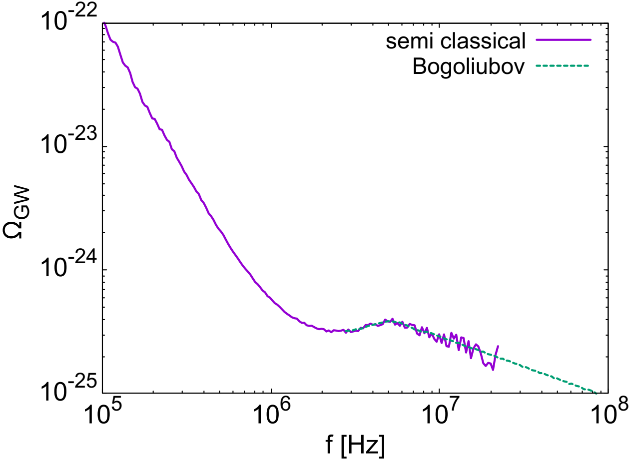
- Renormalized graviton energy density

$$\rho_h(k,\tau) = \frac{k^4}{a^4(\tau)\pi^2}|\beta_k(\tau)|^2.$$

- Singular at horizon crossing $\omega_k = 0$
- Faster for large k modes
- Normalization relation $|\alpha_k(\tau)|^2 - |\beta_k(\tau)|^2 = 1$ can be used to track precision

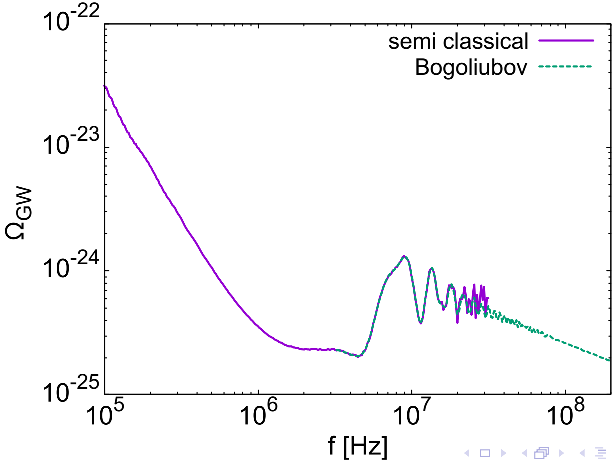
Chaotic inflation $T_{RH} = 10^{10}\,GeV$

$$V = \frac{1}{2}m^2\phi^2, \quad m_\phi \sim 1.4 \times 10^{13}\,\mathrm{GeV}$$



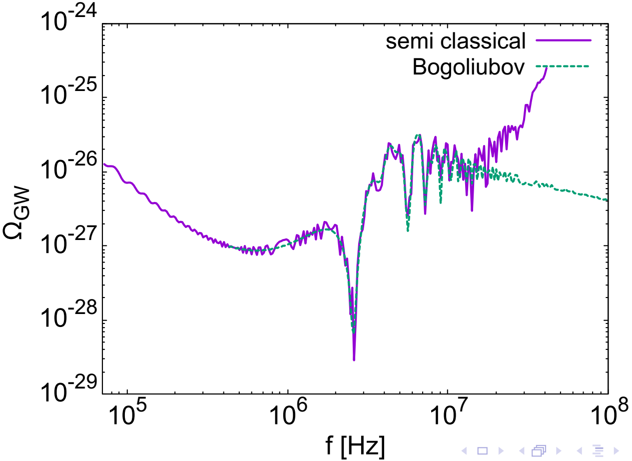
Starobinsky inflation $T_{RH} = 10^{10}\,GeV$

$$V = \frac{3 m_\phi^2 M_{\rm Pl}^2}{4}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}\right)\right]^2, \quad m_\phi \simeq 3\times 10^{13}\,\mathrm{GeV}$$



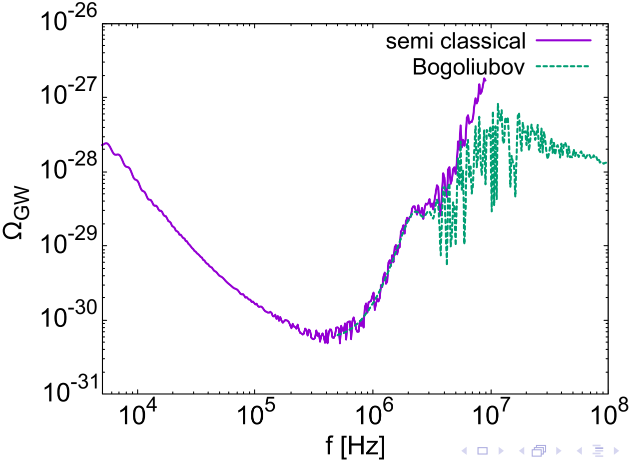
Hilltop inflation $v = M_{\rm pl}, \quad n = 4, \quad T_{RH} = 10^{10}\, GeV$

$$V = \Lambda^4 \left[1 - \left(\frac{\phi}{v}\right)^n\right]^2, \quad \frac{H_{\rm e}}{m_\phi} = \frac{v}{\sqrt{6} n M_{\rm Pl}}$$



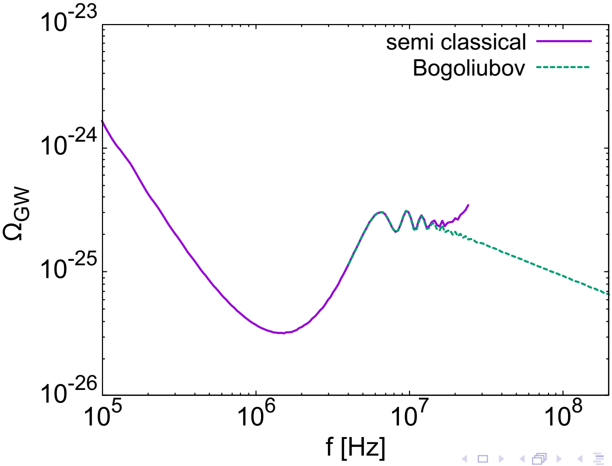
Hilltop inflation $v = 0.1M_{\rm pl}$, $n = 4$, $T_{RH} = 10^{10}GeV$

$$V = \Lambda^4\left[1-\left(\frac{\phi}{v}\right)^n\right]^2, \quad \frac{H_e}{m_\phi} = \frac{v}{\sqrt{6}nM_{\rm Pl}}$$



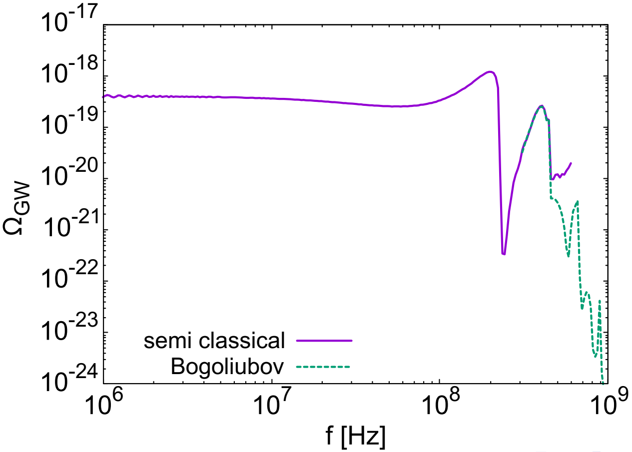
α-attractor T model $n = 2, \quad T_{RH} = 10^{10}\,GeV$

$$V = \frac{\lambda}{n}\left[\Lambda \tanh\left(\frac{|\phi|}{\Lambda}\right)\right]^n, \quad m_\phi \simeq 1.4 \times 10^{13}\,\mathrm{GeV}\left(\frac{60}{N}\right) \text{ for } n = 2$$



α-attractor T model $n = 4, \quad T_{RH} = 10^{10}\,GeV$

$$V = \frac{\lambda}{n}\left[\Lambda \tanh\left(\frac{|\phi|}{\Lambda}\right)\right]^n, \quad w = 1/3$$



Conclusion

- Primordial GW spectrum extends beyond the superhorizon modes
- Nontrivial model specific structure appears when $m_\phi \neq H_e$
- For GWs that reenter the horizon the spectrum reveals w during reheating
- Signal too weak to be detected in near future
- Might overlap with other high frequency GW sources - bremsstrahlung, scatterings in the thermal bath, inflaton self interaction