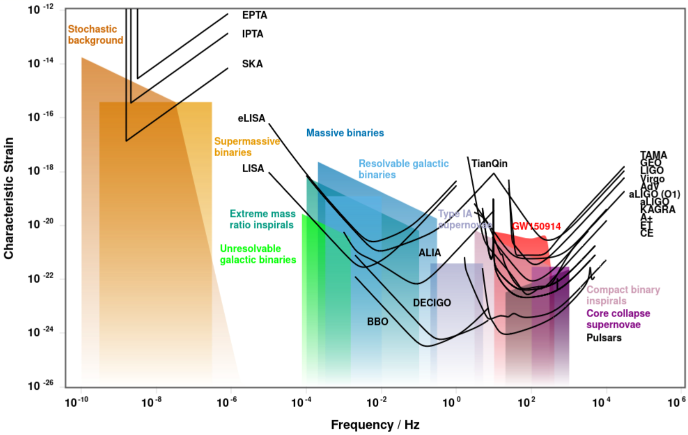
Upper Bound on Thermal Gravitational Wave Backgrounds from Hidden Sectors

Yannis Georis (ヤニス・ジョリス)
based on work in collaboration with M. Drewes, J. Klarić and P. Klose
[arXiv:2312.13855]

Gravitational Waves and the Early Universe: Accelerated Expansion, Dynamical Inhomogeneity, and Beyond
March 13, 2026

Gravitational Waves landscape

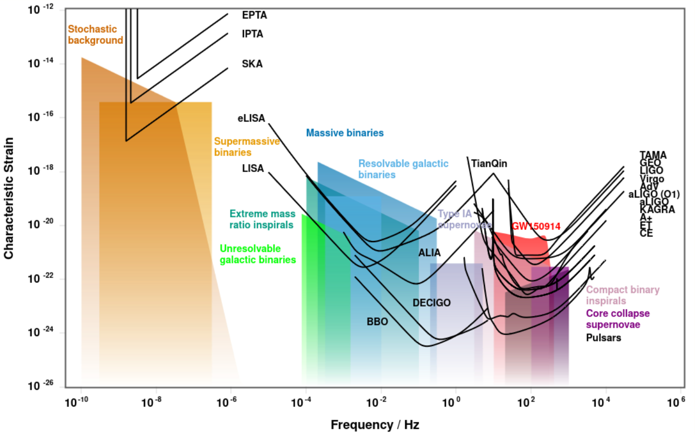


[Moore/Cole/Berry, 1408.0740]

- Combination of ground- and space-based interferometers + PTAs will ≈ cover the frequency band $[10^{-9}, 10^{4}]$ Hz

Gravitational Waves landscape

CMB B-modes



Ultra-high-frequency GW detectors

[Moore/Cole/Berry, 1408.0740]

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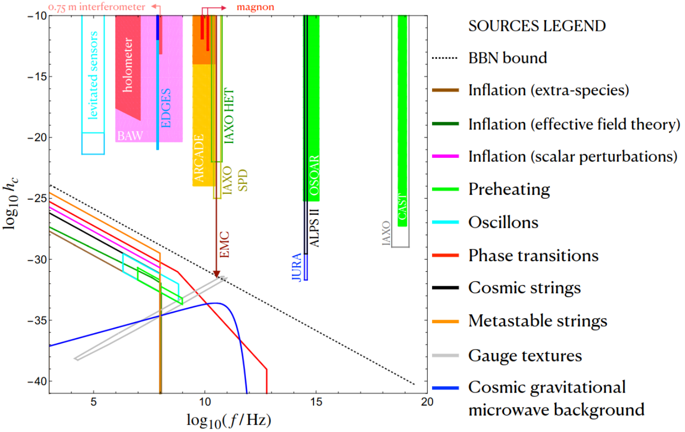
Ultra-High-Frequency Gravitational Waves



[arXiv:2011.12414]

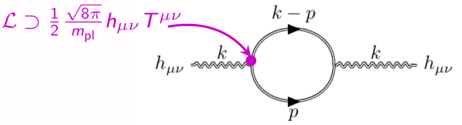
- No known astrophysical background at frequencies above 10^4 Hz
  $\longrightarrow$ Powerful probe of cosmological backgrounds
- Many different sources: inflation, preheating, topological defects, . . .

Gravitational Waves from thermal fluctuations

- Even in equilibrium, thermal plasma emit gravitational waves from microscopic processes ($\sim$ black-body radiation)

$$\frac{d\dot{e}_{gw}}{d\ln k} + 4H\frac{de_{gw}}{d\ln k} = 16\pi^2\left(\frac{k}{2\pi a}\right)^3 \frac{\Pi(k/a)}{m_{pl}^2} \quad \sim \langle T^{\mu\nu} T^{\rho\sigma}\rangle$$

- GW production rate governed by the self-energy



- Assuming standard cosmic history

$$h^2\Omega_{gw}(f) \approx 2.02\cdot 10^{-38}\times\left(\frac{f}{\mathrm{Hz}}\right)^3\times\int_{T_{min}}^{T_{max}}\frac{dT'}{m_{Pl}}\frac{\Pi\left(2\pi f a_0/a'\right)}{8T'^4}\,.$$

- Contribution is small but unavoidable! Act as cosmic GW floor.

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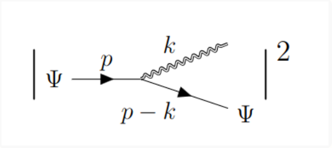
- GW production rate governed by the self-energy

$\mathcal{L} \supset \frac{1}{2}\frac{\sqrt{8\pi}}{m_{\mathrm{pl}}} h$



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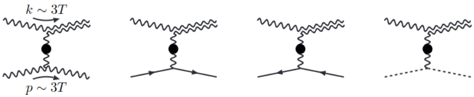
Boltzmann vs hydrodynamic regime

- For hard graviton momentum/frequency $k \sim \pi T$, Π dominated by particle scatterings (Boltzmann regime) *e.g.*

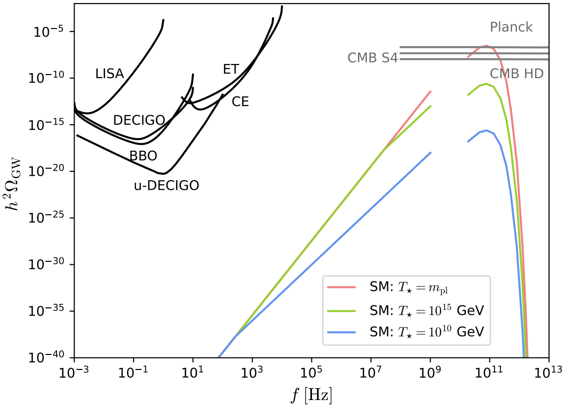


$\longrightarrow$ Computed at LO for the SM by [Ghiglieri/Jackson/Laine/Zhu, 2004.11392]

- For soft momentum $k \ll T$, long-range hydrodynamic fluctuations dominate (hydrodynamic regime)

$\longrightarrow$ Estimated for the SM by [Ghiglieri/Laine, 1504.02569]

Standard Model GW background



[See Ghiglieri/Jackson/Laine/Zhu '15,'20, Ringwald/Schütte-Engel/Tamarit '20]

- GW spectrum peaks for $f \sim 10^{11}$ Hz (Model independent: $k \sim \pi T$)

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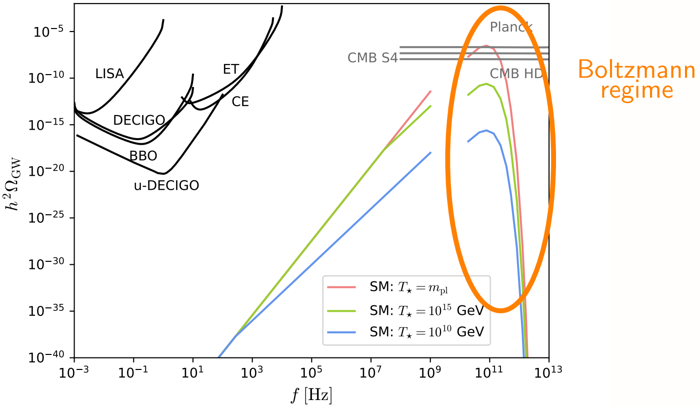

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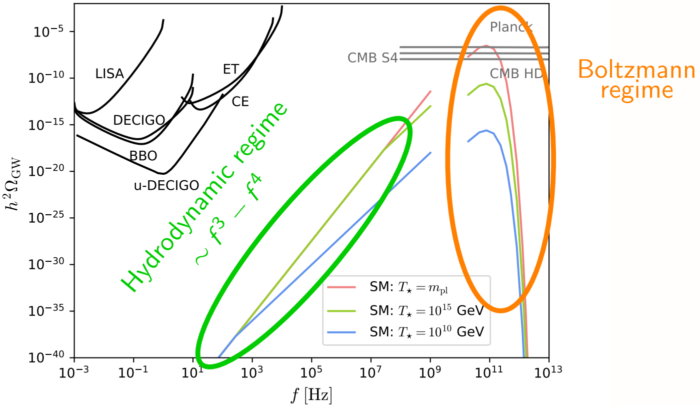

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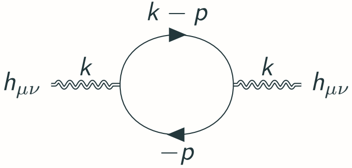
- Stress-energy tensor given at leading order by

$$T_{ij}(x) \supset \frac{c_X}{4}\overline{\psi}_x iD_{ij}\psi_x,\ iD_{ij} = \gamma_i i\partial_j + \gamma_j i\partial_i$$



Gravitational Waves from hidden sectors

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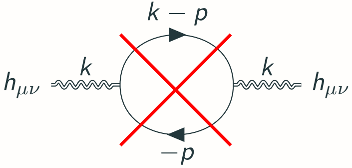

- Tree level contributions vanishes for kinematical reasons: Need to resum (in the hydrodynamic regime)!

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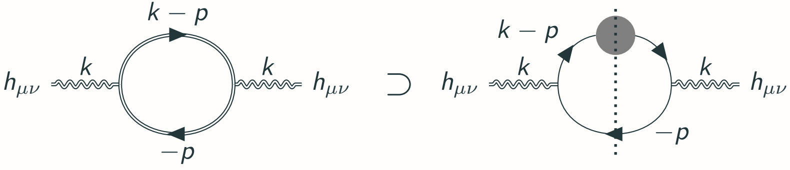

- In hydrodynamic regime, enhanced for feebly interacting particles

Plasma shear viscosity

$$\Pi(k) \sim 8T\eta \sim \frac{T^4}{\Upsilon}$$

Particles' width

$\longrightarrow$ Can dominate SM contribution!

Is it a good way to probe hidden sectors?

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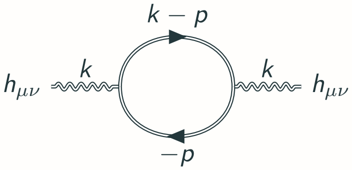

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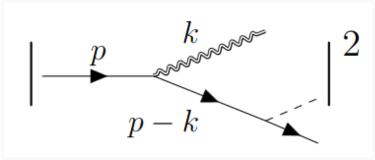

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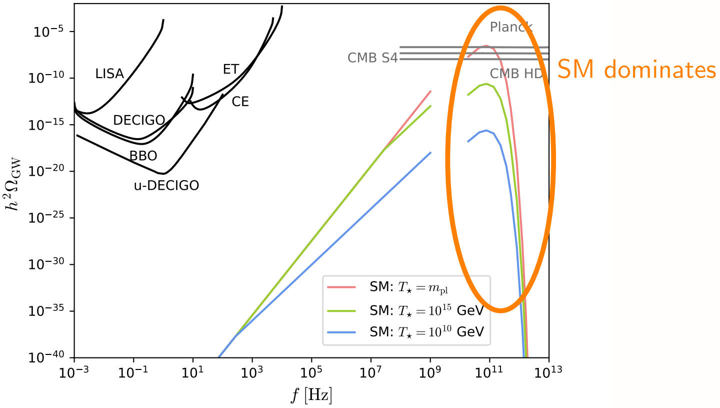

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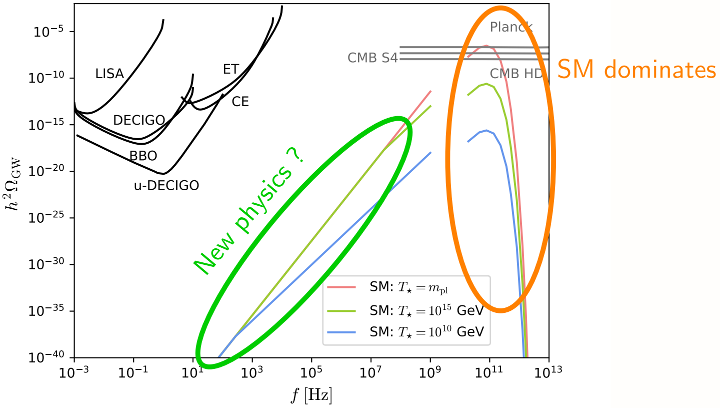

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GW production rate

- Fermionic production rate in real time (in-in) formalism

$$\Pi(k) = -\frac{c_X^2}{8}\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\int \frac{\mathrm{d}p_0}{2\pi}\, \mathbb{L}^{ij;kl}\, p_i p_k \,\mathrm{Tr}[\gamma_j iS_p^{>}\gamma_l iS_{p-k}^{<} + \gamma_j iS_p^{<}\gamma_l iS_{p-k}^{>}]$$

Traceless-transverse projector

Derivative coupling

$\sim (\not{p} + m)\frac{\Gamma_p}{\Omega_p^2+\Gamma_p^2}(1-f)$

- After some algebra,

$$\Pi(k) \overset{m\ll T}{\simeq} g_X \frac{16\pi^2}{225} T^5 \frac{5\Upsilon_{\text{av}}}{k^2 + 10\Upsilon_{\text{av}}^2}\,, \qquad g_X = \begin{cases} 1 & \text{Spin } 0 \\ 2\cdot c_X & \text{Spin } \frac{1}{2} \end{cases}$$

- Hydrodynamic regime:

$$k < \sqrt{10}\Upsilon_{\text{av}}, \Pi(k) \sim \frac{1}{\Upsilon_{\text{av}}},$$

Boltzmann regime:

$$k > \sqrt{10}\Upsilon_{\text{av}}, \Pi(k) \sim \Upsilon_{\text{av}}$$

- For renormalisable interactions $\Upsilon_{\text{av}} = yT$

$$k < \sqrt{10}\Upsilon_{\text{av}} \longleftrightarrow f < f_c = y\cdot 6\cdot 10^{10}\mathrm{Hz}$$

Hubble suppression

- GW evolution equation

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2 h_{ij}}{a^2} = \frac{16\pi\, T_{ij}}{a^2 m_{\rm Pl}^2}$$

- 2 regimes
  1. Super-Horizon ($k < H$) modes are static
  2. Sub-Horizon ($k > H$) modes

- GW production is delayed until $k = \frac{2\pi f a_0}{a} > H = T^2/M_0$

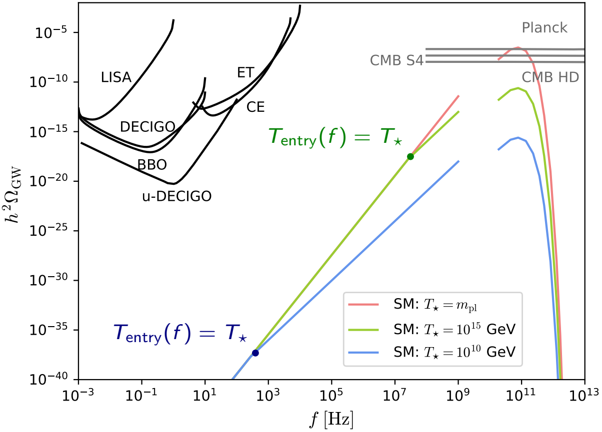
- In terms of temperature, production is delayed until

frequency dependent !

$$T < T_{\rm entry}(f) \approx 4 \cdot 10^7 \mathrm{GeV} \frac{f}{\mathrm{Hz}}$$

→ SM contribution behaves as f^4 at low frequencies, not f^3!

Standard Model GW background



[See Ghiglieri/Jackson/Laine/Zhu '15,'20, Ringwald/Schütte-Engel/Tamarit '20]

Upper bound on GW emission

- Production rate in the relativistic regime

$$\Pi(k) \overset{m \ll T}{\simeq} g_X \frac{16\pi^2}{225} T^5 \frac{5\Upsilon_{\mathrm{av}}}{k^2 + 10\Upsilon_{\mathrm{av}}^2}$$

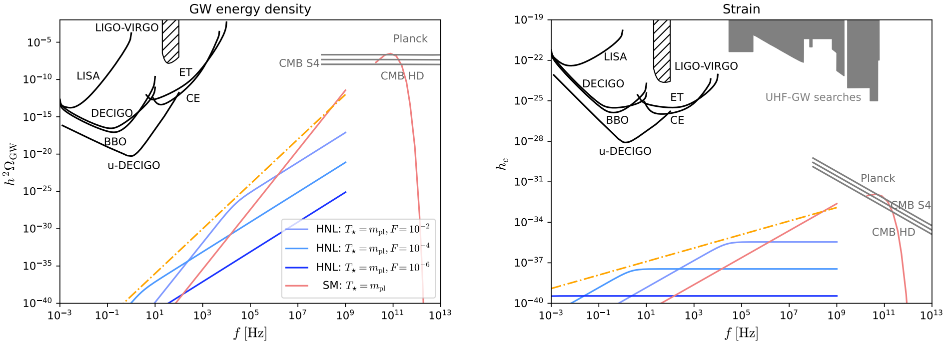
  $\longrightarrow$ Maximised for a width $\Upsilon_{\mathrm{av}} = k/\sqrt{10}$ ($\Upsilon_{\mathrm{av}} = k/2$ in the non-relativistic case)

- Leads to the model-independent upper bound

$$h^2\Omega_{\mathrm{gw}}(f) < 4.9 \cdot 10^{-40} \times g_X \left(\frac{f}{\mathrm{Hz}}\right)^3$$

- Tradeoff: Larger enhancement for smaller Υ but arises for smaller frequencies
- Does not apply in case of
  1. Out-of-equilibrium dynamics
  2. Hidden sector hotter than SM
  3. Beyond radiation domination

Illustration: Right-handed neutrinos



[Drewes/YG/Klarić/Klose, 2312.13855]

- Coupled to SM through Yukawa coupling

$$\mathcal{L} \supset F\psi(\tilde{\phi}^\dagger \ell) + \text{h.c.}$$

- Right-handed neutrino width

$$\Upsilon_{\rm av} \simeq 0.2\frac{F^2 T}{16\pi} \ ,$$

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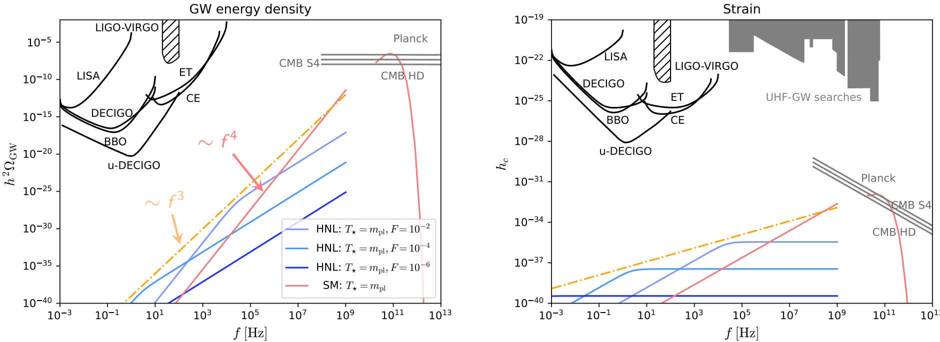

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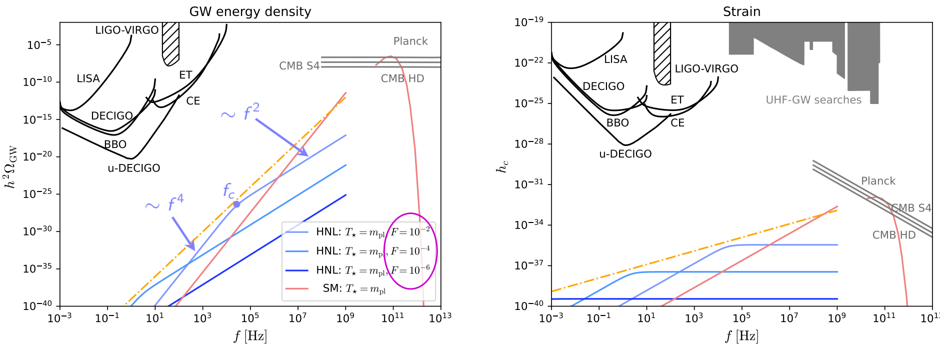

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GWs beyond SM radiation domination

- Can we adapt our formula to GW production if
  1. SM do not dominate energy budget?
  2. entropy exchange between SM and hidden sector?

- If production is still thermal:

$$h^2\Omega_{\rm gw}(f) \approx 2.02 \cdot 10^{-38} \times \left(\frac{f}{\rm Hz}\right)^3 \int_{T_{\min}}^{T_{\max}} \frac{dT'}{m_{\rm Pl}} \frac{\Pi\left(\frac{2\pi f a_0}{a}\right)}{8T'^4} \left|\frac{d\ln a}{d\ln T'}\right| \left(\frac{T'}{\overline{T}'}\right) \left(\frac{\rho_{\rm SM}}{\rho_{\rm tot}}\right)^{\frac{1}{2}}$$

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Radiation domination

Non-standard cosmic history

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Radiation domination

Non-standard cosmic history

- Many works on graviton bremstrahlung during reheating: expect hydrodynamic and Hubble suppression effect to be relevant at low frequencies!

Summary and outlook

- Ultra-High-Frequency GWs are powerful probes of new physics because of lack of astrophysical background
- Can be produced by plasma in thermal equilibrium
  $\rightarrow$ Constitutes an irreducible background for every theories
- Background can be enhanced for feebly interacting particles
- Upper bound on such background is very restrictive
- Inclusion of hydrodynamic and Hubble suppression crucial for accurate estimate of GW emission
- Formalism can also be applied to non-equilibrium situations

Thanks for your attention!

ご清聴ありがとうございました。

Backup slides

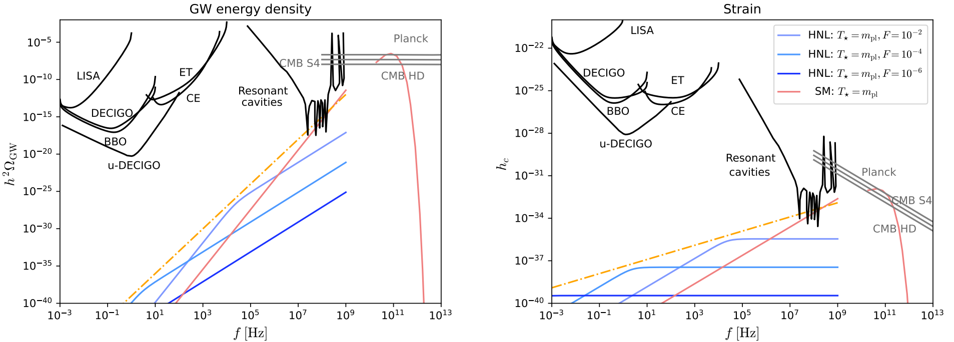
N_{eff} as Big Bang thermometer

	SM	νMSM	SMASH	MSSM
T_{max} [GeV] $<$	$(1.2\text{-}5.1) \times 10^{19}$	$(1.3\text{-}5.4) \times 10^{19}$	$(1.4\text{-}6.0(1)) \times 10^{19}$	$(2.3\text{-}9.4) \times 10^{19}$
$T_{\text{max}}^{\Delta N_{\text{eff}}=0.001}$ [GeV] $<$	2.3×10^{17}	2.4×10^{17}	2.7×10^{17}	4.39×10^{17}

[Ringwald/Schütte-Engel/Tamarit '20]

- Can (in theory) probe the maximal temperature of the SM plasma by measuring N_{eff}

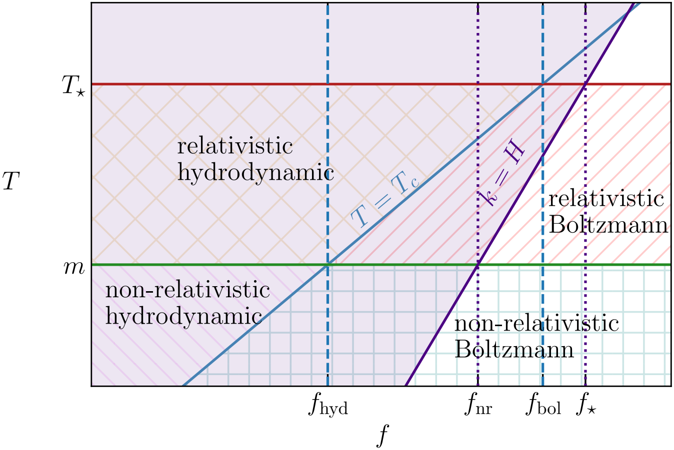
Resonant cavity searches



[Drewes/YG/Klarić/Klose, 2312.13855]

Resonant cavity searches [Herman/Lehoucq/Füzfa, '22] can potentially test these models but rely on unknown technology !

Production regimes



[Drewes/YG/Klarić/Klose, 2312.13855]

- Different possible scenarios depending on Hubble vs hydrodynamics scale

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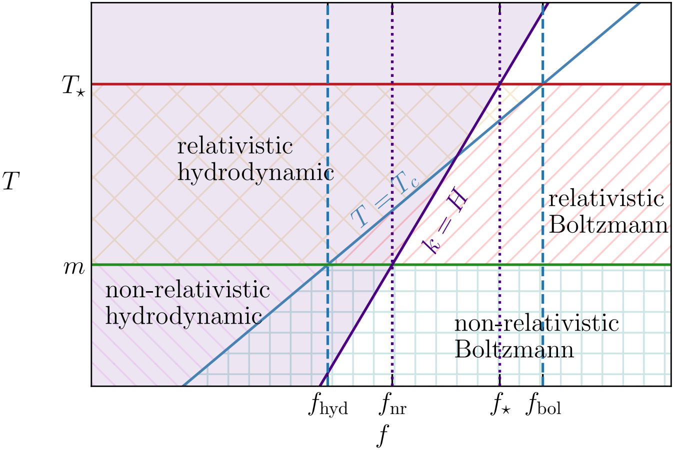

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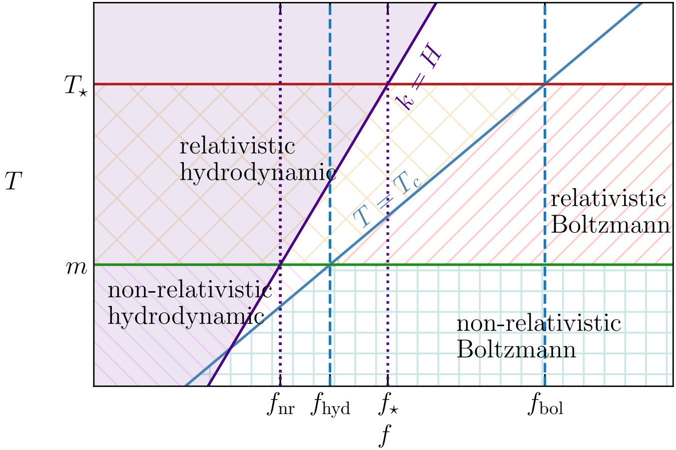


[Drewes/YG/Klarić/Klose, 2312.13855]

- Different possible scenarios depending on Hubble vs hydrodynamics scale

Case 1

$$h^2\Omega_{\rm gw}^{T>m}(f) \overset{f_X>f_{\rm bol}}{\simeq} g_X \frac{1.6\cdot 10^{-40}}{2(d-4)+1}\left(\frac{f}{\rm Hz}\right)^2 \frac{f_{\rm bol}}{\rm Hz}\begin{cases} 0 & \text{for} \quad f<f_{\rm nr}\,, \\ \left(\frac{f}{f_\star}\right)^{2(d-4)} & \text{for} \quad f_{\rm nr}<f<f_\star\,, \\ \frac{f_\star}{f} & \text{for} \quad f_\star<f\,. \end{cases}$$

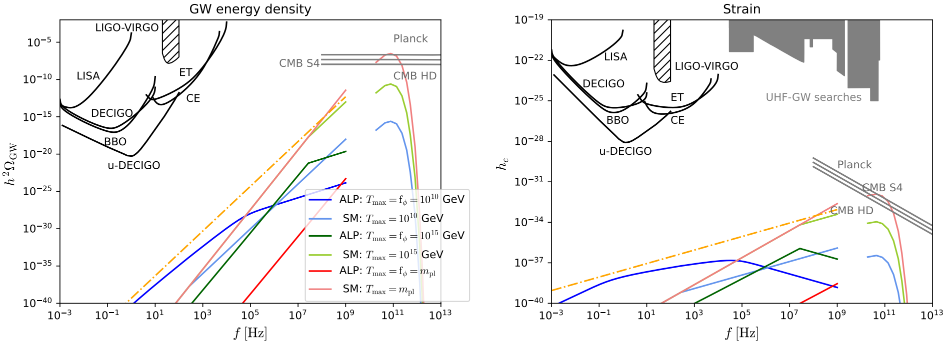
Case 2

$$h^2\Omega_{\rm gw}^{T>m}(f) \overset{f_\star \gg f_{\rm bol}}{\simeq} g_X \frac{1.6\cdot 10^{-40}}{2(d-4)+1}\left(\frac{f}{\rm Hz}\right)^2 \frac{f_{\rm bol}}{\rm Hz}\begin{cases} 0 & \text{for} \quad f < f_{\rm nr}\,, \\ \left(\frac{f}{f_\star}\right)^{2(d-4)} & \text{for} \quad f_{\rm nr} < f < f_x\,, \\ \beta\frac{f_x}{f_{\rm bol}}\left(\frac{f}{f_x}\right)^{\frac{1}{2(d-4)}} & \text{for} \quad f_x < f < f_{\rm bol}\,, \\ \frac{f_\star}{f} & \text{for} \quad f_{\rm bol} < f\,. \end{cases}$$

Case 3

$$h^2\Omega_{\rm gw}^{T>m}(f) \underset{f_{\rm hyd}\gg f_{\rm nr}}{\overset{f_{\rm bol}\gg f_\star}{\simeq}} g x \frac{1.6\cdot 10^{-40}}{2(d-4)+1}\left(\frac{f}{\rm Hz}\right)^2 \frac{f_{\rm bol}}{\rm Hz}\begin{cases} 0 & \text{for} \quad f < f_{\rm hyd}\,, \\ \beta\,\frac{f_x}{f_{\rm bol}}\left(\frac{f}{f_x}\right)^{\frac{1}{2(d-4)}} & \text{for} \quad f_{\rm hyd} < f < f_{\rm bol}\,, \\ \frac{f_\star}{f} & \text{for} \quad f_{\rm bol} < f\,. \end{cases}$$

Example 2: Axion-like particles

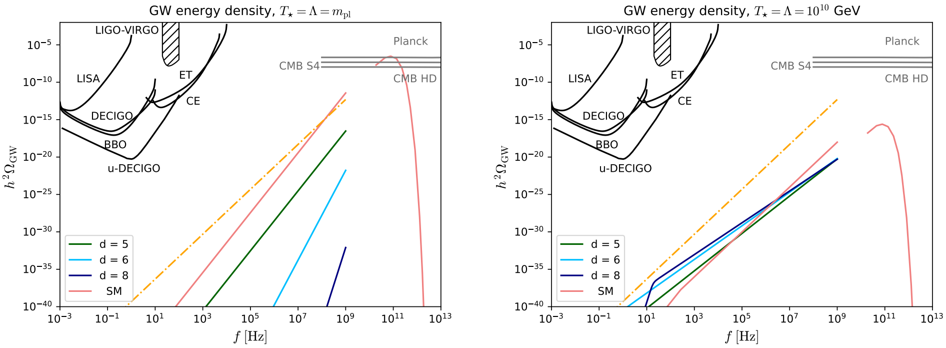


[Drewes/YG/Klarić/Klose, 2312.13855]

- Thermal width

$$\Upsilon_{\rm av} \overset{m \ll T}{=} \kappa\, n_c^3 (n_c^2 - 1) \frac{\alpha^5 T^3}{f_\phi^2} \;, \quad \kappa \approx 1.5 \;, \quad \frac{1}{\alpha} \approx \frac{22 n_c}{12\pi} \ln\left(\frac{2\pi T}{\Lambda_{\rm IR}}\right)$$

Example 3: Higher dimensional operators



[Drewes/YG/Klarić/Klose, 2312.13855]

- Assuming generically that the width scales as

$$\Upsilon_{\rm av} \simeq y\, T \left(\frac{T}{\Lambda}\right)^{2(d-4)} \begin{cases} 1 & T \gg m \\ \left(\frac{m}{T}\right)^n & T \lesssim m \end{cases}, \quad n \leq 1 + 2(d-4)$$

- Unavoidable contribution to the width (at least) at $d = 8$ from graviton exchanges

Frequency dependence of the GW spectrum $T^\star = m_{\rm pl}$

$$h^2\Omega_{\rm gw}^{T>m}(f) \overset{f_x > f_{\rm bol}}{\simeq} g_X \frac{1.6 \cdot 10^{-40}}{2(d-4)+1} \left(\frac{f}{\rm Hz}\right)^2 \frac{f_{\rm bol}}{\rm Hz} \begin{cases} 0 & \text{for} \quad f < f_{\rm nr}\,, \\ \left(\frac{f}{f_\star}\right)^{2(d-4)} & \text{for} \quad f_{\rm nr} < f < f_\star\,, \\ \frac{f_\star}{f} & \text{for} \quad f_\star < f\,. \end{cases}$$

- Such frequency scaling depends on the ratios between $T_\star, \Lambda, m, \ldots$

  $\longrightarrow$ Can extract information on the particle's properties from the scaling !