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Stochastic gravitational wave from graviton bremsstrahlung in inflaton decay into massive spin 3/2 particles

Sourav

In collaboration with

Dr. Diganta Das and Mihika Sanghi

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Center for Computational Natural Sciences and Bioinformatics
International Institute of Information Technology, Hyderabad 500 032, India

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Motivation: Probing the Reheating Era

- **Direct Evidence of Inflation:** While CMB anisotropies provide strong but indirect evidence for inflation, detecting a stochastic GW background would directly probe inflationary dynamics.
- **Reheating Dynamics:** After inflation, the inflaton coherently oscillates about its potential minimum, transferring energy to relativistic particles and setting the stage for BBN.
- **Graviton Bremsstrahlun:** Perturbative decay of the inflaton is universally accompanied by **graviton bremsstrahlung**, sourcing high-frequency GWs.
- **Perturbative Reheating:** We numerically analyze the impact of:
  - Inflaton decays into massive **spin-3/2** Rarita-Schwinger particles.
  - **Potential Shape:** Polynomial forms $V(\phi) \sim \phi^k$ where $k \geq 2$.

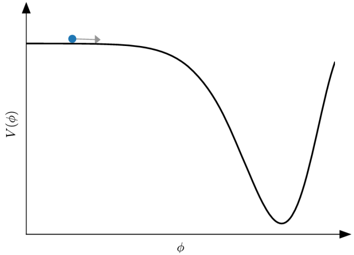


Figure: Inflaton oscillations near the potential minimum $V(\phi) \sim \phi^k$. The exponent k governs the reheating dynamics and GW spectra.

The Gravitational Interaction: An Unavoidable Signal

- **Linearized Theory:** Since the Hubble timescale is much longer than the inflaton oscillation period ($T \ll H^{-1}$), the cosmic expansion can be neglected over the decay timescale. The spacetime can therefore be approximated as Minkowski, and the metric expanded as:

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + \frac{2}{M_{Pl}} h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

- **Quantization:** To compute interactions in a QFT framework, the metric perturbations $h_{\mu\nu}$ are promoted to operators. Their excitations correspond to the two propagating degrees of freedom of the **graviton**.
- **Universal Coupling:** Gravitons emerge as fluctuations of the metric and, by the Equivalence Principle, couple universally to the energy-momentum tensor ($T^{\mu\nu}$) of all fields involved:

$$\mathcal{L}_{\text{int}} \supset \frac{1}{M_{Pl}} h_{\mu\nu} (T_{\phi}^{\mu\nu} + T_{\psi}^{\mu\nu})$$

- **Graviton Bremsstrahlung:** Consequently, any perturbative decay of the inflaton is accompanied by graviton emission, sourcing a stochastic GW background.(*Source: Nakayama and Tang, 2019*)

Action and Lagrangian

The complete dynamics of the reheating phase are governed by the Action:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) + \mathcal{L}_{3/2} + \mathcal{L}_{int}\right]$$

- **Inflaton Potential:** Assumed to be $V(\phi) \sim \phi^k$ near the minimum ($\phi = 0$), where the exponent k dictates the reheating dynamics.
- **RS Field ($\mathcal{L}_{3/2}$):** Describes free massive spin-3/2 particles with mass $m_{3/2}$.
- **Interaction Lagrangian ($\mathcal{L}_{int}$):** Contains the universal gravitational terms and the specific inflaton-RS decay vertices:

$$\sqrt{-g}\mathcal{L}_{int} = \frac{1}{M_{Pl}}(h_{\mu\nu}T_\phi^{\mu\nu} + h_{\mu\nu}T_\psi^{\mu\nu}) + \underbrace{\lambda_s\bar{\psi}_\mu\psi^\mu\phi}_{\text{Scalar}} + \underbrace{i\lambda_p\bar{\psi}_\mu\gamma^5\psi^\mu\phi}_{\text{Pseudoscalar}}$$

The Rarita-Schwinger (RS) Field

- The RS field, denoted as ψ_μ, is a vector-valued spinor field describing a relativistic massive spin-3/2 particle.
- The Lagrangian for a free massive RS field is given by (*Source: Kaneta et al., arXiv:2309.15146*):

$$\mathcal{L}_{3/2} = \overline{\psi}_\mu (i\gamma^{\mu\rho\nu}\partial_\rho + m_{3/2}\gamma^{\mu\nu})\psi_\nu$$

  where $\gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$ and $\gamma^{\mu\nu\rho}$ is the antisymmetrized product of three gamma matrices.
- The resulting equations of motion imply four physical polarization states:

$$(i\,\partial\!\!\!/ - m_{3/2})\psi_\mu = 0, \quad \gamma^\mu\psi_\mu = 0, \quad \partial^\mu\psi_\mu = 0$$

- Calculations require summing over states using the **spin sum projectors**:
  - $\sum u_a\bar{u}_b = (p\!\!\!/ + m_{3/2}) \times \left(\eta_{ab} - \frac{1}{3}\gamma_a\gamma_b - \frac{2}{3}\frac{p_a p_b}{m_{3/2}^2} + \frac{p_a\gamma_b - p_b\gamma_a}{3m_{3/2}}\right)$
  - $\sum v_a\bar{v}_b = (p\!\!\!/ - m_{3/2}) \times \left(\eta_{ab} - \frac{1}{3}\gamma_a\gamma_b - \frac{2}{3}\frac{p_a p_b}{m_{3/2}^2} - \frac{p_a\gamma_b - p_b\gamma_a}{3m_{3/2}}\right)$

Inflaton Dynamics: The Oscillatory Phase

Near the potential minimum $\phi = 0$, we consider a power-law potential $V(\phi) = \lambda M_P^4(\phi/M_P)^k$. The evolution is split into a slowly-varying envelope $\phi_0(t)$ and a rapid oscillatory part $\mathcal{P}(t)$:

$$\phi(t) = \phi_0(t)\mathcal{P}(t)$$

Substituting this into the Klein-Gordon equation leads to a set of coupled equations:

Coupled Equations of Motion

- **Rapid Oscillations:**

$$\ddot{\mathcal{P}} + k\lambda M_p^{4-k}\phi_0^{k-2}\mathcal{P}^{k-1} = 0$$

- **Envelope Evolution:**

$$\dot{\phi}_0 + 3H\dot{\phi}_0 + k\lambda M_p^{4-k}\phi_0^{k-1}\langle\mathcal{P}^{k-1}\dot{\mathcal{P}}\rangle = 0$$

- $\mathcal{P}(t)$ is a periodic function with angular frequency $\omega \propto \phi_0^{(k-2)/2}$.
- The energy density averages to $\rho_\phi \propto \phi_0^k$, dictated by the virial theorem.

(*Source: Jiang and Suyama, arXiv:2410.11175*)

Classical Field Picture

The periodic part $\mathcal{P}(t)$ can be decomposed into a Fourier series, leading to the **Classical Field Picture**:

$$\mathcal{P}(t) = \sum_{n=-\infty}^{\infty} \mathcal{P}_n e^{-in\omega t}$$

Quadratic ($k = 2$)

- Purely sinusoidal; only $n = \pm 1$ exists.
- Equivalent to a collection of particles at rest with mass m_ϕ.
- **Particle Picture** is sufficient.

Anharmonic ($k > 2$)

- Highly non-sinusoidal; higher harmonics ($|n| > 1$) are non-zero.
- Equivalent to a **tower of effective masses** $m_n = n\omega$.
- Requires the **Classical Field Picture**.

Total Decay Rate: Calculated as the weighted sum over these harmonic modes:

$$\Gamma_\phi^{1\to 2} = \sum_{n=1}^{\infty} b_n \Gamma_{\phi_n}^{1\to 2}, \qquad \frac{d\Gamma^{1\to 3}}{dE_w} = \sum_{n=1}^{\infty} b_n \frac{d\Gamma_{\phi_n}^{1\to 3}}{dE_w}$$

Where, weight factor: $b_n = (k+2)(k-1)\left(\frac{\omega}{m_\phi}\right)^2 n^2 |\mathcal{P}_n|^2, \quad \sum_{n=1}^{\infty} b_n = 1$

The Bremsstrahlung Process: Amplitudes & Diagrams

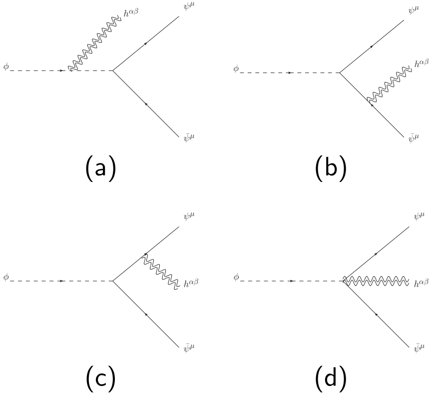


Figure: The four Feynman diagrams contributing to the decay $\phi \to \psi\bar{\psi}h$.

(*Source: Barman et al., arXiv:2301.11345*)

The matrix elements for these processes are:

$$i\mathcal{M}_1 = -i\frac{\lambda}{M_P}\frac{\bar{u}(p)\epsilon^{*\mu\nu}l_\mu l_\nu v(q)}{M_\phi E_w}$$

$$i\mathcal{M}_2 = i\frac{\lambda}{M_P}\frac{\bar{u}(p)\epsilon^{*\mu\nu}p_\mu p_\nu v(q)}{p\cdot\omega}$$

$$i\mathcal{M}_3 = i\frac{\lambda}{M_P}\frac{\bar{u}(p)\epsilon^{*\mu\nu}q_\mu q_\nu v(q)}{q\cdot\omega}$$

$$\mathcal{M}_4 \propto \eta_{\mu\nu}\epsilon^{*\mu\nu}$$

Graviton Polarization ($\epsilon^i_{\mu\nu}$): Symmetric: $\epsilon^{i\mu\nu} = \epsilon^{i\nu\mu}$; Traceless: $\eta^{\mu\nu}\epsilon^i_{\mu\nu} = 0$; Transverse: $\omega_\mu\epsilon^{i\mu\nu} = 0$

Key Simplifications:

- $\mathcal{M}_1 = 0$: The inflaton decays at rest, so its four-momentum is $l = (M_\phi, \mathbf{0})$.
- $\mathcal{M}_4 = 0$: Since the graviton is massless and thus traceless ($\eta_{\mu\nu}\epsilon^{*\mu\nu} = 0$).

Calculating the Decay Rates

Two-Body Rate ($\Gamma_\phi^{1\to 2}$): The primary energy transfer channel to radiation.

$$\Gamma^{1\to 2}_{\phi,(s/p)} = \frac{\lambda_{s,p}^2 m_\phi}{72\pi}\left[(k-1)(k+2)\left(\frac{\omega}{m_\phi}\right)^3 \sum_{n=1}^{\infty} |\mathcal{P}_n|^2 n^3 \times \Phi_{s,p}(y_n)\right]$$

Three-Body Differential Rate ($\frac{d\Gamma^{1\to 3}}{dE_\omega}$): Sources the stochastic GW background.

$$\frac{d\Gamma^{1\to 3}}{dE_\omega} = \frac{\lambda_{s,p}^2 (k+2)(k-1)}{2^{10}\cdot 3^5 \pi^3}\left(\frac{m_\phi}{M_{Pl}}\right)^2\left(\frac{\omega}{m_\phi}\right)^4\left[\sum_{n=1}^{\infty}\frac{n^4|\mathcal{P}_n|^2 \sum T_i^{s,p}(y_n, x_n, \alpha)}{x_n y_n^6(\alpha^2-1)}\right]$$

Kinematic Features:

- **Constraint:** $0 < x_n < \frac{1}{2} - 2y_n^2$, where $x_n = \frac{E_\omega}{n\omega}$, $y_n = \frac{m_{3/2}}{n\omega}$ and $\alpha = \sqrt{1 - \frac{4y_n^2}{1-2x_n}}$.
- **Multi-mode Nature:** The infinite sum over n reflects the decay of harmonic modes of the oscillating inflaton field.
- **Universal Coupling:** Both λ_s and λ_p contribute to graviton bremsstrahlung without interference.

Simulating the Cosmic Evolution: The Boltzmann Equations

We adopt a phenomenological reheating description where spin 3/2 particles (RS field) can decay to Standard Model fields through dimension 5 operator (supressed by Λ) at a rate faster than inverse decay rate of the inflaton i.e. $\Gamma_{3/2} \gg \Gamma_\phi$, while maintaining effective field theory validity with $m_\phi/\Lambda \ll 1$. This allows the use of coupled Boltzmann equations to map inflaton depletion to the subsequent radiation-dominated epoch.

1. Evolution of Inflaton and Radiation:

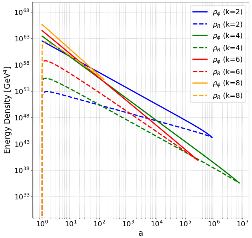
$$\dot{\rho}_\phi + 3H(1+w_\phi)\rho_\phi = -(1+w_\phi)\rho_\phi\left(\Gamma_\phi^{1\to2} + \Gamma_\phi^{1\to3}\right)$$

$$\dot{\rho}_R + 4H\rho_R = (1+w_\phi)\rho_\phi\left(\Gamma_\phi^{1\to2} + \int \frac{d\Gamma_\phi^{1\to3}}{dE_w} dE_w\right)$$

Solved subject to $H^2 = \frac{1}{3M_{Pl}^2}(\rho_\phi + \rho_R)$ to find T_{rh}.

2. Post-hoc GW Production:

$$\frac{d}{dt}\left(\frac{d\rho_{gw}}{dE_w}\right) + 4H\frac{d\rho_{gw}}{dE_w} = (1+w_\phi)\frac{\rho_\phi}{m_\phi}\frac{d\Gamma_\phi^{1\to3}}{dE_w}E_w$$



- Initial conditions: $\rho_R = 0$ at ϕ_{end}.
- $\rho_{\rm end} = \frac{3}{2}\lambda M_{Pl}^4 \left(\frac{\phi_{\rm end}}{M_{Pl}}\right)^k$.
- Intersection of ρ_ϕ and ρ_R defines reheating.

From Simulation to Observable Spectrum

Our Boltzmann simulation yields the energy densities at the time of reheating. To predict the signal we might observe today, we must account for cosmic expansion.

1. Define the Observable: The standard measure for a stochastic GW background is the dimensionless energy density spectrum, Ω_{GW}:

$$\Omega_{GW}(f) \equiv \frac{1}{\rho_{c,0}} \frac{d\rho_{GW,0}}{d(\ln f)} = \Omega_{\gamma}, 0 \frac{d(\rho_{\mathrm{gw}}(T_0)/\rho_R(T_0))}{d \ln f}$$

where $\rho_{c,0}$ is the critical energy density today, and f is the observed frequency.

2. Connect Past to Present: The spectrum today is related to the spectrum at reheating (T_{rh}) by the redshift of energy and the conservation of entropy:

$$\Omega_{GW}(f) = \Omega_{\gamma,0} \left[\frac{g_*(T_{rh})}{g_*(T_0)}\right] \left[\frac{g_{s*}(T_0)}{g_{s*}(T_{rh})}\right]^{4/3} \times \left.\frac{d(\rho_{GW}/\rho_R)}{d(\ln E_w)}\right|_{T_{rh}}$$

- $\Omega_{\gamma,0}$ is the present photon energy density fraction.
- g_* and g_{s*} are the relativistic degrees of freedom for energy and entropy.
- The final term, the ratio of GW to radiation energy, is the direct output of our Boltzmann simulation.

Result 1: Spectral Hierarchy and Multi-peak Features

The stochastic GW background is formed by the superposition of signals from individual harmonic modes of the oscillating inflaton.

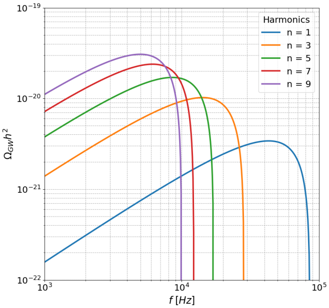


Figure: GW spectra from the first nine harmonics ($k = 6$).

Observations:

- **Hierarchy:** There is a distinct hierarchy in the GW amplitude across harmonics, related to their respective decay rates.
- **Saturation:** The total spectrum effectively saturates within the first nine harmonics for the k values studied.
- **Signature:** Superimposing these harmonics results in a unique **multi-peak feature**.
- **Invariance:** The qualitative features are insensitive to the specific type of coupling (λ_s vs λ_p).

Result 2: Impact of Potential Shape and Detectability

The resulting GW spectra are highly sensitive to the inflaton potential index k, which dictates the reheating dynamics.

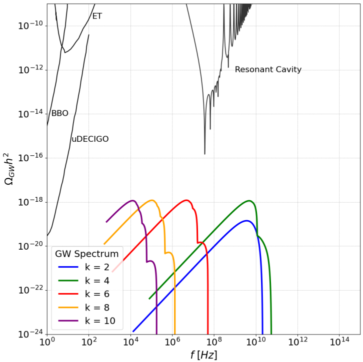


Figure: Present-day GW spectra for $k = 2, 4, 6, 8, 10$ compared with future detector sensitivities.

Key Observations:

- **Potential Index (k):** The choice of k determines the time of reheating; increasing k shifts the peak frequency and modifies the spectral amplitude.
- **Characteristic Shape:** For $k > 2$, the spectra exhibit unique multi-peak features from harmonic superpositions.
- **Future Sensitivity:** The predicted signals currently remain below the sensitivity curves of **BBO**, **uDECIGO**, **ET**, and **Resonant Cavities**.
- **Diagnostic Value:** If detected, the multi-peak signature and peak position would uniquely determine the inflationary potential and the RS field mass.

Result 3: Sensitivity to Couplings and RS Mass

The observable GW spectrum is highly sensitive to the microscopic parameters of the Lagrangian.

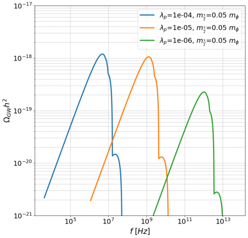


Figure: Effect of coupling λ_p ($k = 6$).

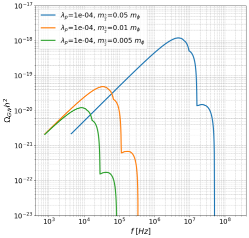


Figure: Effect of RS mass $m_{3/2}$ ($k = 6$).

Coupling ($\lambda_{s,p}$):

- Decreasing the coupling constant makes the spectrum peak at a **higher frequency**.
- The spectra are largely insensitive to whether the coupling is scalar (λ_s) or pseudoscalar (λ_p).

RS Particle Mass ($m_{3/2}$):

- The spectrum exhibits significant sensitivity to the mass of the spin-3/2 field.
- Benchmark values $m_{3/2} \leq 0.05 m_\phi$ ensure production in the relativistic regime.

Summary and Conclusion

- **Probing inflation Dynamics:** We have studied the stochastic GW spectra sourced by the perturbative decay of the inflaton into massive Rarita-Schwinger fields.
- **Parameter Sensitivity:** The characteristic spectra are highly sensitive to the inflaton potential exponent k, the RS mass $m_{3/2}$, and the Lagrangian coupling constants $\lambda_{s,p}$.
- **Unique Spectral Signatures:** For anharmonic potentials ($k > 2$), the superposition of harmonic modes produces a unique multi-peak feature that acts as a fingerprint for the inflationary potential and RS particle production.
- **Microscopic Window:** These high-frequency signals provide a fundamental window into the microscopic physics of inflation and the reheating era, potentially accessible to future GHz-band detectors.
- **Observational Potential:** While currently below future detector limits, an observation would uniquely identify the spin and potential shape governing the early universe.

The stochastic GW background remains a prime candidate for a direct probe into the dynamics of reheating.

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Thank You

Questions?