国立天文台 NAOJ

Gravitational Waves and the Early Universe: Accelerated Expansion, Dynamical Inhomogeneity, and Beyond

The Imprints of Primordial Non-Gaussianity on the Scalar-Induced Gravitational-Wave Background

Based on arXiv:2505.16820

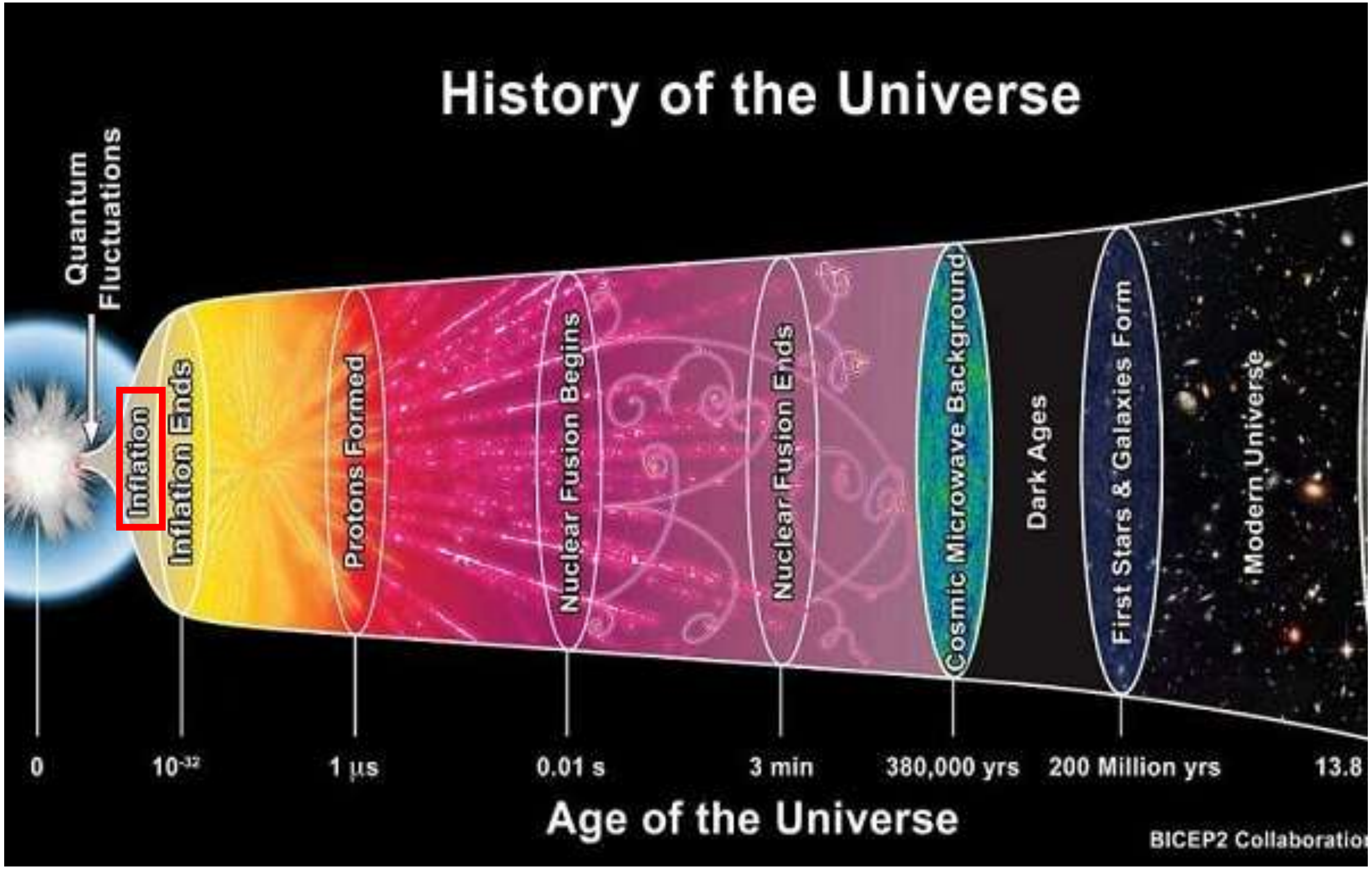
by Jun-Peng Li, Sai Wang, Zhi-Chao Zhao, and Kazunori Kohri

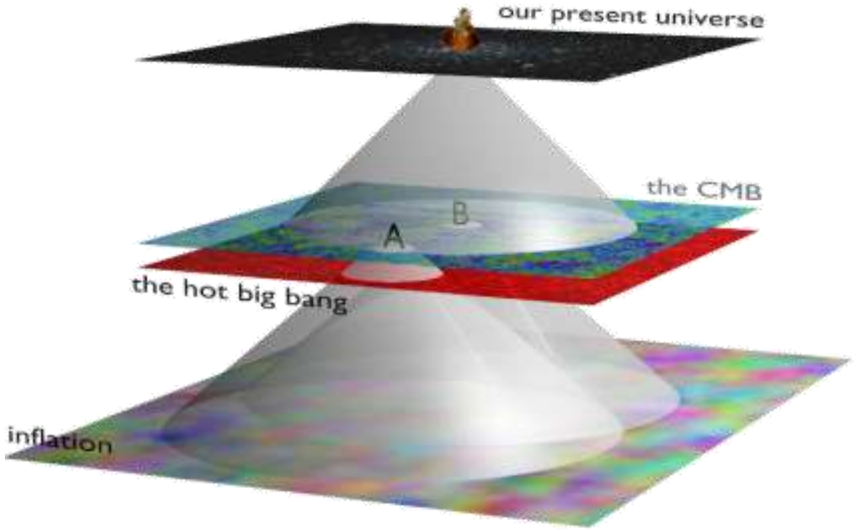
Speaker: Jun-Peng Li (李俊蓬)
National Astronomical Observatory of Japan

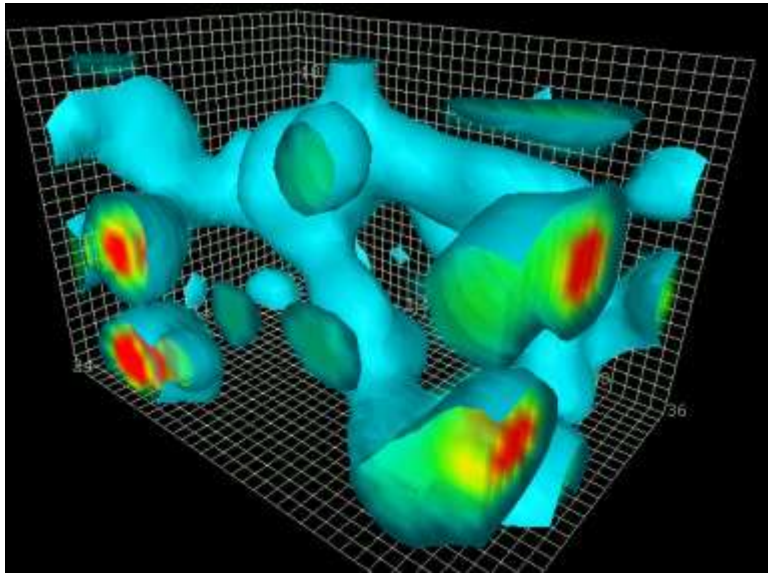
Nagoya U, March 12th, 2026

OUTLINE

Primordial Curvature Perturbations







primordial curvature perturbations ⟷ quantum fluctuations

Primordial Non-Gaussianity

Metric during inflation: $\mathrm{d}s^2 = a^2(\eta)\left[-e^{2\zeta}\mathrm{d}\eta^2 + e^{-2\zeta}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]$

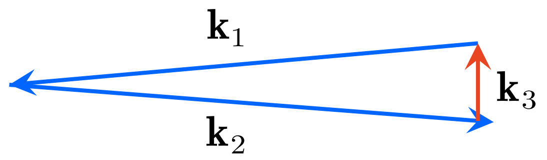
Statistics: $\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = \delta^{(2)}(\mathbf{k}_1+\mathbf{k}_2)P_\zeta(k_1)$ $\quad \mathrm{D}(X) = \mathrm{E}\left[(X-\mathrm{E}(X))^2\right]$

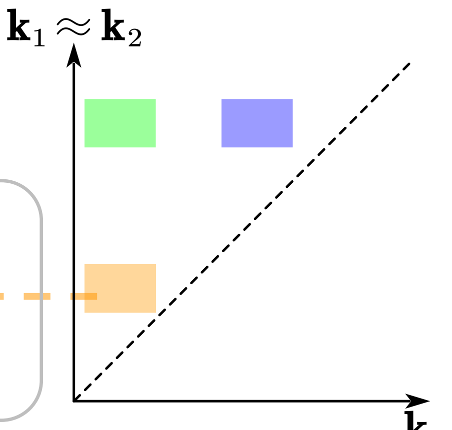
$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = \delta^{(2)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\zeta(k_1,k_2,k_3)$ $\quad \mathrm{Skew}(X) = \mathrm{E}\left[\left((X-\mathrm{E}(X))/\sqrt{\mathrm{D}(X)}\right)^3\right]$

$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle_c = \delta^{(2)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)T_\zeta(k_1,k_2,k_3,k_4)$ $\quad \mathrm{Kurt}(X) = \mathrm{E}\left[\left((X-\mathrm{E}(X))/\sqrt{\mathrm{D}(X)}\right)^4\right]$

local-type

$$\zeta = \zeta_{\mathrm{g}} + \frac{3}{5}f_{\mathrm{NL}}\left(\zeta_{\mathrm{g}}^2 - \langle\zeta_{\mathrm{g}}\rangle^2\right) + \frac{9}{25}g_{\mathrm{NL}}\zeta_{\mathrm{g}}^3 + \cdots$$



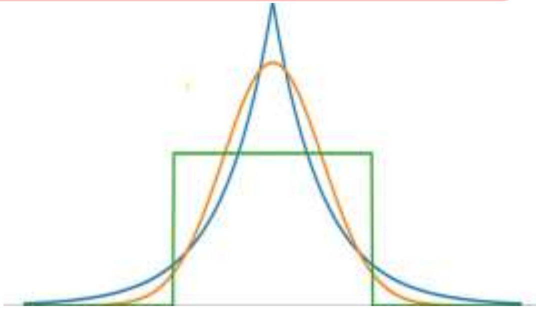


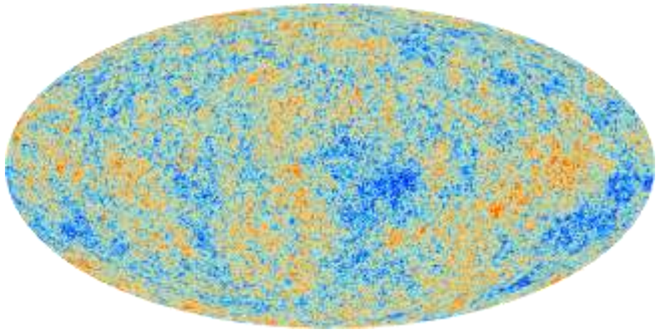
$f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$

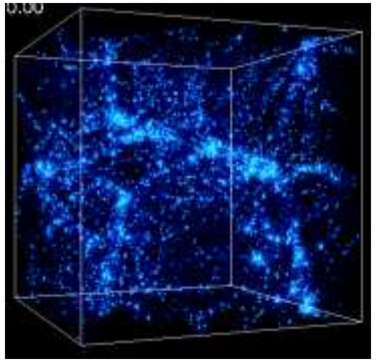
Planck 2018 Results

$f_{\mathrm{NL}} = -0.9 \pm 5.1$ (68% CL)

$g_{\mathrm{NL}} = (-5.8 \pm 6.5) \times 10^4$ (68% CL)

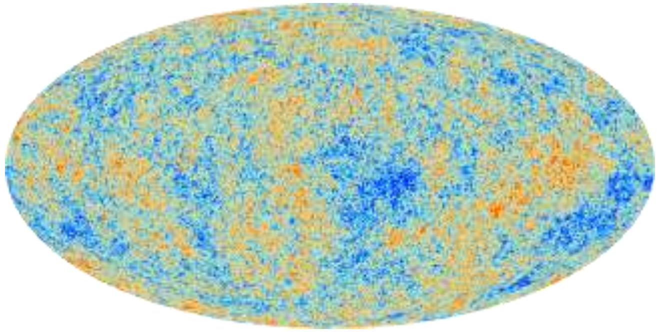




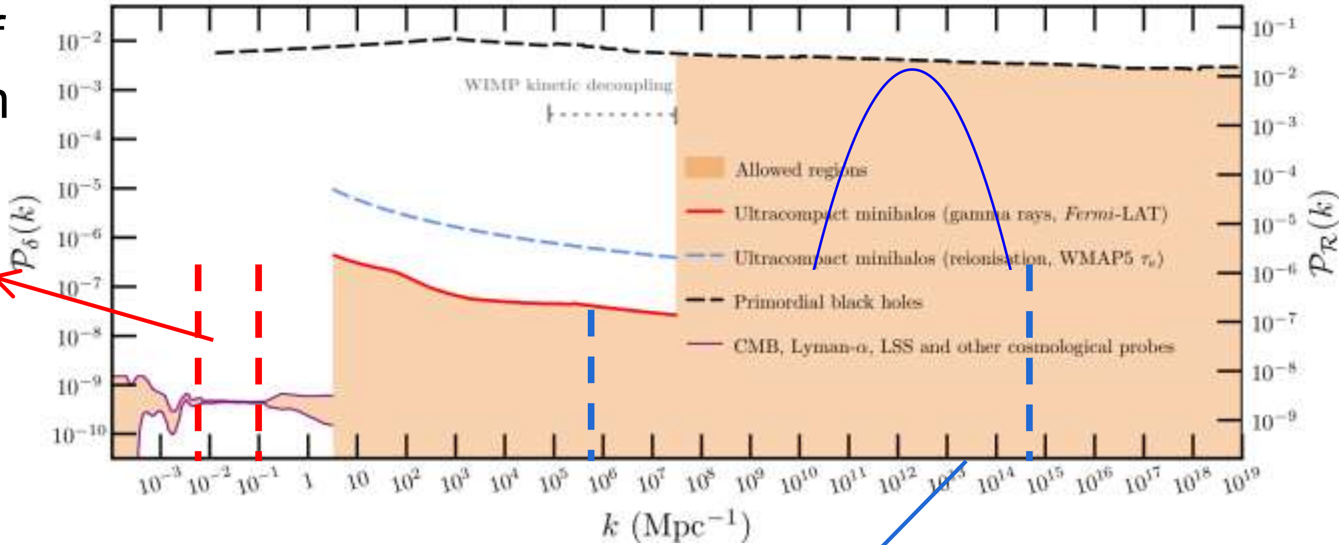


CMB & LSS: $\sim 10^{-16}\mathrm{Hz}$

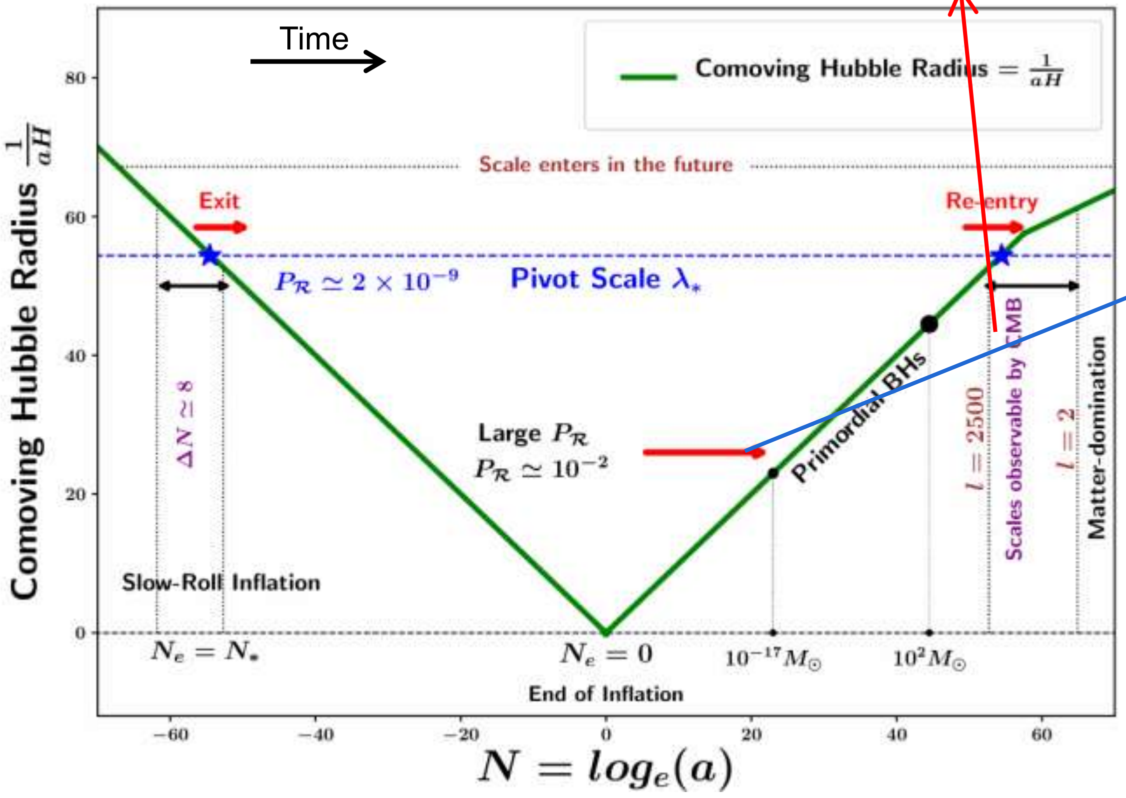
Small- & Large-Scale Decomposition $\zeta = \zeta_S + \zeta_L$



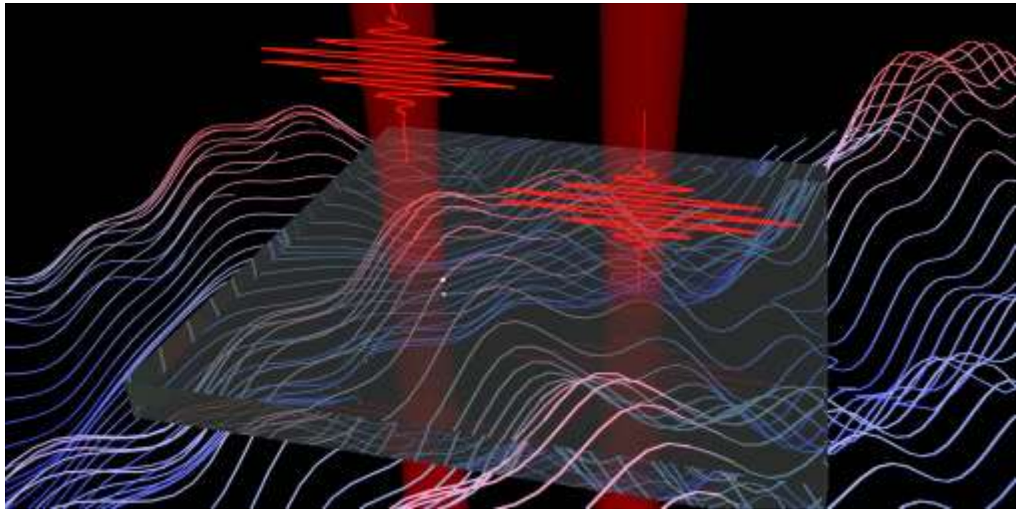
A : Amplitude of power spectrum



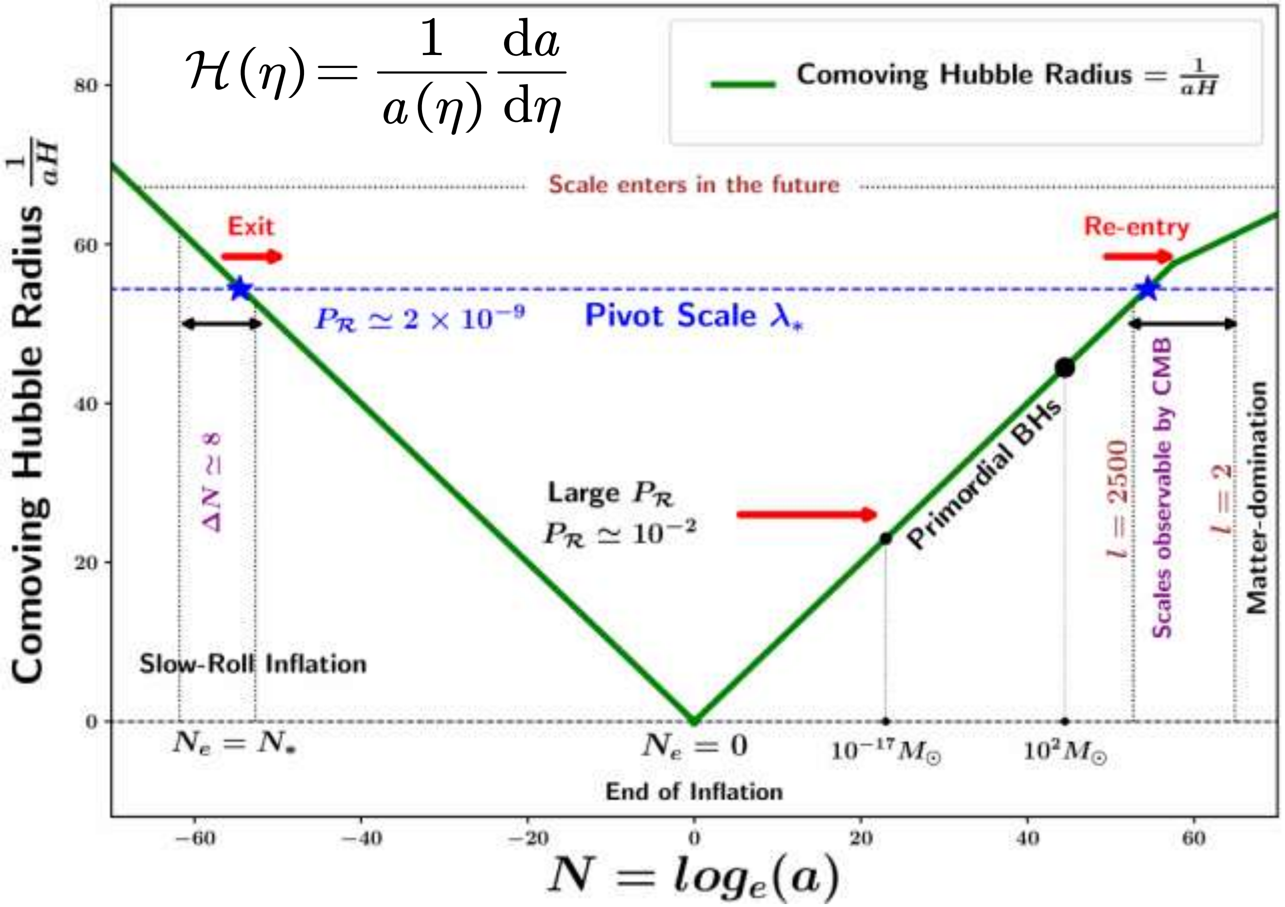
Large-Scale:
$\sim 10^{-16}$ Hz
$A_L \sim 10^{-9}$



Small-Scale: $10^{-9} \sim 1$ Hz
Assum peak: $A_S \lesssim 10^{-2}$



Scalar-Induced Gravitational Waves



$$ds^2 = a^2\left[-e^{2\Phi}d\eta^2 + \left(e^{-2\Phi}\delta_{ij} + \frac{h_{ij}}{2}\right)dx^i dx^j\right]$$

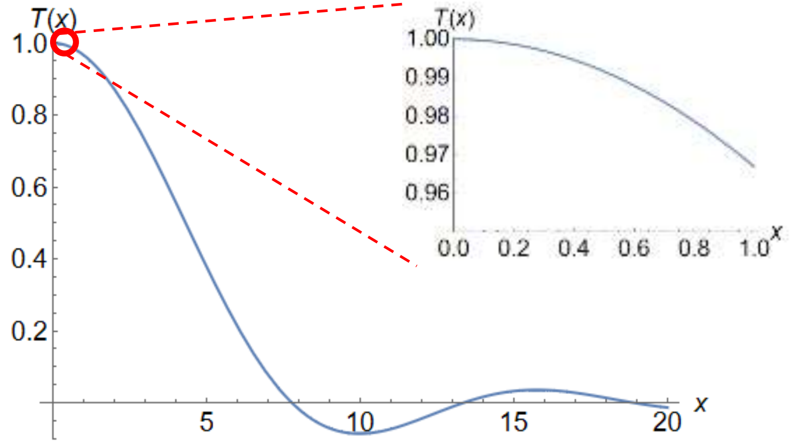
Perfect Fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + P g_{\mu\nu}$

SIGW Motion Equation

$$\partial_\eta^2 h_\lambda(\eta,\mathbf{q}) + 2\mathcal{H}\partial_\eta h_\lambda(\eta,\mathbf{q}) + q^2 h_\lambda(\eta,\mathbf{q}) = 4S_\lambda(\eta,\mathbf{q})$$

$$S_\lambda \sim Q_\lambda \Phi\Phi$$

$$\Phi(\eta,\mathbf{q}) = \left(\frac{3+3w}{5+3w}\right)T(q\eta)\zeta(\mathbf{q})$$



$$h_\lambda(\eta,\mathbf{q}) = 4\int \frac{d^3\mathbf{q}_a}{(2\pi)^{3/2}}\zeta(\mathbf{q}_a)\zeta(\mathbf{q}-\mathbf{q}_a)Q_\lambda(\mathbf{q},\mathbf{q}_a)\hat{I}(|\mathbf{q}-\mathbf{q}_a|,q,\eta)$$

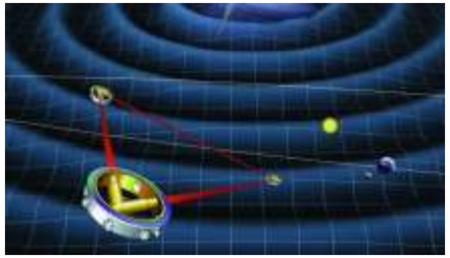
$$\hat{I}(|\mathbf{q}-\mathbf{q}_a|,q,\eta) \sim T(|\mathbf{q}-\mathbf{q}_a|\eta)T(q\eta)$$

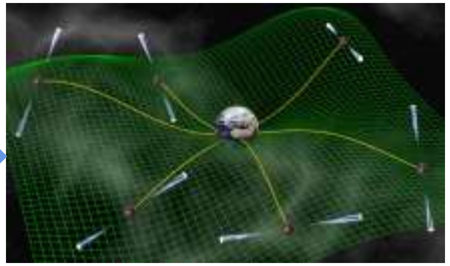
arxiv: 1804.08577

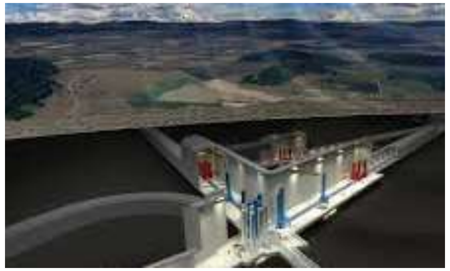
Energy-density Full Spectrum

Definition: $\omega_{\mathrm{gw}}(\eta,\mathbf{x},\mathbf{q}) = \dfrac{1}{\rho_{\mathrm{c}}(\eta)} \dfrac{\mathrm{d}\rho_{\mathrm{gw}}(\eta,\mathbf{x})}{\mathrm{d}\ln q\, \mathrm{d}^2\mathbf{n}}$ $\quad \mathbf{q} = q\,\mathbf{n}$

Density contrast: $\omega_{\mathrm{gw}}(\eta,\mathbf{x},\mathbf{q}) = \bar{\omega}_{\mathrm{gw}}(\eta,q)\,(1+\delta_{\mathrm{gw}}(\eta,\mathbf{x},\mathbf{q}))$







Initial moment (SIGW production) $(\eta_{\mathrm{in}},\mathbf{x}_{\mathrm{in}})$

GW energy-density on subhorizon scales

$$\rho_{\mathrm{gw,in}} = \frac{m_{\mathrm{Pl}}^2}{16 a_{\mathrm{in}}} \overline{\partial_l h_{ij} \partial_l h_{ij}}$$

Energy-density full spectrum

$$\bar{\omega}_{\mathrm{gw,in}} \sim \langle\zeta\zeta\zeta\zeta\rangle$$

$$\delta_{\mathrm{gw,in}} \sim \langle\zeta\zeta\zeta\zeta\rangle$$

Cosmic Expansion

Boltzmann equation (low multipoles)

Present-day (observation) $(\eta_0,\mathbf{x}_0)$

$q = 2\pi\nu \qquad \mathbf{n} = (\mathbf{x}_0 - \mathbf{x}_{\mathrm{in}})/(\eta_0 - \eta_{\mathrm{in}})$

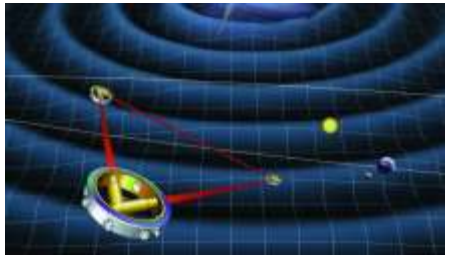
$$\bar{\Omega}_{\mathrm{gw,0}}(\nu) \simeq 4.2\times 10^{-5} \times 4\pi \bar{\omega}_{\mathrm{gw,in}}(q)$$

$$\delta_{\mathrm{gw,0}}(\mathbf{q}) = \delta_{\mathrm{gw,in}}(\mathbf{q}) + (4 - n_{\mathrm{gw}}(\nu))\Phi_{\mathrm{in}}$$

Energy-density Full Spectrum

Definition: $$\omega_{\rm gw}(\eta,\mathbf{x},\mathbf{q}) = \frac{1}{\rho_{\rm c}(\eta)}\frac{\mathrm{d}\rho_{\rm gw}(\eta,\mathbf{x})}{\mathrm{d}\ln q\,\mathrm{d}^2\mathbf{n}} \qquad \mathbf{q}=q\,\mathbf{n}$$

Density contrast: $\omega_{\rm gw}(\eta,\mathbf{x},\mathbf{q}) = \bar{\omega}_{\rm gw}(\eta,q)\left(1+\delta_{\rm gw}(\eta,\mathbf{x},\mathbf{q})\right)$



Initial moment (SIGW production) $(\eta_{\rm in},\mathbf{x}_{\rm in})$

GW energy-density on subhorizon scales (including inflationary initial condition: 2407.09405)

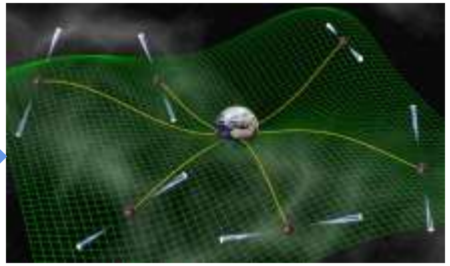
$$\tilde{\rho}_{\rm gw,in} = \frac{m_{\rm Pl}^2 e^{2\Phi_{\rm L,in}}}{16 a_{\rm in}}\overline{\partial_l h_{ij}\partial_l h_{ij}}$$

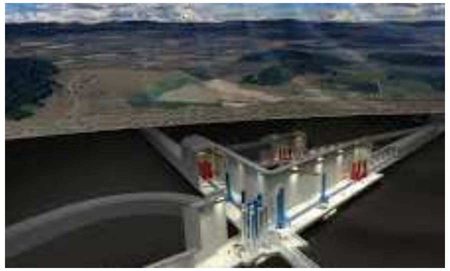
Energy-density full spectrum

$$\bar{\omega}_{\rm gw,in} \sim \langle\zeta\zeta\zeta\zeta\rangle$$

$$\tilde{\delta}_{\rm gw,in} = \delta_{\rm gw,in} + 2\Phi_{\rm L,in}$$

$$\delta_{\rm gw,in} \sim \langle\zeta\zeta\zeta\zeta\rangle$$





Cosmic Expansion

Boltzmann equation (low multipoles)

Present-day (observation) $(\eta_0,\mathbf{x}_0)$

$$q = 2\pi\nu \qquad \mathbf{n} = (\mathbf{x}_0-\mathbf{x}_{\rm in})/(\eta_0-\eta_{\rm in})$$

$$\bar{\Omega}_{\rm gw,0}(\nu) \simeq 4.2\times10^{-5}\times 4\pi\bar{\omega}_{\rm gw,in}(q)$$

$$\delta_{\rm gw,0}(\mathbf{q}) = \delta_{\rm gw,in}(\mathbf{q}) + (4-n_{\rm gw}(\nu))\Phi_{\rm in}$$

$$\delta_{\rm gw,0}(\mathbf{q}) = \delta_{\rm gw,in}(\mathbf{q}) + (6-n_{\rm gw}(\nu))\Phi_{\rm in}$$

The SIGW Background

The SIGW Background

$$\bar{\Omega}_{\mathrm{gw},0}(\nu) \propto 4\pi \bar{\omega}_{\mathrm{gw,in}}(q)$$

$$\delta_{\mathrm{gw},0}(\mathbf{q}) = \delta_{\mathrm{gw,in}}(\mathbf{q}) + (6 - n_{\mathrm{gw}}(\nu))\,\Phi_{\mathrm{in,L}}$$

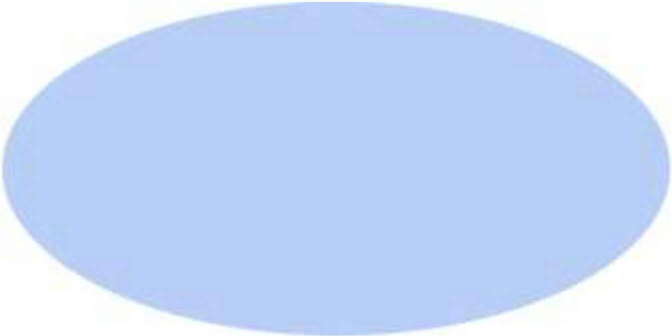
$\sim \langle \zeta\zeta\zeta\zeta \rangle$

Isotropic Component

$$\bar{\omega}_{\mathrm{gw}}(\eta, q) = \langle \omega_{\mathrm{gw}}(\eta, \mathbf{x}, \mathbf{q}) \rangle$$

$$\bar{\Omega}_{\mathrm{gw},0}(\nu) \propto 4\pi \bar{\omega}_{\mathrm{gw,in}}(q)$$

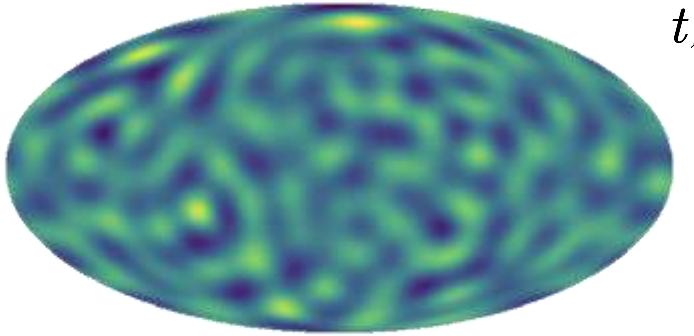
Energy density fraction spectrum
1-pt



Anisotropies

$$\tilde{C}_\ell(\nu) \sim \left\langle \prod_{i=1}^{2} \delta_{\mathrm{gw},0,\ell_i m_i}(2\pi\nu) \right\rangle$$

Angular power spectrum
2-pt



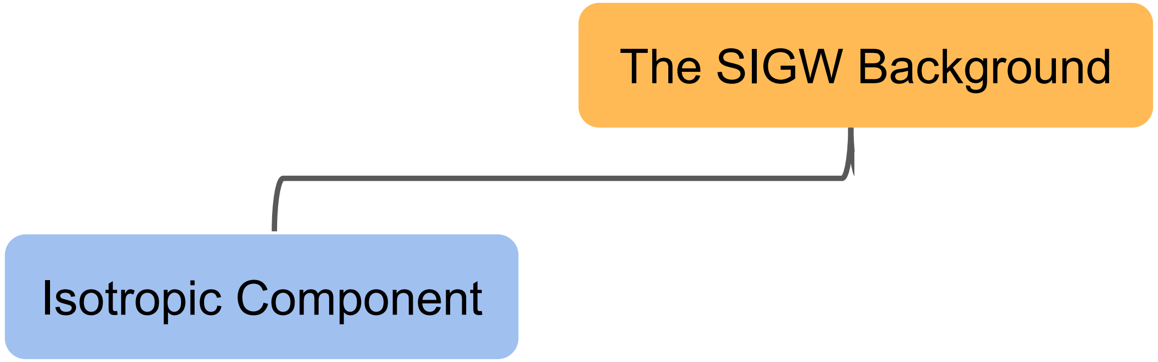
Non-Gaussianity

$$B_\ell(\nu) \sim \left\langle \prod_{i=1}^{3} \delta_{\mathrm{gw},0,\ell_i m_i}(2\pi\nu) \right\rangle$$

$$t^{\ell_1 \ell_2}_{\ell_3 \ell_4}(L, \nu) \sim \left\langle \prod_{i=1}^{4} \delta_{\mathrm{gw},0,\ell_i m_i}(2\pi\nu) \right\rangle_{\mathrm{c}}$$

Angular bispectrum and trispectrum
3-pt, 4-pt, …

Isotropic Component



R.-G. Cai, S. Pi, M. Sasaki, PRL 122 (2019) 201101

P. Adshead, K. D. Lozanov, Z. J. Weiner, JCAP 10 (2021), 080

K. T. Abe, R. Inui, Y. Tada, S. Yokoyama, JCAP 5 (2023), 044

JL, S. Wang, Z.-C. Zhao, and K. Kohri, JCAP 10 (2023), 056

C. Yuan, D.-S. Meng, Q.-G. Huang, JCAP 12 (2023), 036

JL, S. Wang, Z.-C. Zhao, and K. Kohri, JCAP 06 (2024) 039

Y.-H. Yu, S. Wang, PRD 109 (2024) 083501

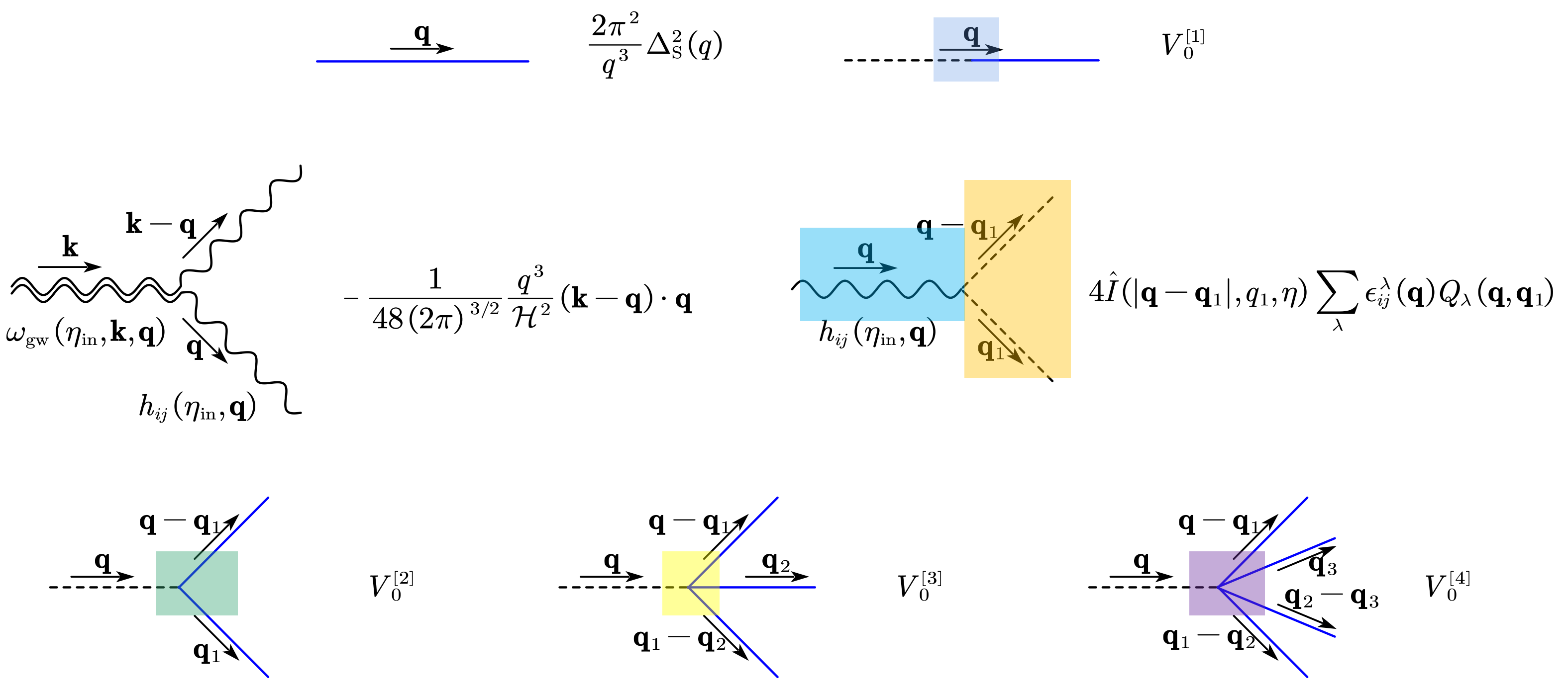
……

Energy density fraction spectrum

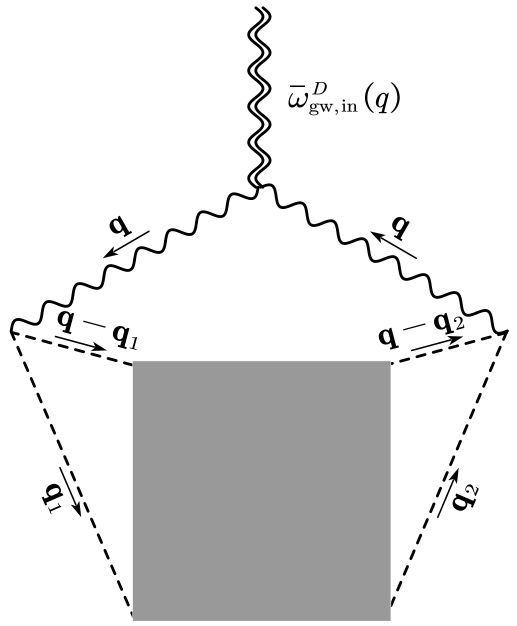
$$\bar{\Omega}_{\mathrm{gw},0}(\nu) = 4\pi h^2 \Omega_{\mathrm{rad},0} \bar{\omega}_{\mathrm{gw,in}}(q) \left(\frac{g_{*,\rho}(T_{\mathrm{in}})}{g_{*,\rho}(T_0)}\right) \left(\frac{g_{*,s}(T_0)}{g_{*,s}(T_{\mathrm{in}})}\right)^{4/3}$$

$$\bar{\omega}_{\mathrm{gw,in}}(q) = \frac{q^5}{3(2\pi)^3 \mathcal{H}^2} \left\{\prod_{i=1}^{n} \int \frac{\mathrm{d}^3 \mathbf{q}_i}{(2\pi)^3}\right\} \langle \zeta_{\mathrm{S}}(\mathbf{q}_1) \zeta_{\mathrm{S}}(\mathbf{q}-\mathbf{q}_1) \zeta_{\mathrm{S}}(\mathbf{q}_2) \zeta_{\mathrm{S}}(\mathbf{q}-\mathbf{q}_2) \rangle' \times \sum_{\lambda=+,\times} Q_\lambda(\mathbf{q},\mathbf{q}_1) Q_\lambda(\mathbf{q},\mathbf{q}_j) \overline{\hat{I}(|\mathbf{q}-\mathbf{q}_1|, q_1, \eta) \hat{I}(|\mathbf{q}-\mathbf{q}_j|, q_j, \eta)}$$

Basic Diagrammatic Approach



P. Adshead, K. D. Lozanov, Z. J. Weiner, JCAP 10 (2021), 080
K. T. Abe, R. Inui, Y. Tada, S. Yokoyama, JCAP 5 (2023), 044
JL, S. Wang, Z.-C. Zhao, and K. Kohri, JCAP 10 (2023), 056
JL, S. Wang, Z.-C. Zhao, and K. Kohri, JCAP 06 (2024) 039
……



$$\zeta = \zeta_g + F_{NL}\left(\zeta_g^2 - \langle\zeta_g^2\rangle\right) + G_{NL}\zeta_g^3 + H_{NL}\left(\zeta_g^4 - 3\langle\zeta_g^2\rangle^2\right) + \cdots$$

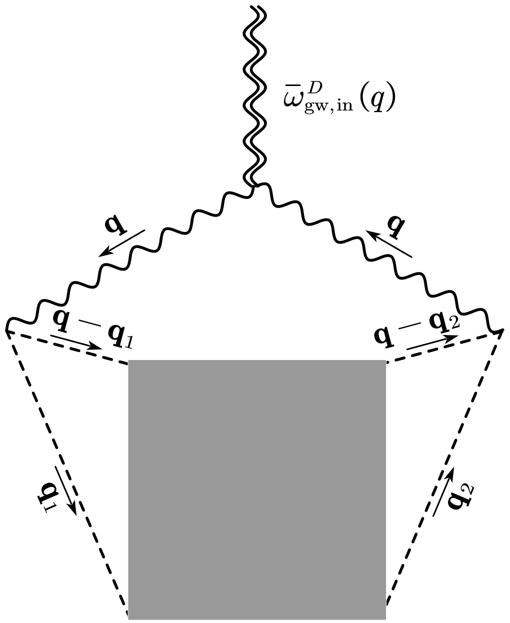
$$h_\lambda(\eta_{in},\mathbf{q}) = 4\int\frac{d^3\mathbf{q}_a}{(2\pi)^{3/2}}\zeta(\mathbf{q}_a)\zeta(\mathbf{q}-\mathbf{q}_a)Q_\lambda(\mathbf{q},\mathbf{q}_a)\hat{I}(|\mathbf{q}-\mathbf{q}_a|,q_a,\eta_{in})$$

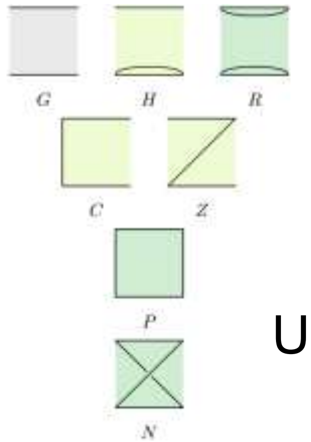
$$\bar{\omega}^D_{gw,in}(q) = \frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{d^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times}Q_\lambda Q_\lambda\overline{\hat{I}\hat{I}}\right)\mathcal{P}^D$$

$$\bar{\omega}_{gw,in}(q) = \sum_D \bar{\omega}^D_{gw,in}(q)$$

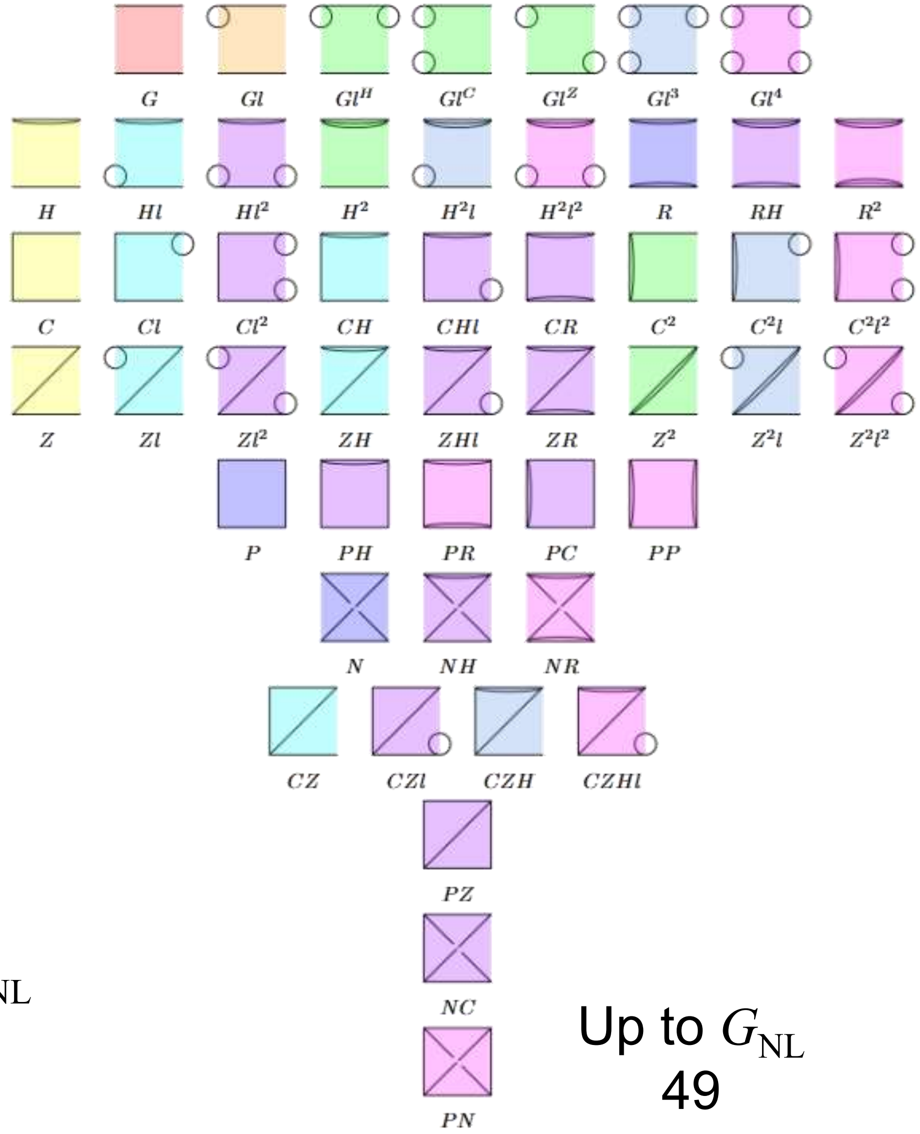
Basic Diagrammatic Approach: Diagrams

JL, S. Wang, Z.-C. Zhao, and K. Kohri, arXiv: 2505.16820

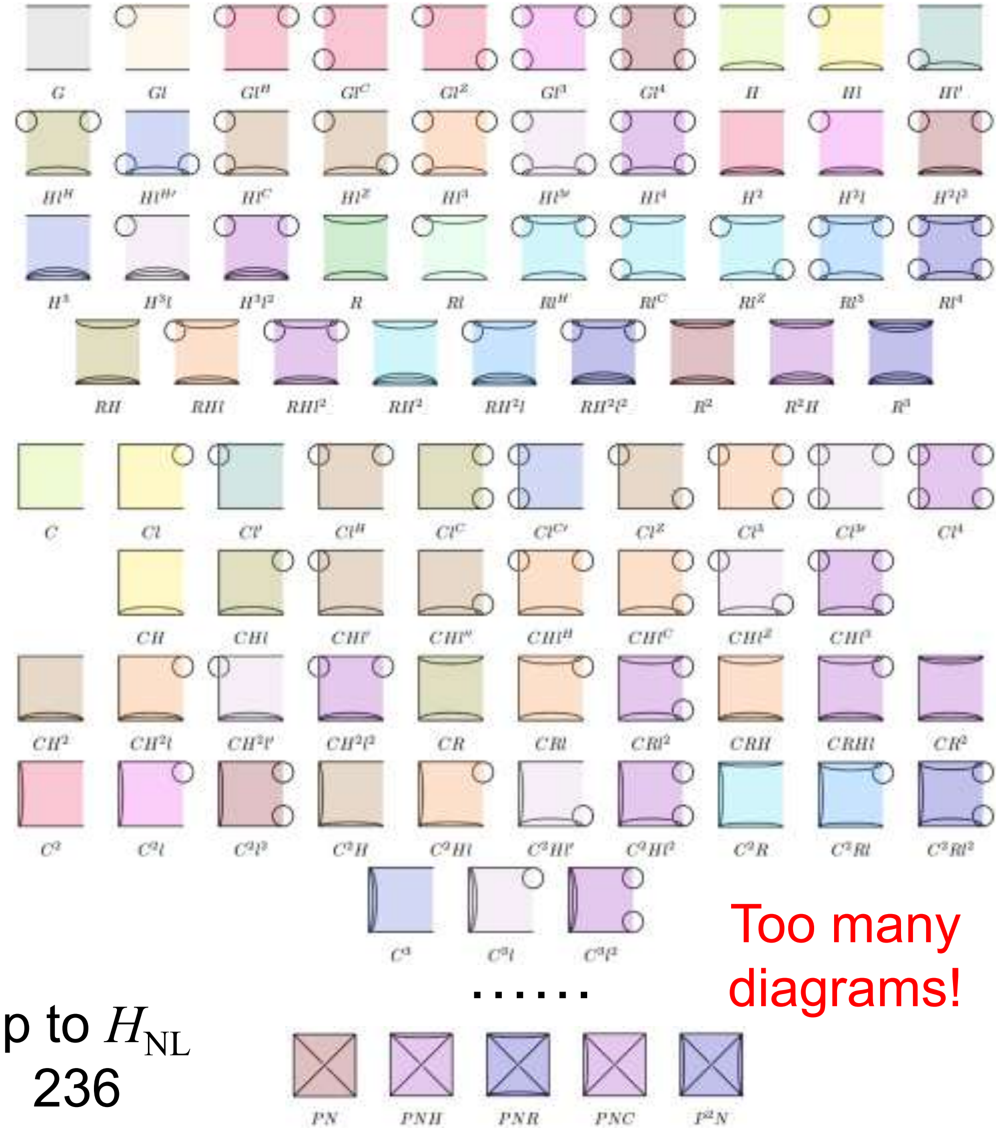




Up to F_{NL}
7



Up to G_{NL}
49

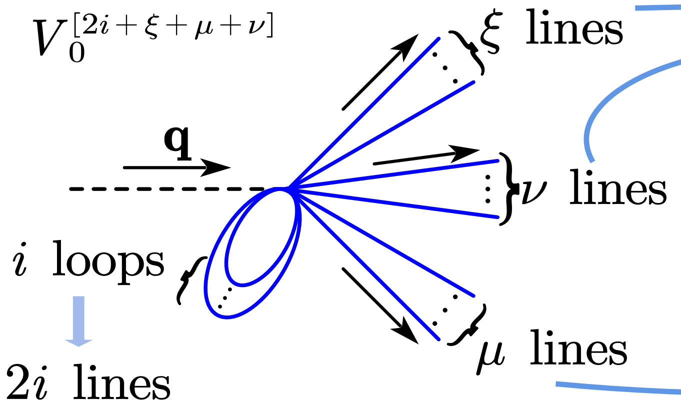


Up to H_{NL}
236

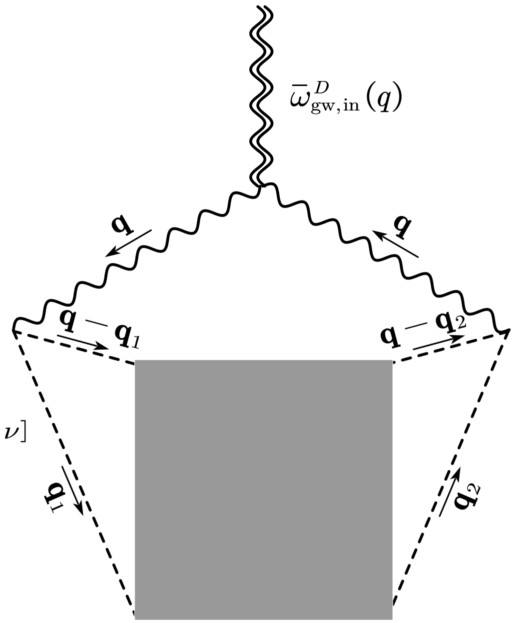
Too many diagrams!

"Renormalized" Diagrammatic Approach

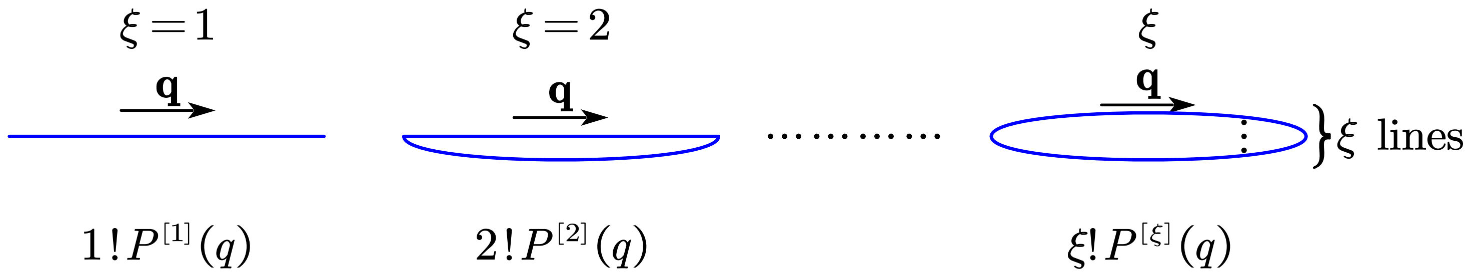
$$\bar{\omega}_{\mathrm{gw,in}}^{X-\mathrm{like}}(q)=\frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times}Q_\lambda Q_\lambda\overline{\hat{I}\hat{I}}\right)\mathcal{P}^{X-\mathrm{like}}$$



$$\frac{(2i)!}{2^i i!}\binom{2i+\xi+\mu+\nu}{\nu}\binom{2i+\xi+\mu}{\mu}\binom{2i+\xi}{\xi}A_{\mathrm{S}}^{i}V_0^{[2i+\xi+\mu+\nu]}$$



All loops are interchangeable, and likewise the two lines in every loop.



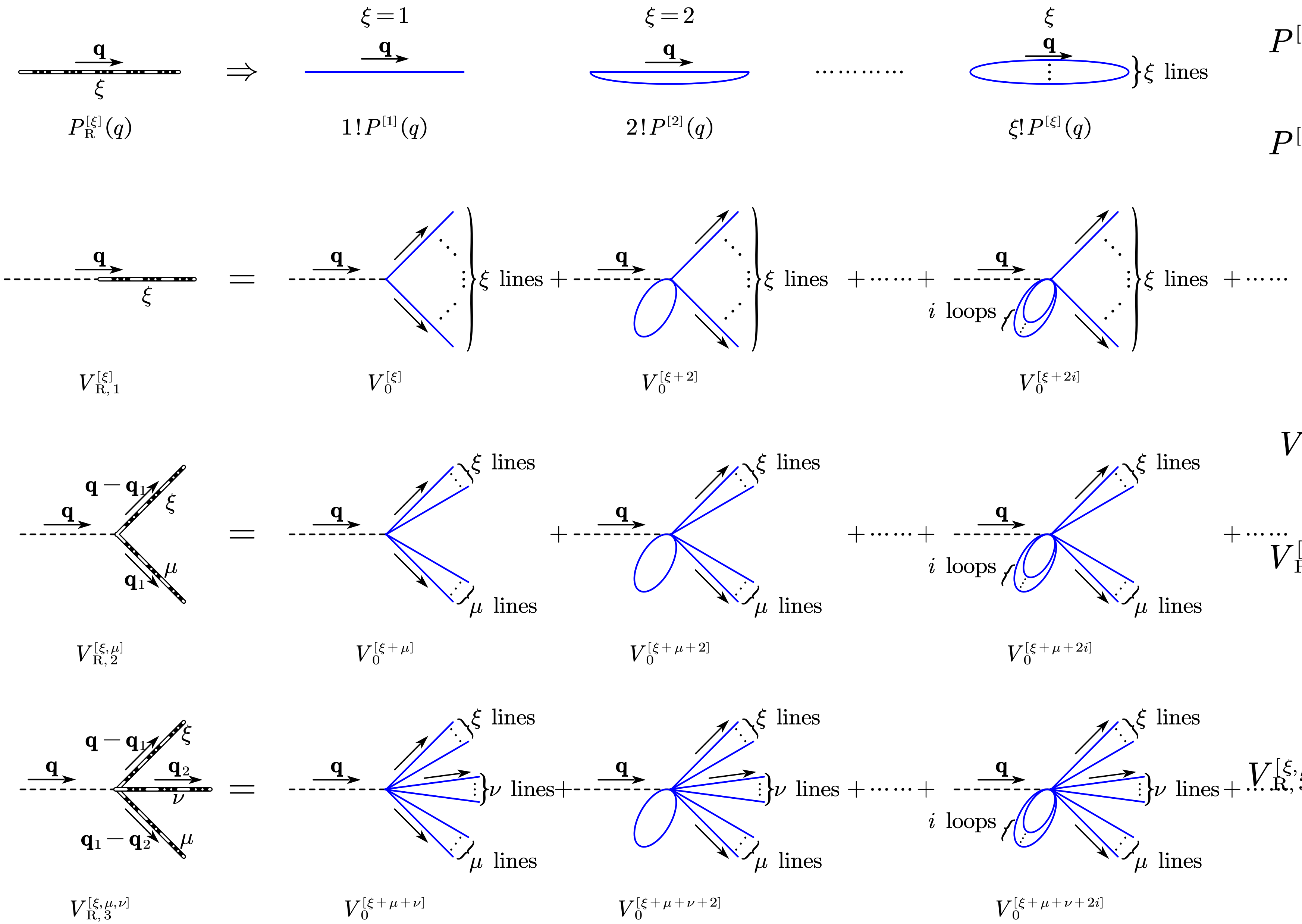
$$P^{[1]}(q)=\frac{2\pi^2}{q^3}\Delta_{\mathrm{S}}^2(q)$$

$$P^{[\xi]}(q)=\int\frac{\mathrm{d}^3\tilde{\mathbf{q}}}{(2\pi)^3}P^{[\xi-1]}(\tilde{q})P^{[1]}(|\mathbf{q}-\tilde{\mathbf{q}}|),$$

where $\xi\geq 2,\xi\in\mathbb{N}_+$

$$P_{\mathrm{R},a}^{[\xi]}=\xi!P_a^{[\xi]}$$

“Renormalized” Diagrammatic Approach



$$P^{[1]}(q) = \frac{2\pi^2}{q^3}\Delta_{\mathrm{S}}^2(q)$$

$$P^{[\xi]}(q) = \int \frac{\mathrm{d}^3\tilde{\mathbf{q}}}{(2\pi)^3} P^{[\xi-1]}(\tilde{q})\, P^{[1]}(|\mathbf{q}-\tilde{\mathbf{q}}|),$$

where $\xi \geq 2, \xi \in \mathbb{N}_+$

$$P_{\mathrm{R},a}^{[\xi]} = \xi!\, P_a^{[\xi]}$$

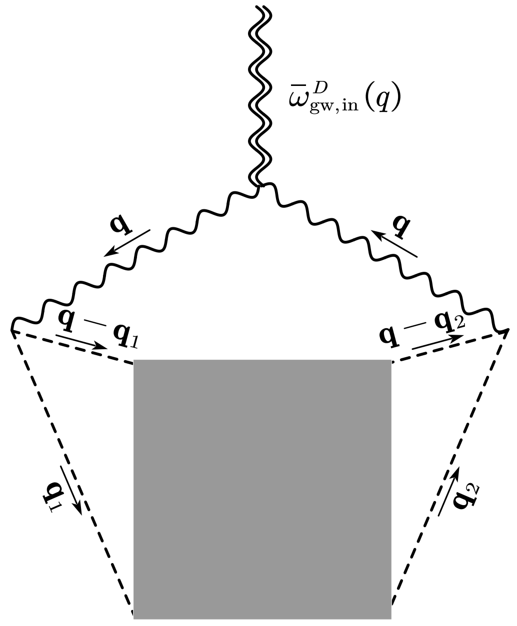
$$V_{\mathrm{R},3}^{[\xi]} = \sum_{i=0}^{\lfloor (o-\xi)/2 \rfloor} \frac{(2i)!}{2^i i!} \binom{2i+\xi}{\xi} A_{\mathrm{S}}^i V_0^{[2i+\xi+\mu+\nu]}$$

$$V_{\mathrm{R},2}^{[\xi,\mu]} = \sum_{i=0}^{\lfloor (o-\xi-\mu)/2 \rfloor} \frac{(2i)!}{2^i i!} \binom{2i+\xi+\mu}{\mu} \times \binom{2i+\xi}{\xi} A_{\mathrm{S}}^i V_0^{[2i+\xi+\mu+\nu]}$$

$$V_{\mathrm{R},3}^{[\xi,\mu,\nu]} = \sum_{i=0}^{\lfloor (o-\xi-\mu-\nu)/2 \rfloor} \frac{(2i)!}{2^i i!} \binom{2i+\xi+\mu+\nu}{\nu} \times \binom{2i+\xi+\mu}{\mu} \binom{2i+\xi}{\xi} A_{\mathrm{S}}^i V_0^{[2i+\xi+\mu+\nu]}$$

"Renormalized" Diagrammatic Approach: Diagrams

$$\bar{\omega}^{X-\text{like}}_{\text{gw,in}}(q) = \frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times}Q_\lambda Q_\lambda\,\overline{\hat{I}\hat{I}}\right)\mathcal{P}^{X-\text{like}}$$

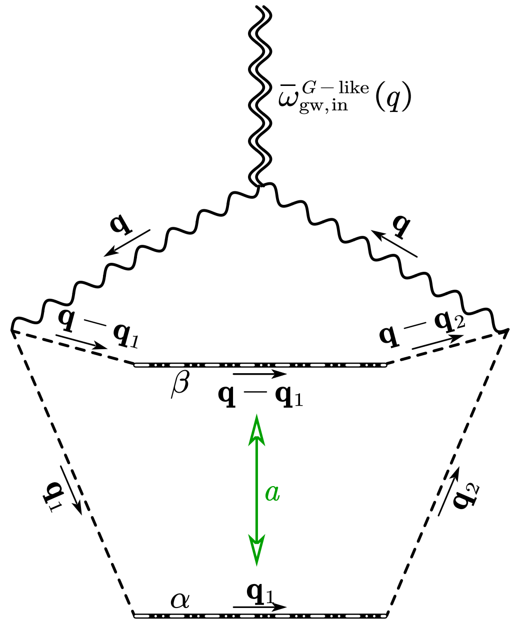


“Renormalized” Diagrammatic Approach: Diagrams

$$\bar{\omega}^{X-\text{like}}_{\text{gw,in}}(q) = \frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times} Q_\lambda Q_\lambda\,\overline{\hat{I}\hat{I}}\right)\mathcal{P}^{X-\text{like}}$$

$$\mathcal{P}^{G-\text{like}} = 2\left(\sum_{\alpha=1}^{o} V^{[\alpha]}_{\text{R},1} P^{[\alpha]}_{\text{R},a} V^{[\alpha]}_{\text{R},1}\right)\left(\sum_{\beta=1}^{o} V^{[\beta]}_{\text{R},1} P^{[\beta]}_{\text{R},a} V^{[\beta]}_{\text{R},1}\right)$$

Disconnected component



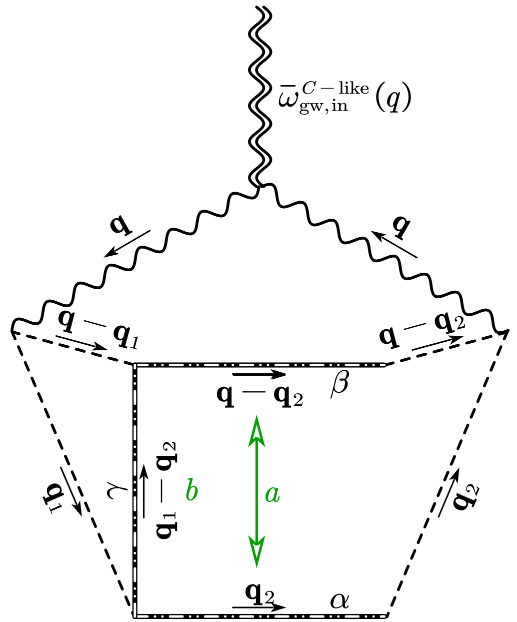
“Renormalized” Diagrammatic Approach: Diagrams

$$\bar{\omega}^{X-\mathrm{like}}_{\mathrm{gw,in}}(q) = \frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times} Q_\lambda Q_\lambda \overline{\hat{I}\hat{I}}\right)\mathcal{P}^{X-\mathrm{like}}$$

$$\mathcal{P}^{G-\mathrm{like}} = 2\left(\sum_{\alpha=1}^{o} V^{[\alpha]}_{\mathrm{R},1} P^{[\alpha]}_{\mathrm{R},a} V^{[\alpha]}_{\mathrm{R},1}\right)\left(\sum_{\beta=1}^{o} V^{[\beta]}_{\mathrm{R},1} P^{[\beta]}_{\mathrm{R},a} V^{[\beta]}_{\mathrm{R},1}\right)$$

Disconnected component

$$\mathcal{P}^{C-\mathrm{like}} = 4\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{o-\gamma}\sum_{\alpha=1}^{o-\gamma} V^{[\alpha]}_{\mathrm{R},1} P^{[\alpha]}_{\mathrm{R},a} V^{[\alpha,\gamma]}_{\mathrm{R},2} P^{[\gamma]}_{\mathrm{R},b} V^{[\gamma,\beta]}_{\mathrm{R},2} P^{[\beta]}_{\mathrm{R},a} V^{[\beta]}_{\mathrm{R},1}$$



Primordial trispectrum

“Renormalized” Diagrammatic Approach: Diagrams

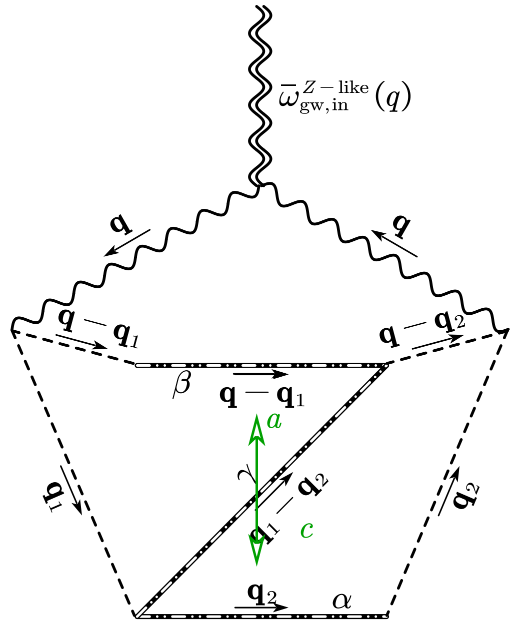
$$\bar{\omega}^{X-\text{like}}_{\text{gw,in}}(q) = \frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times} Q_\lambda Q_\lambda \overline{\hat{I}\hat{I}}\right)\mathcal{P}^{X-\text{like}}$$

$$\mathcal{P}^{G-\text{like}} = 2\left(\sum_{\alpha=1}^{o} V^{[\alpha]}_{\mathrm{R},1} P^{[\alpha]}_{\mathrm{R},a} V^{[\alpha]}_{\mathrm{R},1}\right)\left(\sum_{\beta=1}^{o} V^{[\beta]}_{\mathrm{R},1} P^{[\beta]}_{\mathrm{R},a} V^{[\beta]}_{\mathrm{R},1}\right)$$

Disconnected component

$$\mathcal{P}^{C-\text{like}} = 4\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{o-\gamma}\sum_{\alpha=1}^{o-\gamma} V^{[\alpha]}_{\mathrm{R},1} P^{[\alpha]}_{\mathrm{R},a} V^{[\alpha,\gamma]}_{\mathrm{R},2} P^{[\gamma]}_{\mathrm{R},b} V^{[\gamma,\beta]}_{\mathrm{R},2} P^{[\beta]}_{\mathrm{R},a} V^{[\beta]}_{\mathrm{R},1}$$

$$\mathcal{P}^{Z-\text{like}} = 4\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{o-\gamma}\sum_{\alpha=1}^{o-\gamma} V^{[\alpha]}_{\mathrm{R},1} P^{[\alpha]}_{\mathrm{R},a} V^{[\alpha,\gamma]}_{\mathrm{R},2} P^{[\gamma]}_{\mathrm{R},c} V^{[\gamma,\beta]}_{\mathrm{R},2} P^{[\beta]}_{\mathrm{R},a} V^{[\beta]}_{\mathrm{R},1}$$



Primordial trispectrum

“Renormalized” Diagrammatic Approach: Diagrams

$$\bar{\omega}_{\mathrm{gw,in}}^{X-\mathrm{like}}(q) = \frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times}Q_\lambda Q_\lambda\,\overline{\hat{I}\hat{I}}\right)\mathcal{P}^{X-\mathrm{like}}$$

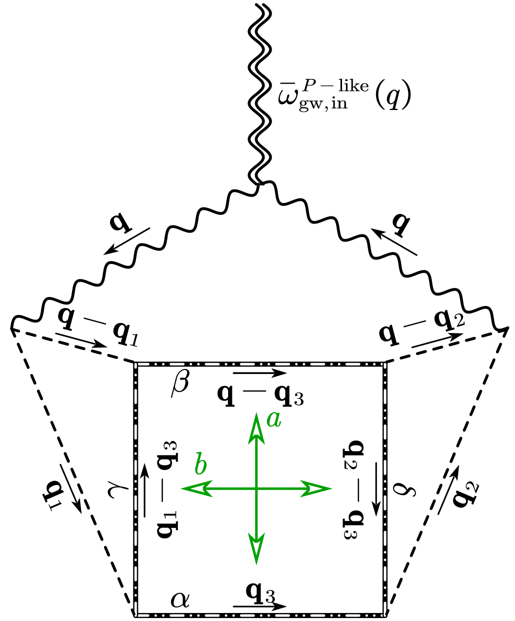
$$\mathcal{P}^{G-\mathrm{like}} = 2\left(\sum_{\alpha=1}^{o}V_{\mathrm{R},1}^{[\alpha]}P_{\mathrm{R},a}^{[\alpha]}V_{\mathrm{R},1}^{[\alpha]}\right)\left(\sum_{\beta=1}^{o}V_{\mathrm{R},1}^{[\beta]}P_{\mathrm{R},a}^{[\beta]}V_{\mathrm{R},1}^{[\beta]}\right)$$

Disconnected component

$$\mathcal{P}^{C-\mathrm{like}} = 4\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{o-\gamma}\sum_{\alpha=1}^{o-\gamma}V_{\mathrm{R},1}^{[\alpha]}P_{\mathrm{R},a}^{[\alpha]}V_{\mathrm{R},2}^{[\alpha,\gamma]}P_{\mathrm{R},b}^{[\gamma]}V_{\mathrm{R},2}^{[\gamma,\beta]}P_{\mathrm{R},a}^{[\beta]}V_{\mathrm{R},1}^{[\beta]}$$

$$\mathcal{P}^{Z-\mathrm{like}} = 4\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{o-\gamma}\sum_{\alpha=1}^{o-\gamma}V_{\mathrm{R},1}^{[\alpha]}P_{\mathrm{R},a}^{[\alpha]}V_{\mathrm{R},2}^{[\alpha,\gamma]}P_{\mathrm{R},c}^{[\gamma]}V_{\mathrm{R},2}^{[\gamma,\beta]}P_{\mathrm{R},a}^{[\beta]}V_{\mathrm{R},1}^{[\beta]}$$

$$\mathcal{P}^{P-\mathrm{like}} = 2\sum_{\delta=1}^{o-1}\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{\min(o-\gamma,o-\delta)}\sum_{\alpha=1}^{\min(o-\gamma,o-\delta)}P_{\mathrm{R},a}^{[\alpha]}V_{\mathrm{R},2}^{[\alpha,\gamma]}P_{\mathrm{R},b}^{[\gamma]}V_{\mathrm{R},2}^{[\gamma,\beta]}P_{\mathrm{R},a}^{[\beta]}V_{\mathrm{R},2}^{[\beta,\delta]}P_{\mathrm{R},b}^{[\delta]}V_{\mathrm{R},2}^{[\delta,\alpha]}$$



Primordial trispectrum

“Renormalized” Diagrammatic Approach: Diagrams

$$\bar{\omega}^{X-\text{like}}_{\text{gw,in}}(q) = \frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times}Q_\lambda Q_\lambda\,\overline{\hat{I}\hat{I}}\right)\mathcal{P}^{X-\text{like}}$$

$$\mathcal{P}^{G-\text{like}} = 2\left(\sum_{\alpha=1}^{o}V^{[\alpha]}_{\text{R},1}P^{[\alpha]}_{\text{R},a}V^{[\alpha]}_{\text{R},1}\right)\left(\sum_{\beta=1}^{o}V^{[\beta]}_{\text{R},1}P^{[\beta]}_{\text{R},a}V^{[\beta]}_{\text{R},1}\right)$$

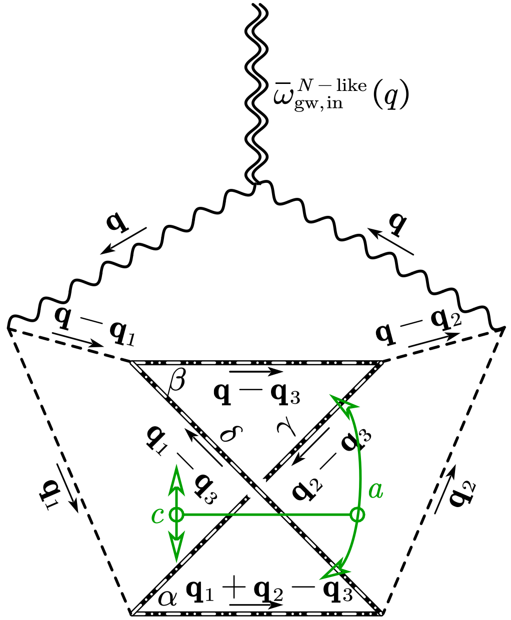
Disconnected component

$$\mathcal{P}^{C-\text{like}} = 4\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{o-\gamma}\sum_{\alpha=1}^{o-\gamma}V^{[\alpha]}_{\text{R},1}P^{[\alpha]}_{\text{R},a}V^{[\alpha,\gamma]}_{\text{R},2}P^{[\gamma]}_{\text{R},b}V^{[\gamma,\beta]}_{\text{R},2}P^{[\beta]}_{\text{R},a}V^{[\beta]}_{\text{R},1}$$

$$\mathcal{P}^{Z-\text{like}} = 4\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{o-\gamma}\sum_{\alpha=1}^{o-\gamma}V^{[\alpha]}_{\text{R},1}P^{[\alpha]}_{\text{R},a}V^{[\alpha,\gamma]}_{\text{R},2}P^{[\gamma]}_{\text{R},c}V^{[\gamma,\beta]}_{\text{R},2}P^{[\beta]}_{\text{R},a}V^{[\beta]}_{\text{R},1}$$

$$\mathcal{P}^{P-\text{like}} = 2\sum_{\delta=1}^{o-1}\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{\min(o-\gamma,o-\delta)}\sum_{\alpha=1}^{\min(o-\gamma,o-\delta)}P^{[\alpha]}_{\text{R},a}V^{[\alpha,\gamma]}_{\text{R},2}P^{[\gamma]}_{\text{R},b}V^{[\gamma,\beta]}_{\text{R},2}P^{[\beta]}_{\text{R},a}V^{[\beta,\delta]}_{\text{R},2}P^{[\delta]}_{\text{R},b}V^{[\delta,\alpha]}_{\text{R},2}$$

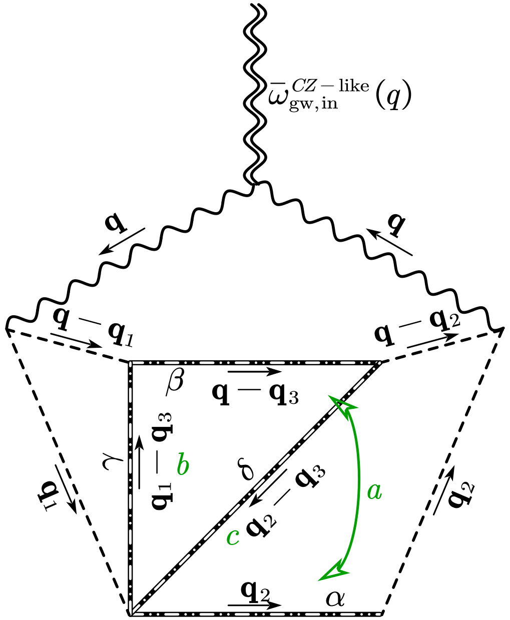
$$\mathcal{P}^{N-\text{like}} = \sum_{\delta=1}^{o-1}\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{\min(o-\gamma,o-\delta)}\sum_{\alpha=1}^{\min(o-\gamma,o-\delta)}P^{[\alpha]}_{\text{R},a}V^{[\alpha,\gamma]}_{\text{R},2}P^{[\gamma]}_{\text{R},c}V^{[\gamma,\beta]}_{\text{R},2}P^{[\beta]}_{\text{R},a}V^{[\beta,\delta]}_{\text{R},2}P^{[\delta]}_{\text{R},c}V^{[\delta,\alpha]}_{\text{R},2}$$



Primordial trispectrum

“Renormalized” Diagrammatic Approach: Diagrams

$$\bar{\omega}^{X-\text{like}}_{\text{gw,in}}(q) = \frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times} Q_\lambda Q_\lambda\,\overline{\hat{I}\hat{I}}\right)\mathcal{P}^{X-\text{like}}$$



$$\mathcal{P}^{G-\text{like}} = 2\left(\sum_{\alpha=1}^{o} V^{[\alpha]}_{\text{R},1}P^{[\alpha]}_{\text{R},a}V^{[\alpha]}_{\text{R},1}\right)\left(\sum_{\beta=1}^{o} V^{[\beta]}_{\text{R},1}P^{[\beta]}_{\text{R},a}V^{[\beta]}_{\text{R},1}\right)$$

Disconnected component

$$\mathcal{P}^{C-\text{like}} = 4\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{o-\gamma}\sum_{\alpha=1}^{o-\gamma} V^{[\alpha]}_{\text{R},1}P^{[\alpha]}_{\text{R},a}V^{[\alpha,\gamma]}_{\text{R},2}P^{[\gamma]}_{\text{R},b}V^{[\gamma,\beta]}_{\text{R},2}P^{[\beta]}_{\text{R},a}V^{[\beta]}_{\text{R},1}$$

$$\mathcal{P}^{Z-\text{like}} = 4\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{o-\gamma}\sum_{\alpha=1}^{o-\gamma} V^{[\alpha]}_{\text{R},1}P^{[\alpha]}_{\text{R},a}V^{[\alpha,\gamma]}_{\text{R},2}P^{[\gamma]}_{\text{R},c}V^{[\gamma,\beta]}_{\text{R},2}P^{[\beta]}_{\text{R},a}V^{[\beta]}_{\text{R},1}$$

$$\mathcal{P}^{P-\text{like}} = 2\sum_{\delta=1}^{o-1}\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{\min(o-\gamma,o-\delta)}\sum_{\alpha=1}^{\min(o-\gamma,o-\delta)} P^{[\alpha]}_{\text{R},a}V^{[\alpha,\gamma]}_{\text{R},2}P^{[\gamma]}_{\text{R},b}V^{[\gamma,\beta]}_{\text{R},2}P^{[\beta]}_{\text{R},a}V^{[\beta,\delta]}_{\text{R},2}P^{[\delta]}_{\text{R},b}V^{[\delta,\alpha]}_{\text{R},2}$$

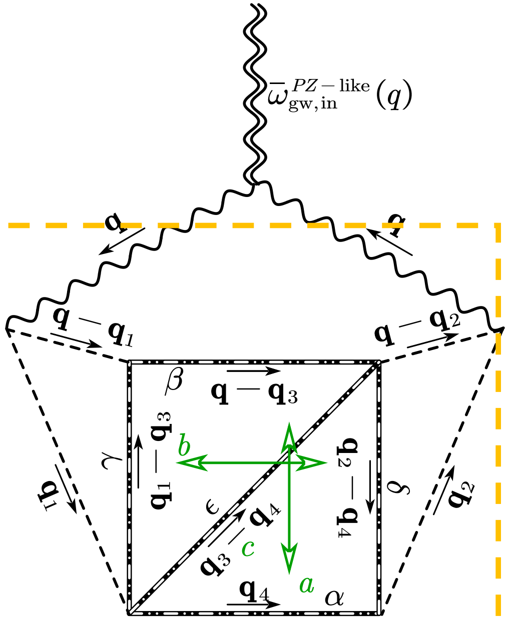
$$\mathcal{P}^{N-\text{like}} = \sum_{\delta=1}^{o-1}\sum_{\gamma=1}^{o-1}\sum_{\beta=1}^{\min(o-\gamma,o-\delta)}\sum_{\alpha=1}^{\min(o-\gamma,o-\delta)} P^{[\alpha]}_{\text{R},a}V^{[\alpha,\gamma]}_{\text{R},2}P^{[\gamma]}_{\text{R},c}V^{[\gamma,\beta]}_{\text{R},2}P^{[\beta]}_{\text{R},a}V^{[\beta,\delta]}_{\text{R},2}P^{[\delta]}_{\text{R},c}V^{[\delta,\alpha]}_{\text{R},2}$$

$$\mathcal{P}^{CZ-\text{like}} = 8\sum_{\delta=1}^{o-2}\sum_{\gamma=1}^{o-2}\sum_{\alpha=1}^{o-\gamma-\delta}\sum_{\beta=1}^{\min(o-\gamma,o-\delta)} P^{[\alpha]}_{\text{R},a}V^{[\alpha]}_{\text{R},1}P^{[\beta]}_{\text{R},b}V^{[\beta,\gamma]}_{\text{R},2}P^{[\gamma]}_{\text{R},c}V^{[\alpha,\gamma,\delta]}_{\text{R},3}P^{[\delta]}_{\text{R},d}V^{[\delta,\beta]}_{\text{R},2}$$

Primordial trispectrum

"Renormalized" Diagrammatic Approach: Diagrams

$$\bar{\omega}_{\mathrm{gw,in}}^{X-\mathrm{like}}(q) = \frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times}Q_\lambda Q_\lambda\overline{\hat{I}\hat{I}}\right)\mathcal{P}^{X-\mathrm{like}}$$



$$\mathcal{P}^{PZ-\mathrm{like}} = 4\sum_{\epsilon=1}^{o-2}\sum_{\delta=1}^{o-\epsilon-1}\sum_{\gamma=1}^{o-\epsilon-1}\sum_{\beta=1}^{\min(o-\gamma,o-\delta-\epsilon)}\sum_{\alpha=1}^{\min(o-\gamma-\epsilon,o-\delta)} P_{\mathrm{R},a}^{[\alpha]}V_{\mathrm{R},3}^{[\alpha,\gamma,\epsilon]}P_{\mathrm{R},b}^{[\gamma]}V_{\mathrm{R},2}^{[\beta,\gamma]}P_{\mathrm{R},a}^{[\beta]}$$

$$\times V_{\mathrm{R},3}^{[\beta,\delta,\epsilon]}P_{\mathrm{R},b}^{[\delta]}V_{\mathrm{R},2}^{[\alpha,\delta]}P_{\mathrm{R},c}^{[\epsilon]}$$

$$\bar{\omega}_{\mathrm{gw,in}}(q) = \sum_{X-\mathrm{like}}\bar{\omega}_{\mathrm{gw,in}}^{X-\mathrm{like}}(q)$$

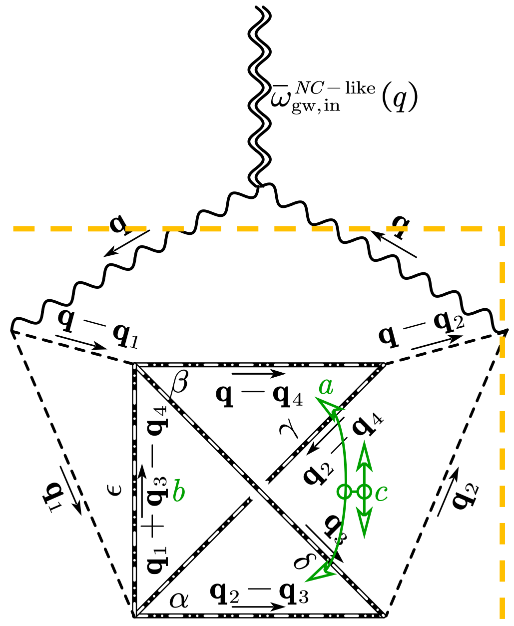
Primordial trispectrum

“Renormalized” Diagrammatic Approach: Diagrams

$$\bar{\omega}^{X-\text{like}}_{\text{gw,in}}(q) = \frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times} Q_\lambda Q_\lambda \overline{\hat{I}\hat{I}}\right)\mathcal{P}^{X-\text{like}}$$

$$\mathcal{P}^{PZ-\text{like}} = 4\sum_{\epsilon=1}^{o-2}\sum_{\delta=1}^{o-\epsilon-1}\sum_{\gamma=1}^{o-\epsilon-1}\sum_{\beta=1}^{\min(o-\gamma,o-\delta-\epsilon)}\sum_{\alpha=1}^{\min(o-\gamma-\epsilon,o-\delta)} P^{[\alpha]}_{\mathrm{R},a}V^{[\alpha,\gamma,\epsilon]}_{\mathrm{R},3}P^{[\gamma]}_{\mathrm{R},b}V^{[\beta,\gamma]}_{\mathrm{R},2}P^{[\beta]}_{\mathrm{R},a} \times V^{[\beta,\delta,\epsilon]}_{\mathrm{R},3}P^{[\delta]}_{\mathrm{R},b}V^{[\alpha,\delta]}_{\mathrm{R},2}P^{[\epsilon]}_{\mathrm{R},c}$$

$$\mathcal{P}^{NC-\text{like}} = 2\sum_{\epsilon=1}^{o-2}\sum_{\delta=1}^{o-\epsilon-1}\sum_{\gamma=1}^{o-\epsilon-1}\sum_{\beta=1}^{\min(o-\gamma,o-\delta-\epsilon)}\sum_{\alpha=1}^{\min(o-\gamma-\epsilon,o-\delta)} P^{[\alpha]}_{\mathrm{R},a}V^{[\alpha,\gamma,\epsilon]}_{\mathrm{R},3}P^{[\gamma]}_{\mathrm{R},c}V^{[\beta,\gamma]}_{\mathrm{R},2}P^{[\beta]}_{\mathrm{R},a} \times V^{[\beta,\delta,\epsilon]}_{\mathrm{R},3}P^{[\delta]}_{\mathrm{R},c}V^{[\alpha,\delta]}_{\mathrm{R},2}P^{[\epsilon]}_{\mathrm{R},b}$$

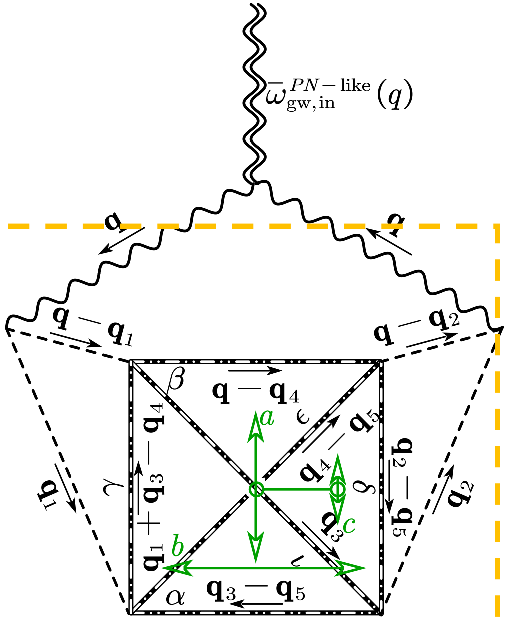


$$\bar{\omega}_{\text{gw,in}}(q) = \sum_{X-\text{like}}\bar{\omega}^{X-\text{like}}_{\text{gw,in}}(q)$$

Primordial trispectrum

"Renormalized" Diagrammatic Approach: Diagrams

$$\bar{\omega}^{X-\text{like}}_{\text{gw,in}}(q) = \frac{q^5}{3(2\pi)^3\mathcal{H}^2}\left\{\prod_{i=1}^{n}\int\frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\right\}\left(\sum_{\lambda=+,\times}Q_\lambda Q_\lambda\overline{\hat{I}\hat{I}}\right)\mathcal{P}^{X-\text{like}}$$



$$\mathcal{P}^{PZ-\text{like}} = 4\sum_{\epsilon=1}^{o-2}\sum_{\delta=1}^{o-\epsilon-1}\sum_{\gamma=1}^{o-\epsilon-1}\sum_{\beta=1}^{\min(o-\gamma,o-\delta-\epsilon)}\sum_{\alpha=1}^{\min(o-\gamma-\epsilon,o-\delta)} P^{[\alpha]}_{\mathrm{R},a}V^{[\alpha,\gamma,\epsilon]}_{\mathrm{R},3}P^{[\gamma]}_{\mathrm{R},b}V^{[\beta,\gamma]}_{\mathrm{R},2}P^{[\beta]}_{\mathrm{R},a} \times V^{[\beta,\delta,\epsilon]}_{\mathrm{R},3}P^{[\delta]}_{\mathrm{R},b}V^{[\alpha,\delta]}_{\mathrm{R},2}P^{[\epsilon]}_{\mathrm{R},c}$$

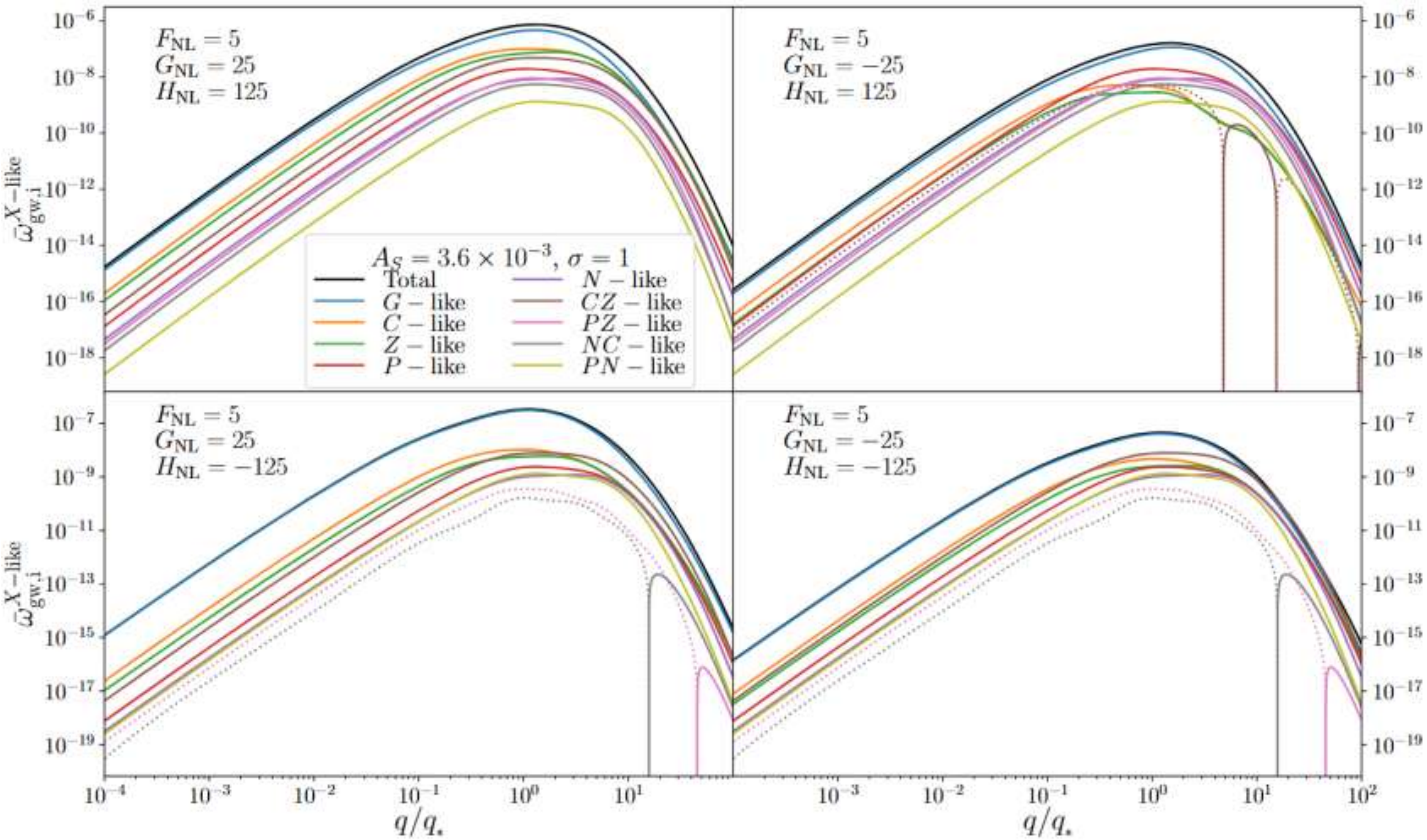
$$\mathcal{P}^{NC-\text{like}} = 2\sum_{\epsilon=1}^{o-2}\sum_{\delta=1}^{o-\epsilon-1}\sum_{\gamma=1}^{o-\epsilon-1}\sum_{\beta=1}^{\min(o-\gamma,o-\delta-\epsilon)}\sum_{\alpha=1}^{\min(o-\gamma-\epsilon,o-\delta)} P^{[\alpha]}_{\mathrm{R},a}V^{[\alpha,\gamma,\epsilon]}_{\mathrm{R},3}P^{[\gamma]}_{\mathrm{R},c}V^{[\beta,\gamma]}_{\mathrm{R},2}P^{[\beta]}_{\mathrm{R},a} \times V^{[\beta,\delta,\epsilon]}_{\mathrm{R},3}P^{[\delta]}_{\mathrm{R},c}V^{[\alpha,\delta]}_{\mathrm{R},2}P^{[\epsilon]}_{\mathrm{R},b}$$

$$\mathcal{P}^{PN-\text{like}} = \sum_{\iota=1}^{o-2}\sum_{\epsilon=1}^{o-2}\sum_{\delta=1}^{\min(o-\epsilon,o-\iota)-1}\sum_{\gamma=1}^{\min(o-\epsilon,o-\iota)-1}\sum_{\beta=1}^{\min(o-\gamma-\epsilon,o-\delta-\iota)}\sum_{\alpha=1}^{\min(o-\gamma-\iota,o-\delta-\epsilon)} P^{[\alpha]}_{\mathrm{R},a}V^{[\alpha,\gamma,\epsilon]}_{\mathrm{R},3}P^{[\gamma]}_{\mathrm{R},b}V^{[\beta,\gamma,\iota]}_{\mathrm{R},3}P^{[\beta]}_{\mathrm{R},a} \times V^{[\beta,\delta,\epsilon]}_{\mathrm{R},3}P^{[\delta]}_{\mathrm{R},b}V^{[\alpha,\delta,\iota]}_{\mathrm{R},3}P^{[\epsilon]}_{\mathrm{R},c}P^{[\iota]}_{\mathrm{R},c}$$

$$\bar{\omega}_{\text{gw,in}}(q) = \sum_{X-\text{like}}\bar{\omega}^{X-\text{like}}_{\text{gw,in}}(q)$$

Primordial trispectrum

Numerical Results



$$\Delta_S^2(q)=\frac{A_S}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\ln^2(q/q_*)}{2\sigma^2}\right)$$

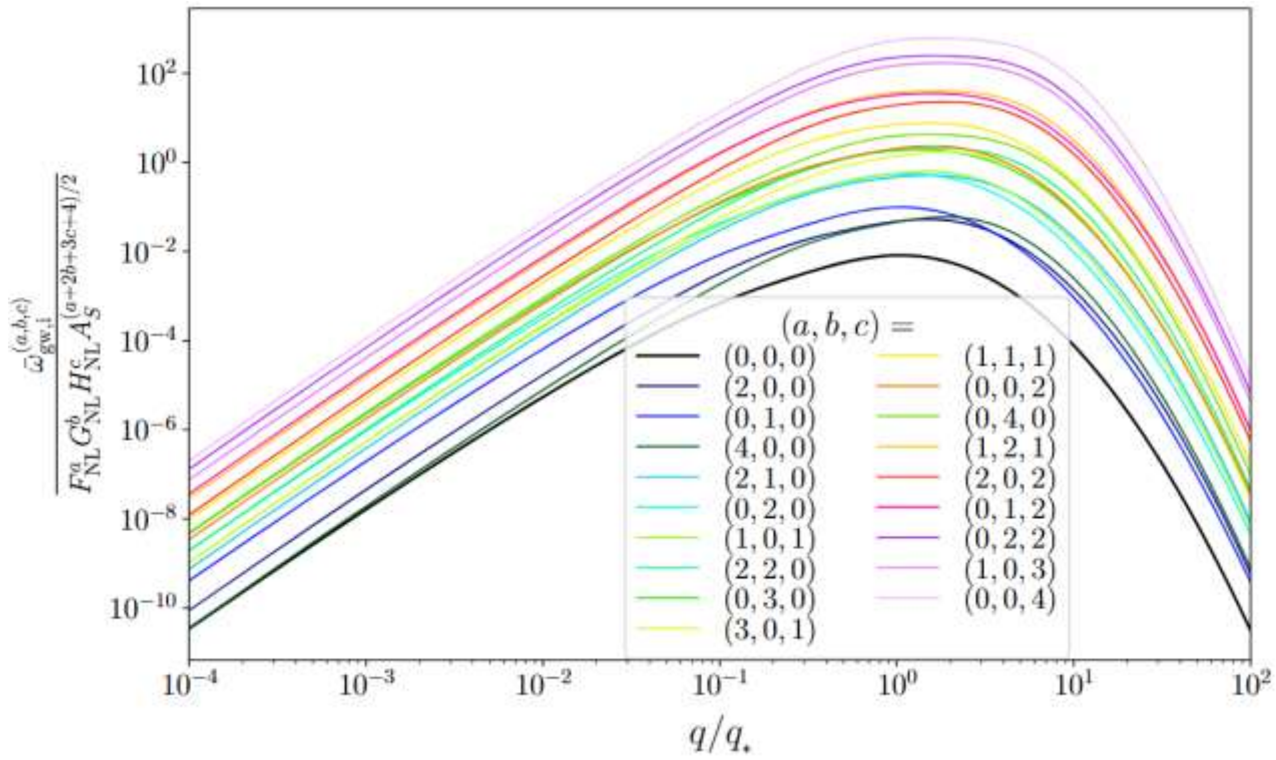
$$\sigma=1$$

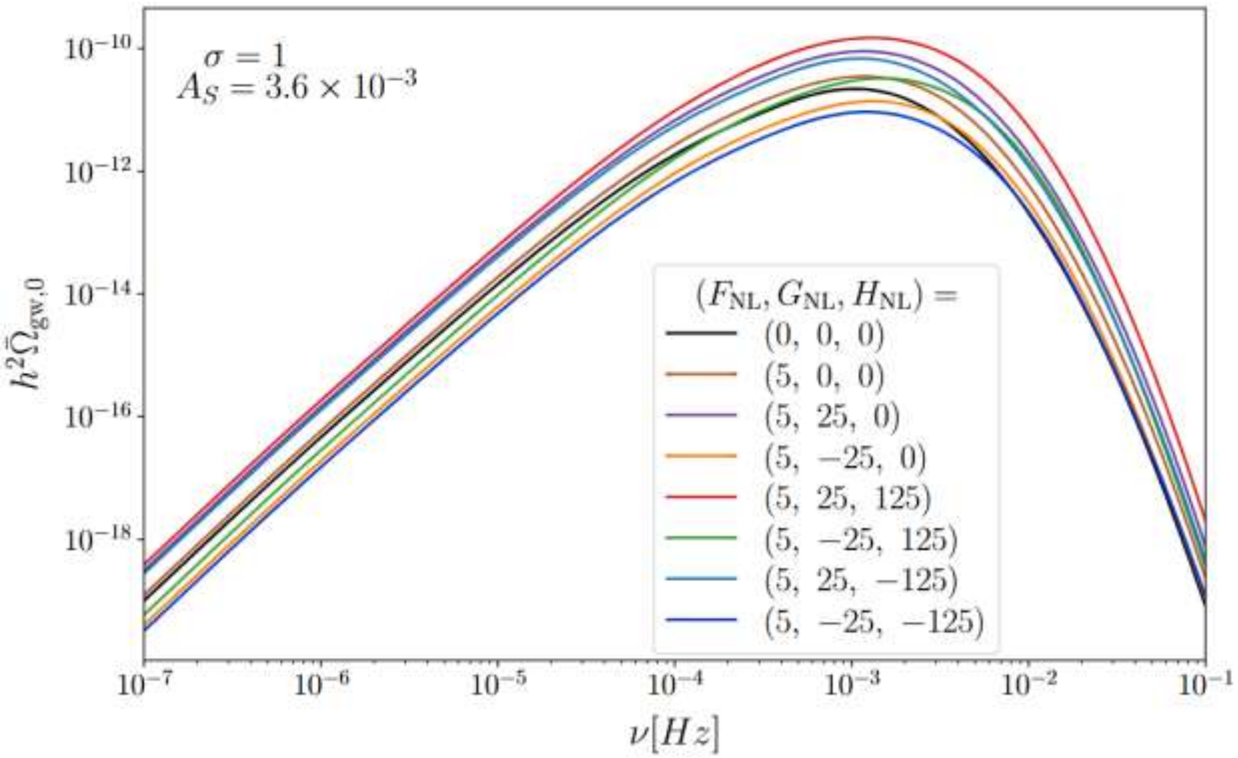
$$\bar{\omega}_{\text{gw,in}}(q)=\sum_{X-\text{like}}\bar{\omega}^{X-\text{like}}_{\text{gw,in}}(q)$$

Numerical Results

$$\Delta_S^2(q) = \frac{A_S}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\ln^2(q/q_*)}{2\sigma^2}\right)$$

$$\sigma = 1$$



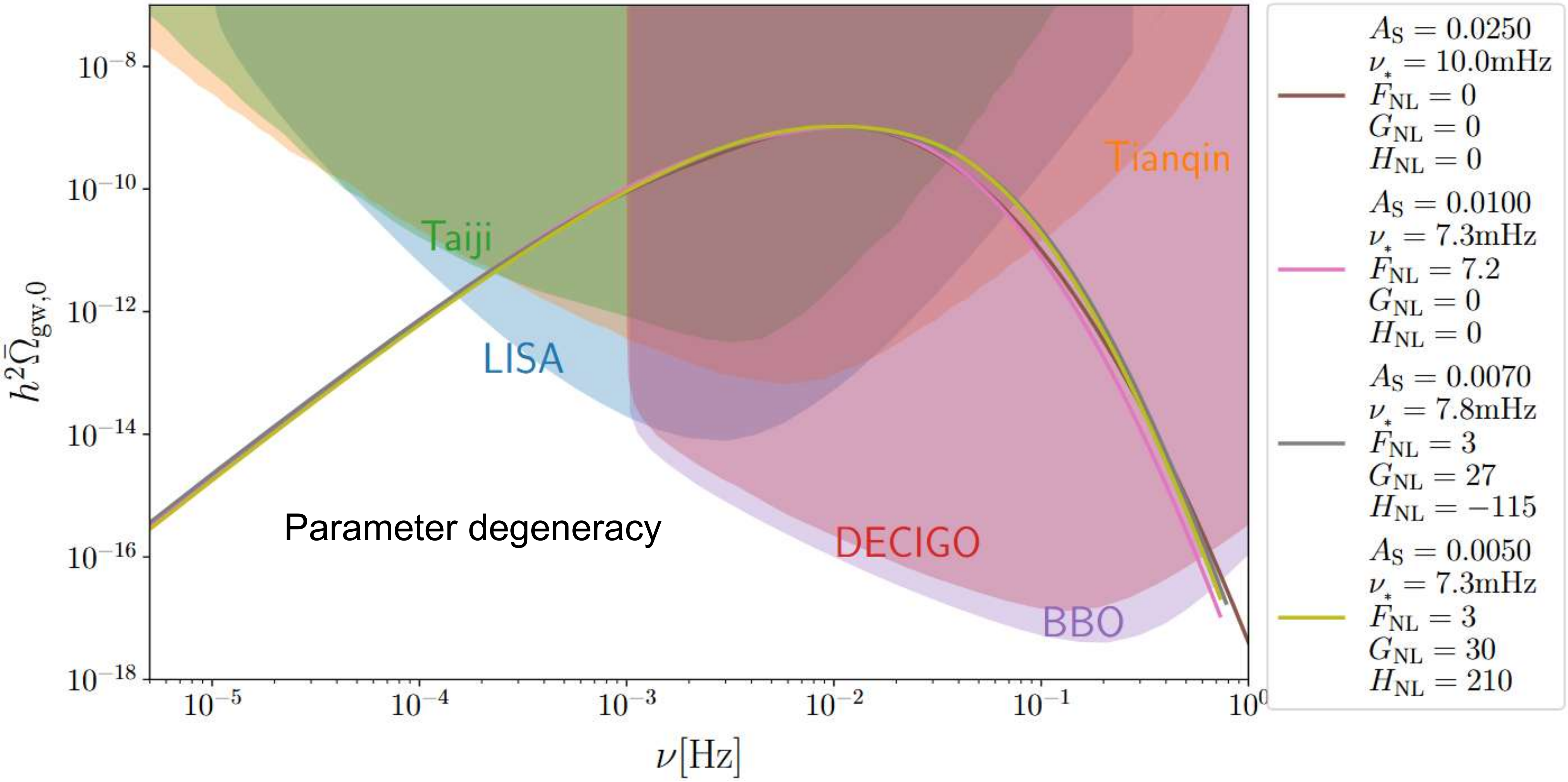


$$\bar{\omega}_{\mathrm{gw,in}}(q) = \sum_{(a,b,c)} \bar{\omega}^{(a,b,c)}_{\mathrm{gw,in}}(q) \qquad \bar{\omega}^{(a,b,c)}_{\mathrm{gw,in}} \propto F^a_{\mathrm{NL}} G^b_{\mathrm{NL}} H^c_{\mathrm{NL}} A^{(a+2b+3c)/2}$$

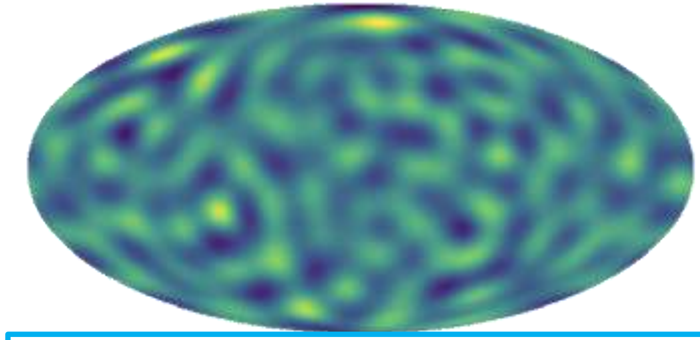
Numerical Results

$$\Delta_S^2(q) = \frac{A_S}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\ln^2(q/q_*)}{2\sigma^2}\right)$$

$$\sigma = 1$$



Anisotropies and Non-Gaussianity



The SIGW Background

N. Bartolo, D. Bertacca, V. De Luca, JCAP 02 (2020) 028
JL, S. Wang, Z.-C. Zhao, and K. Kohri, JCAP 06 (2024) 039
JL, S. Wang, Z.-C. Zhao, and K. Kohri, JCAP 05 (2024) 109……

Anisotropies

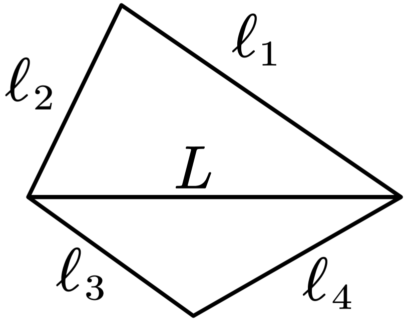
Non-Gaussianity

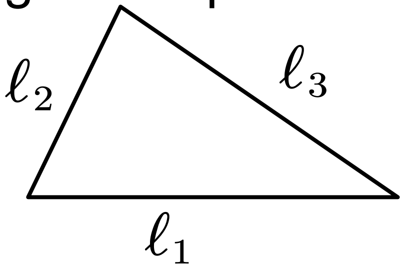
Statistical isotropy and parity invariance

$$\delta_{\mathrm{gw},0,\ell m}(2\pi\nu) = \int \mathrm{d}^2\mathbf{n}\, \delta_{\mathrm{gw},0}(\mathbf{q})\, Y^*_{\ell m}(\mathbf{n})$$

Angular power spectrum $\left\langle \prod_{i=1}^{2} \delta_{\mathrm{gw},0,\ell_i m_i}(2\pi\nu) \right\rangle = \delta_{\ell_1\ell_2}\delta_{m_1 m_2}(-1)^{m_1}\widetilde{C}_\ell(\nu)$

Angular bispectrum $\left\langle \prod_{i=1}^{3} \delta_{\mathrm{gw},0,\ell_i m_i}(2\pi\nu) \right\rangle = \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_\ell(\nu)$

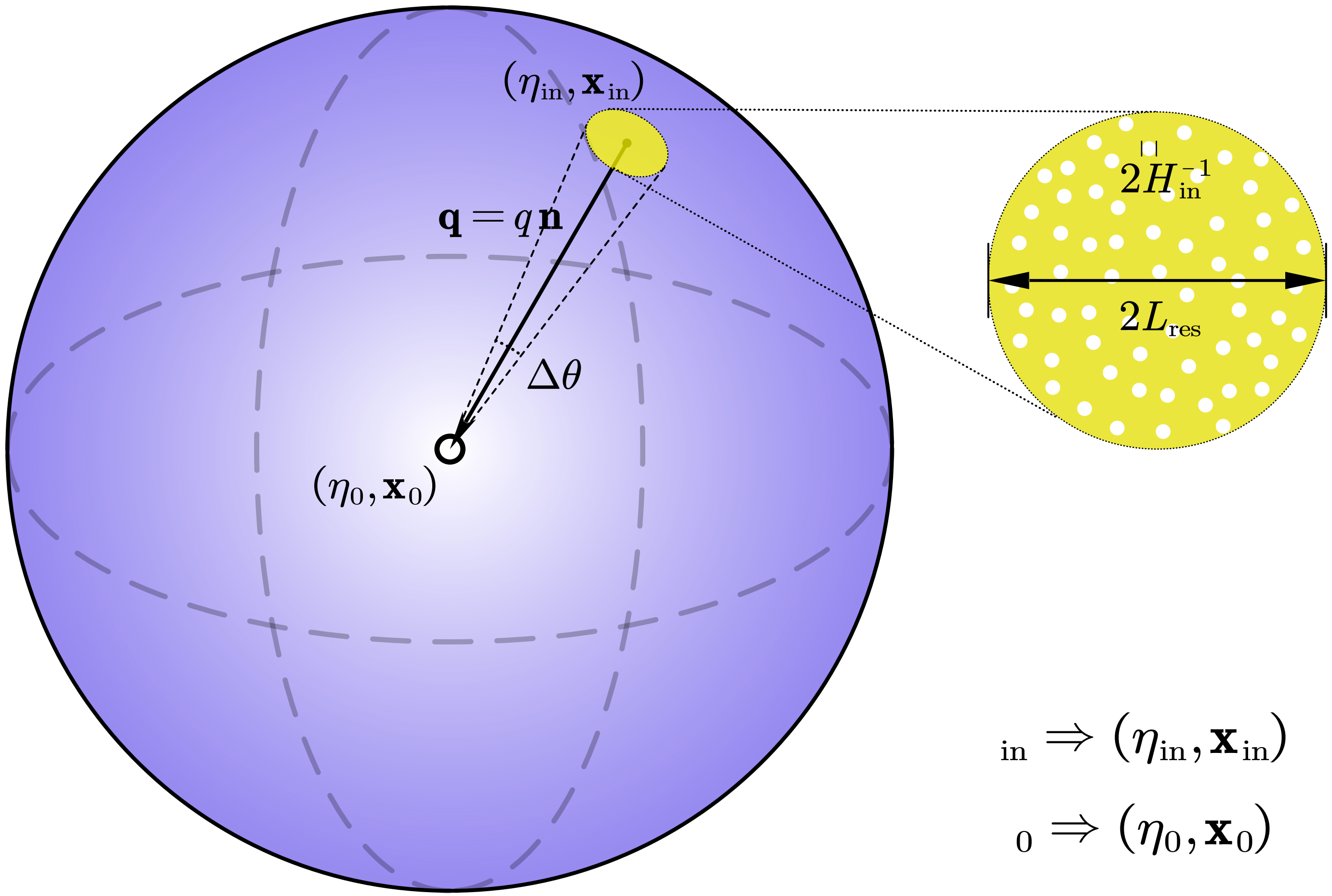




Angular trispectrum $\left\langle \prod_{i=1}^{4} \delta_{\mathrm{gw},0,\ell_i m_i}(2\pi\nu) \right\rangle_{\mathrm{c}} = \sum_{LM} (-1)^M \begin{pmatrix} \ell_1 & \ell_2 & L \\ m_1 & m_2 & -M \end{pmatrix} \begin{pmatrix} \ell_3 & \ell_4 & L \\ m_3 & m_4 & M \end{pmatrix} t^{\ell_1\ell_2}_{\ell_3\ell_4}(L,\nu) + 11\,\mathrm{perms}$

“Coarse-Grained“ Landscape

$$\delta_{\mathrm{gw},0}(\mathbf{q}) = \delta_{\mathrm{gw,in}}(\mathbf{q}) + (6 - n_{\mathrm{gw}}(\nu))\,\Phi_{\mathrm{in}} \qquad \mathbf{q} = q\,\mathbf{n}$$



$\mathrm{in} \Rightarrow (\eta_{\mathrm{in}}, \mathbf{x}_{\mathrm{in}})$

$0 \Rightarrow (\eta_0, \mathbf{x}_0)$

$$\delta_{\mathrm{gw},0}(q, \mathbf{n}) = \delta_{\mathrm{gw,in}}(q) + (6 - n_{\mathrm{gw}}(\nu))\,\Phi_{\mathrm{in}} \qquad \mathbf{n} = \frac{\mathbf{x}_0 - \mathbf{x}_{\mathrm{in}}}{\eta_0 - \eta_{\mathrm{in}}}$$

SIGWs in RD: Extremely small Hubble horizons

Detectors limit on angular resolution

↓

The resolution element covers numerous Hubble-horizon patches along a line-of-sight: $H_{\mathrm{in}}^{-1} \ll L_{\mathrm{res}}$

↓

Superposition of SIGWs

Assumption of statistical isotropy

↓

Initial anisotropies vanish

↓

$\delta_{\mathrm{gw,in}}$ is independent of the direction $\mathbf{n}$

Initial Inhomogeneities

Total initial Inhomogeneities   SW effect

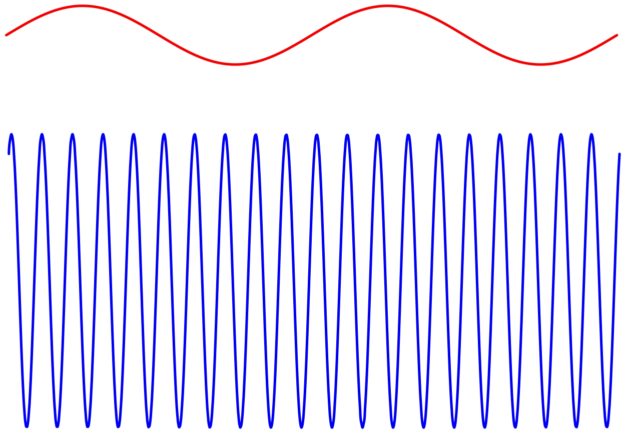
$$\delta_{\mathrm{gw},0}(q,\mathbf{n}) = \tilde{\delta}_{\mathrm{gw,in}}(q) + (4 - n_{\mathrm{gw}}(\nu))\Phi_{\mathrm{L,in}}$$

$$\delta_{\mathrm{gw},0}(q,\mathbf{n}) = \delta_{\mathrm{gw,in}}(q) + (6 - n_{\mathrm{gw}}(\nu))\Phi_{\mathrm{L,in}}$$

PNG-induced Inhomogeneity

ζ_{gL}

ζ_{gS}



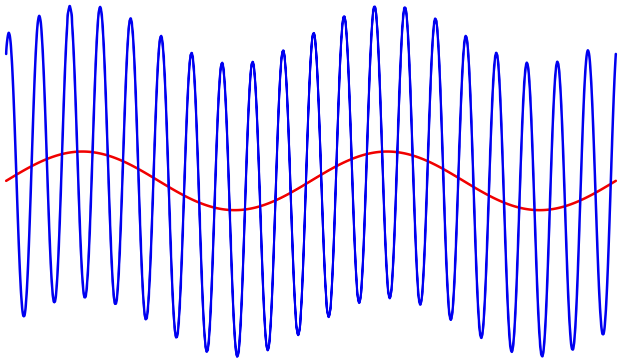
Small-scale: SIGW production

Large-scale: energy-density modulation

Gaussian scenario:

$$\tilde{\delta}_{\mathrm{gw,in}} = 2\Phi_{\mathrm{L,in}}$$

$$\delta_{\mathrm{gw,in}} = 0$$

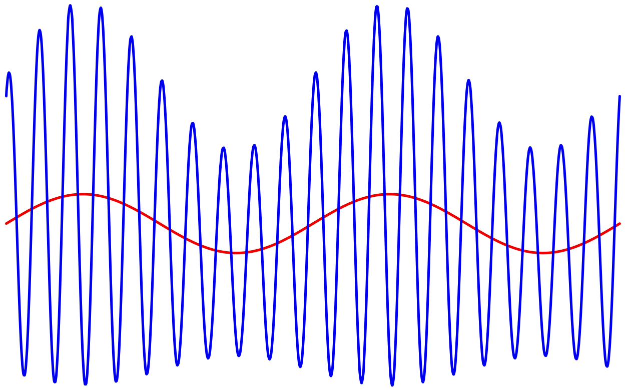


$$\zeta_{\mathrm{g}} = \zeta_{\mathrm{gS}} + \zeta_{\mathrm{gL}}$$

Non-Gaussian scenario:

$$\tilde{\delta}_{\mathrm{gw,in}} = \delta_{\mathrm{gw,in}} + 2\Phi_{\mathrm{L,in}}$$

$$\delta_{\mathrm{gw,in}} \sim \langle\zeta\zeta\zeta\zeta\rangle$$



$$\zeta = \zeta_{\mathrm{gS}} + \zeta_{\mathrm{gL}} + f_{\mathrm{NL}}\zeta_{\mathrm{gS}}\zeta_{\mathrm{gL}}$$

Non-Gaussianity: Coupling between short- and long-wavelength modes

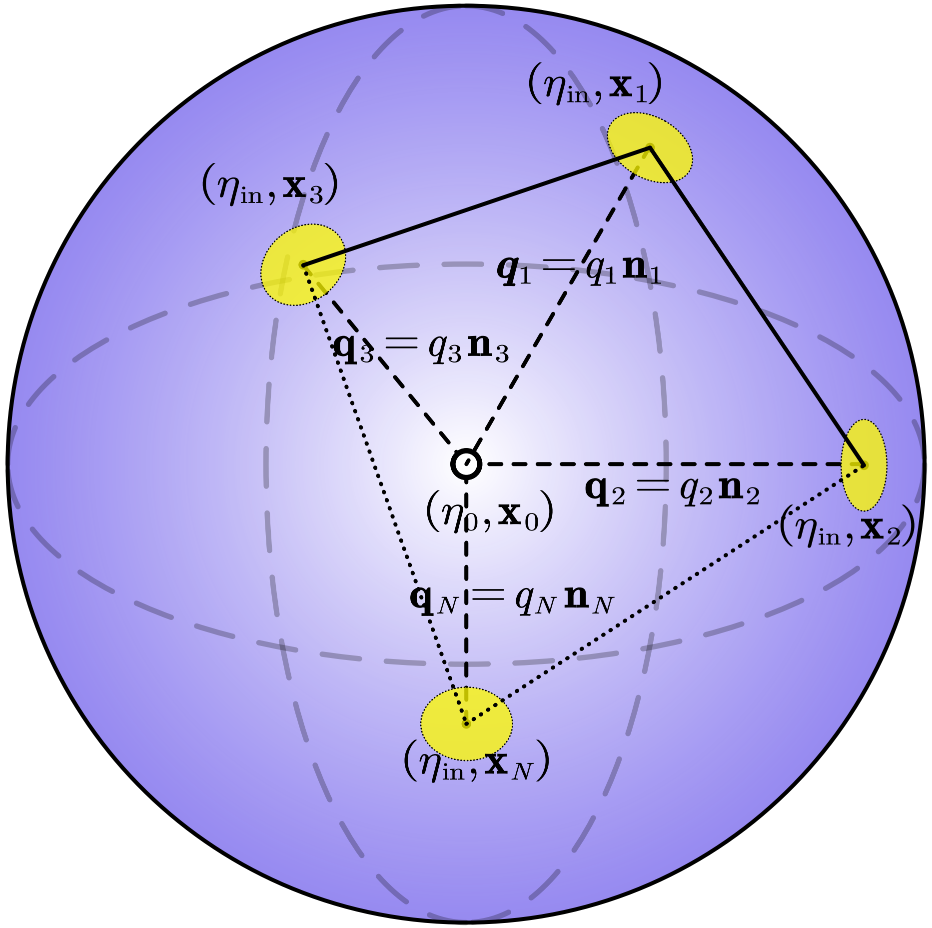
Multi-Point Correlators

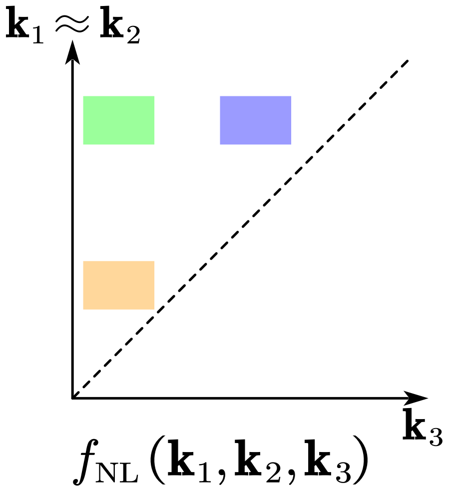
$$\delta_{\mathrm{gw},0}(q,\mathbf{n}) = \delta_{\mathrm{gw,in}}(q) + (6 - n_{\mathrm{gw}}(\nu))\,\Phi_{\mathrm{in}}$$

$$\mathbf{n} = \frac{\mathbf{x}_0 - \mathbf{x}_{\mathrm{in}}}{\eta_0 - \eta_{\mathrm{in}}}$$

$\mathrm{in} \Rightarrow (\eta_{\mathrm{in}}, \mathbf{x}_{\mathrm{in}})$

$0 \Rightarrow (\eta_0, \mathbf{x}_0)$





$$\left\langle \prod_{i=1}^{N} \omega_{\mathrm{gw,in},i}(q) \right\rangle$$

$$\delta_{\mathrm{gw,in}}(q) = \frac{\omega_{\mathrm{gw,in}}(q)}{\bar{\omega}_{\mathrm{gw,in}}(q)} - 1$$

$$\left\langle \prod_{i=1}^{N} \delta_{\mathrm{gw},0}(\mathbf{q}) \right\rangle = \left\langle \prod_{i=1}^{N} \delta_{\mathrm{gw,in},i}(q) \right\rangle + (6 - n_{\mathrm{gw}}(\nu)) \left\langle \Phi_{\mathrm{L,in},N} \prod_{i=1}^{N-1} \delta_{\mathrm{gw,in},i}(q) \right\rangle$$

$$+ \cdots + (6 - n_{\mathrm{gw}}(\nu))^{N-1} \left\langle \delta_{\mathrm{gw,in},1}(q) \prod_{i=2}^{N} \Phi_{\mathrm{L,in},i} \right\rangle$$

$$+ (6 - n_{\mathrm{gw}}(\nu))^{N} \left\langle \prod_{i=1}^{N} \Phi_{\mathrm{L,in},i} \right\rangle$$

Diagrammatic Representation

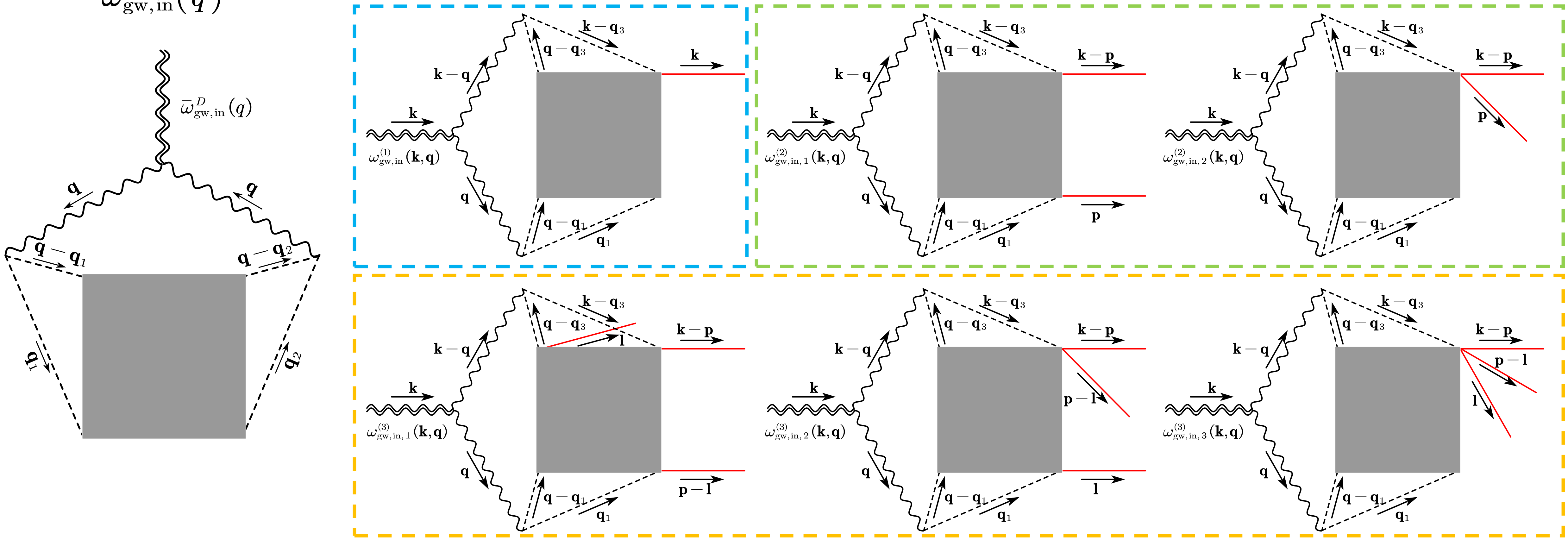
$$\omega_{\rm gw,in}(q) = \bar{\omega}_{\rm gw,in}(q) + \omega^{(1)}_{\rm gw,in}(q) + \omega^{(2)}_{\rm gw,in}(q) + \omega^{(3)}_{\rm gw,in}(q) + \cdots$$

$$\zeta_{\rm g} = \zeta_{\rm gS} + \zeta_{\rm gL} \quad \langle \zeta_{\rm gX}(\mathbf{k})\zeta_{\rm gX'}(\mathbf{k}')\rangle = \delta_{\rm XX'}\delta(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}\Delta^2_{\rm gX}(k), \;\; {\rm X} = {\rm S,L}$$

$\bar{\omega}_{\rm gw,in}(q)$

$\omega^{(1)}_{\rm gw,in}(q) = \omega^{(1)}_{\rm ng}(q)\zeta_{\rm gL}$

$\omega^{(2)}_{\rm gw,in}(q) = \omega^{(2)}_{\rm ng}(q)\zeta^2_{\rm gL}$

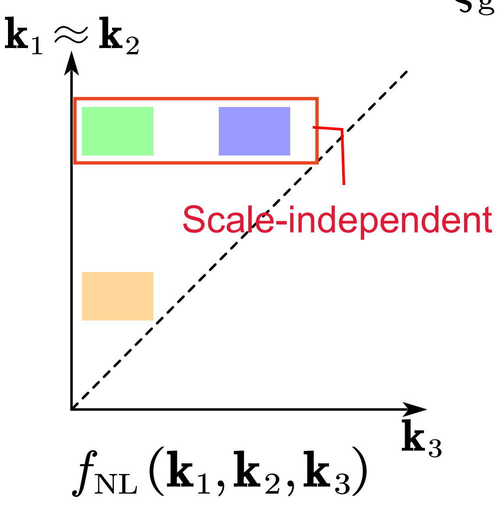


$$\omega^{(3)}_{\rm gw,in}(q) = \omega^{(3)}_{\rm ng}(q)\zeta^3_{\rm gL}$$

Diagrammatic Approach

$$\zeta = \zeta_{\rm g} + F_{\rm NL}\left(\zeta_{\rm g}^2 - \langle\zeta_{\rm g}^2\rangle\right) + G_{\rm NL}\zeta_{\rm g}^3 + H_{\rm NL}\left(\zeta_{\rm g}^4 - 3\langle\zeta_{\rm g}^2\rangle^2\right) + \cdots$$

$$\zeta_{\rm g} = \zeta_{\rm gS} + \zeta_{\rm gL}$$

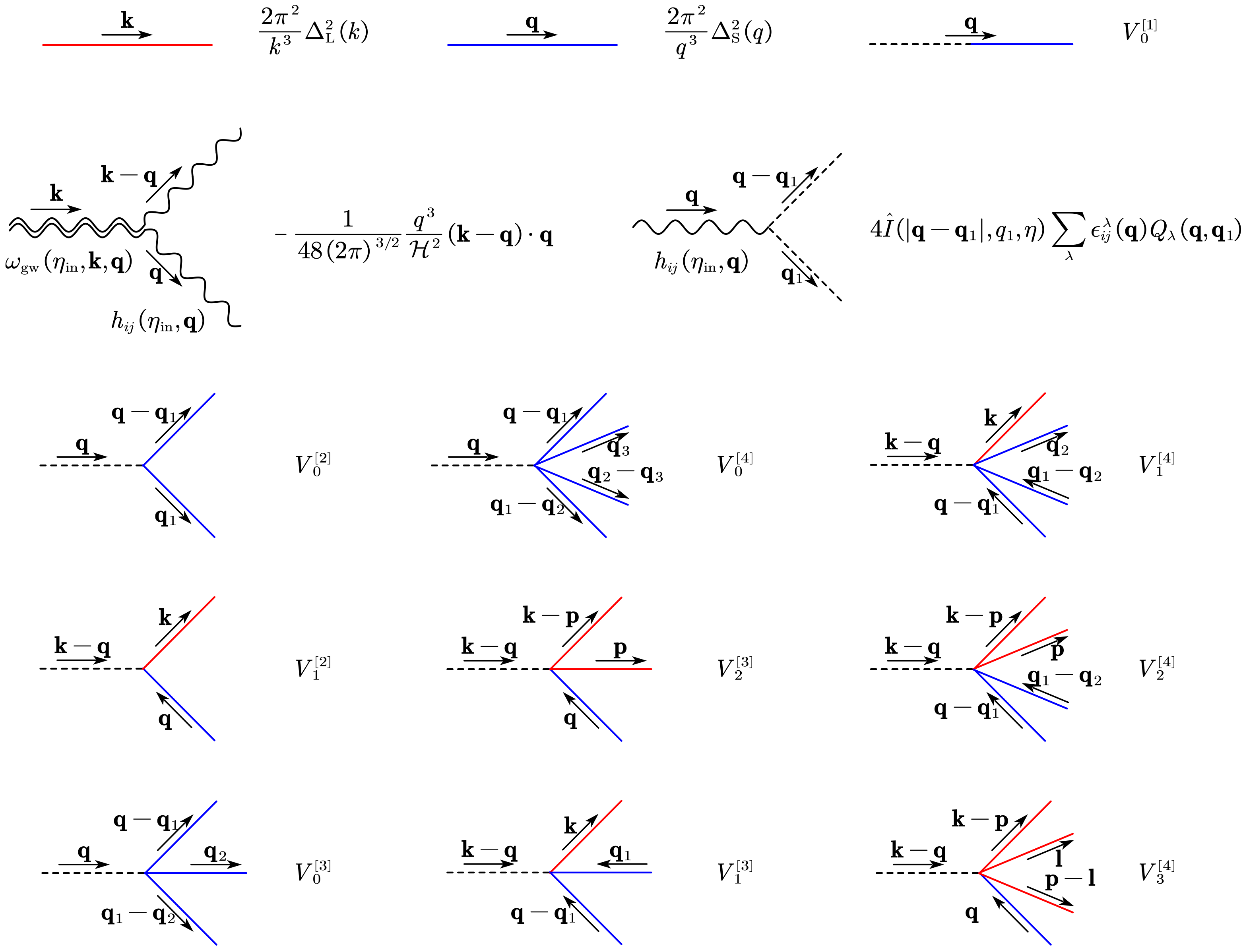


$$V_0^{[1]} = 1$$

$$V_0^{[2]} = V_1^{[2]} = F_{\rm NL}$$

$$V_0^{[3]} = V_1^{[3]} = V_2^{[3]} = G_{\rm NL}$$

$$V_0^{[4]} = V_1^{[4]} = V_2^{[4]} = V_3^{[4]} = H_{\rm NL}$$

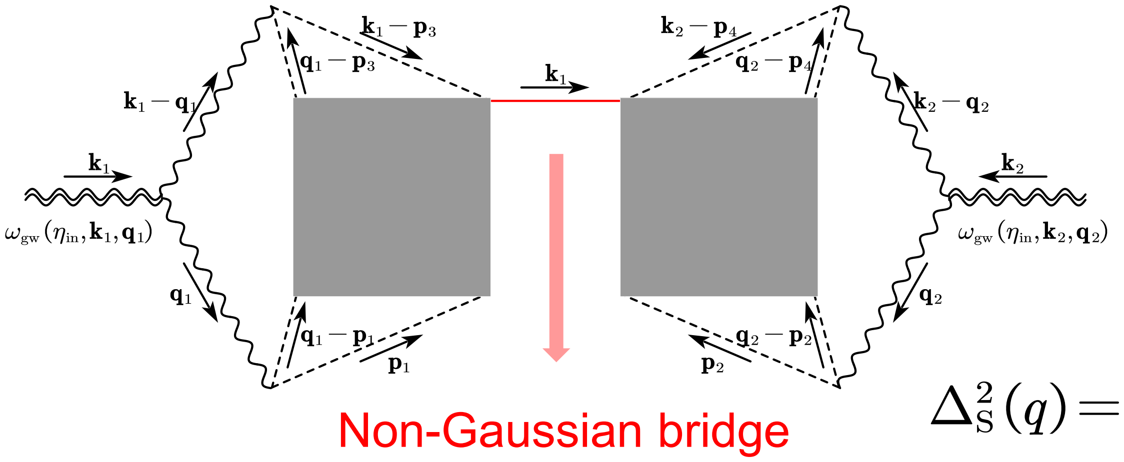


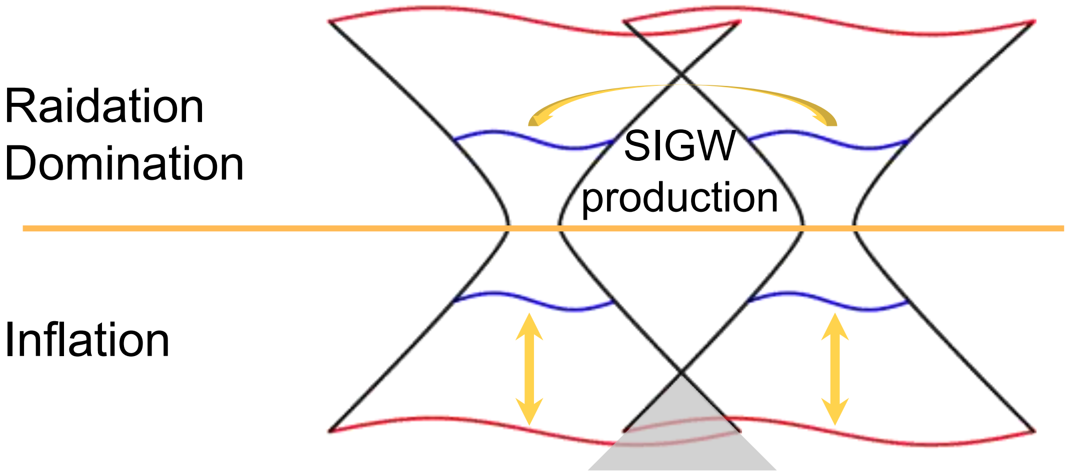
Two-point Angular Correlator

$$\zeta = \zeta_g + F_{NL}(\zeta_g^2 - \langle \zeta_g^2 \rangle) + G_{NL}\zeta_g^3 + H_{NL}(\zeta_g^4 - 3\langle \zeta_g^2 \rangle^2) + \cdots$$

$$\zeta_g = \zeta_{gS} + \zeta_{gL}$$

$$\langle \zeta_{gX}(\mathbf{k})\zeta_{gX'}(\mathbf{k}')\rangle = \delta_{XX'}\delta(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}\Delta^2_{gX}(k),\ \ X = S, L$$





$$\Delta^2_S(q) = \frac{A_S}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\ln^2(q/q_*)}{2\sigma^2}\right)$$

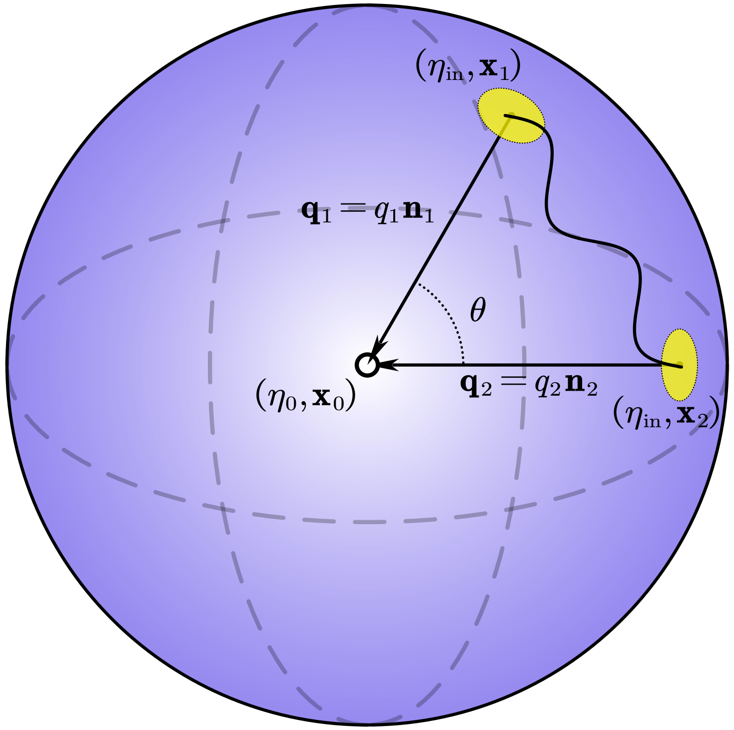
$$\Delta^2_L(q) = A_L \simeq 2.1 \times 10^{-9}$$

$$\langle \omega_{gw}(\eta_{in}, \mathbf{k}_1, q_1)\omega_{gw}(\eta_{in}, \mathbf{k}_2, q_2)\rangle$$

$$\sim \langle \langle \zeta_{gS}\cdots\zeta_{gS}\zeta_{gL}\rangle \langle \zeta_{gS}\cdots\zeta_{gS}\zeta_{gL}\rangle\rangle$$

$$\sim \langle \zeta_{gS}\cdots\zeta_{gS}\rangle \langle \zeta_{gL}\zeta_{gL}\rangle \langle \zeta_{gS}\cdots\zeta_{gS}\rangle$$

ζ_{gS} at $\mathbf{x}_1$ and $\mathbf{x}_2$ are not correlated.



1st-order PNG-induced Initial Inhomogeneity

$$\omega^{(1)}_{\mathrm{gw,in}}(q) = \omega^{(1)}_{\mathrm{ng}}(q) \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} \zeta_{\mathrm{gL}}(\mathbf{k})$$

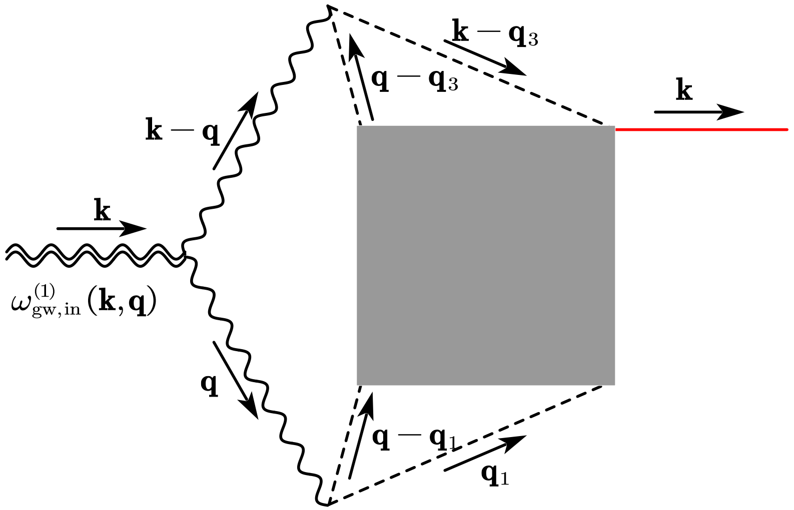
JL, S. Wang, Z.-C. Zhao, and K. Kohri, JCAP 10 (2023) 056
JL, S. Wang, Z.-C. Zhao, and K. Kohri, JCAP 06 (2024) 039

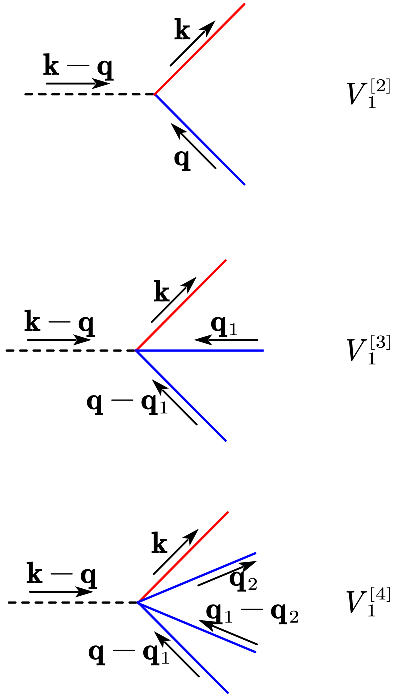
$$\omega^{(1)}_{\mathrm{ng}}(q) = \sum_{c=0}^{4} \sum_{b=0}^{\lfloor 4-c \rfloor} \sum_{a=0}^{\lfloor 4-b-c \rfloor} \bar{\omega}^{(a,b,c)}_{\mathrm{gw,in}}(q) \sum_{i=1}^{o-1} \frac{N_i (i+1) V_1^{[i+1]}}{V_0^{[i]}}$$

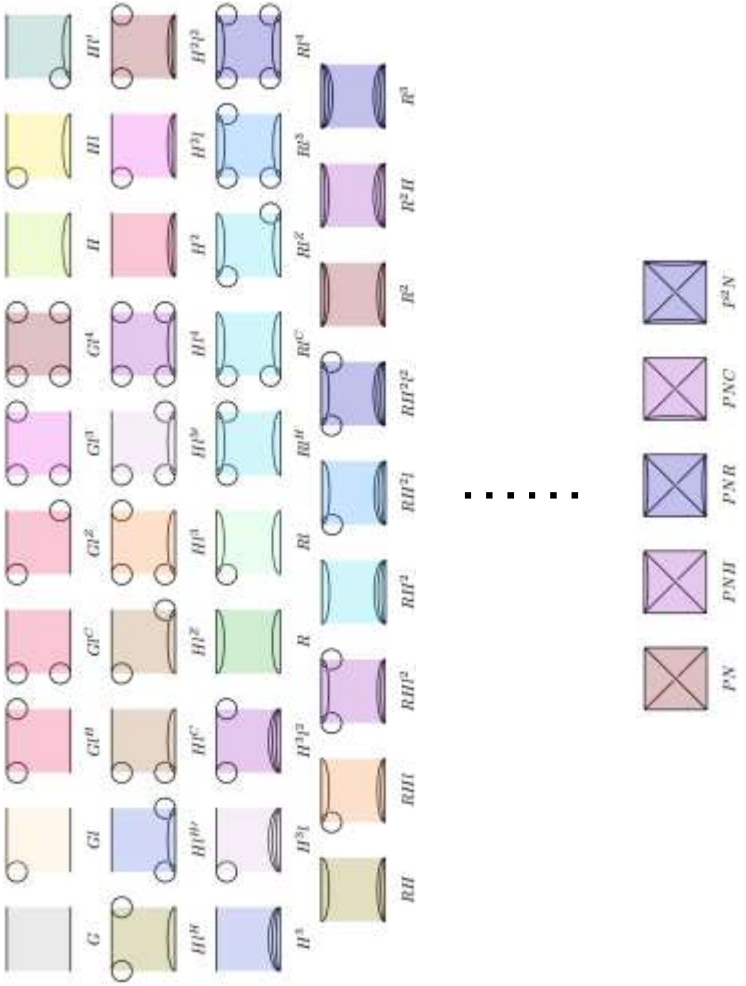
$$\bar{\omega}^{(a,b,c)}_{\mathrm{gw,in}} \propto F^a_{\mathrm{NL}} G^b_{\mathrm{NL}} H^c_{\mathrm{NL}} A^{(a+2b+3c)/2}$$

N_i : number of $V_0^{[i]}$-vertices. $N_1 = 4 - a - b - c$

$N_2 = a \quad N_3 = b \quad N_4 = c$





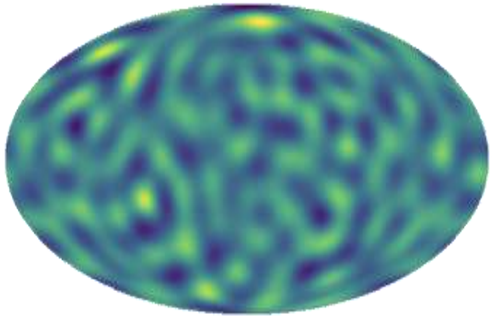


Reduced Angular Power Spectrum

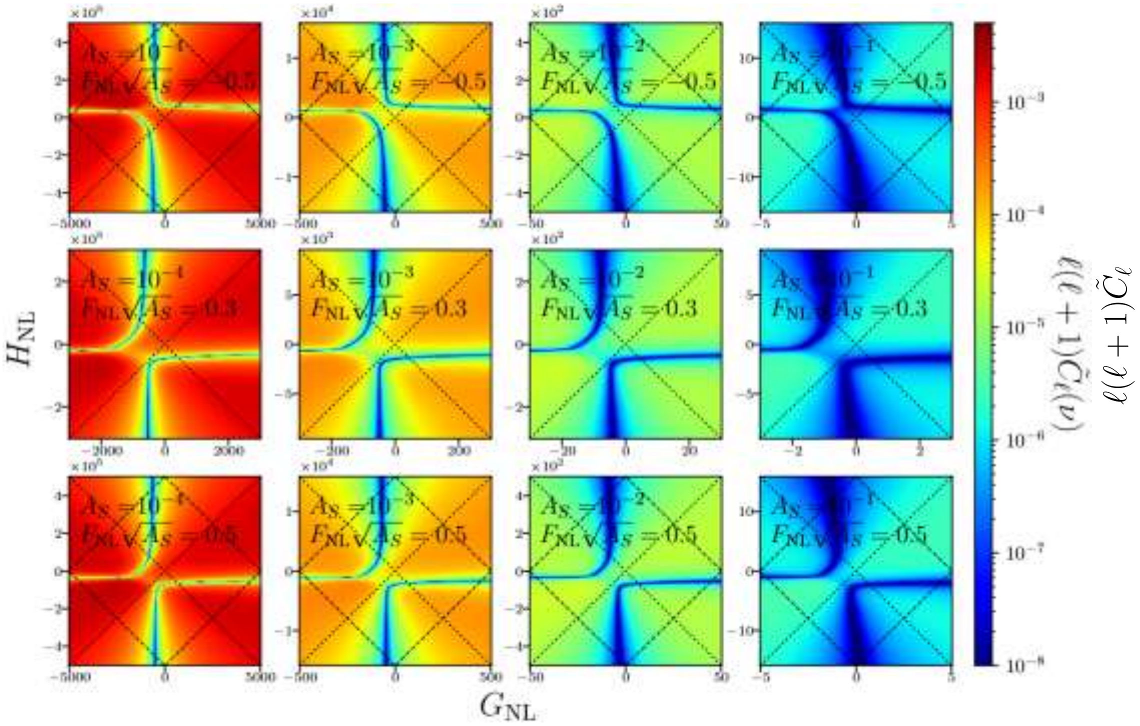
PNG-induced Inhomogeneity SW effect & IIC

JL, S. Wang, Z.-C. Zhao, and K. Kohri, JCAP 10 (2023), 056
JL, S. Wang, Z.-C. Zhao, and K. Kohri, JCAP 06 (2024) 039

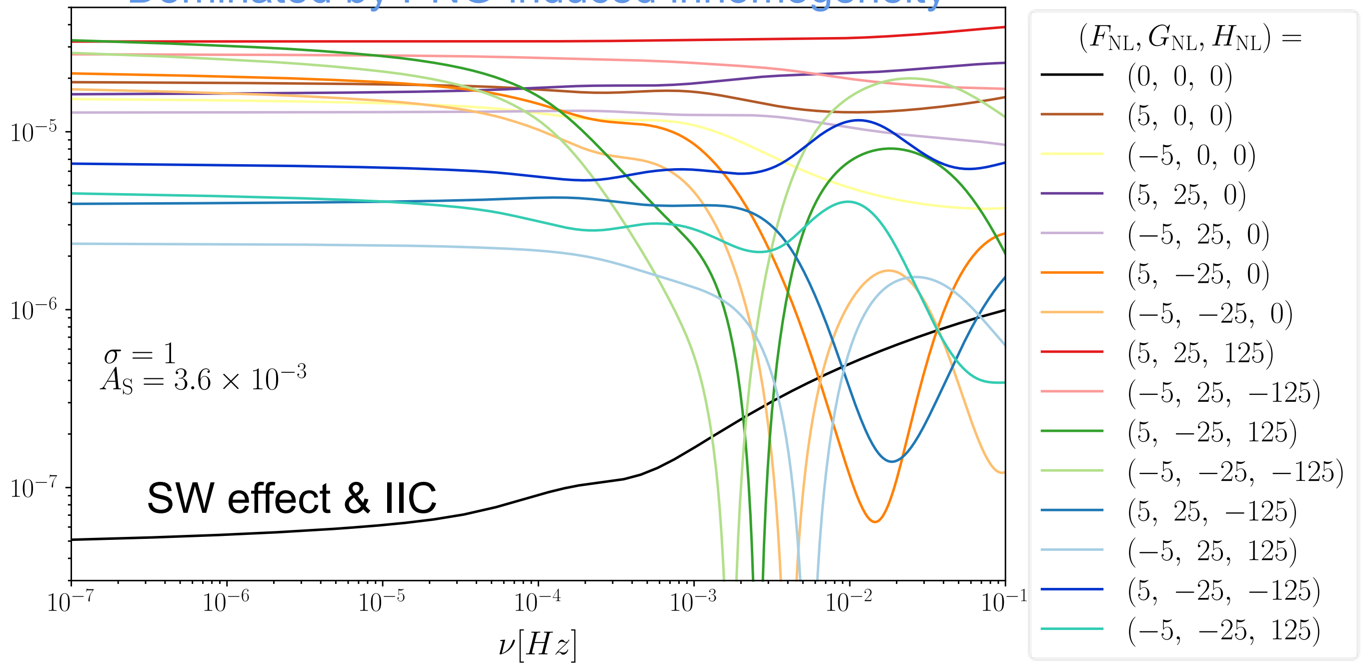
$$\delta^{(1)}_{\mathrm{gw},0}(q) = \left[\frac{\omega^{(1)}_{\mathrm{ng}}(q)}{\bar{\omega}_{\mathrm{gw,in}}(q)} + \frac{3}{5}\left(6 - n_{\mathrm{gw}}(\nu)\right)\right]\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}}\zeta_{\mathrm{gL}}(\mathbf{k})$$



$$\tilde{C}_\ell(\nu) = \frac{2\pi A_{\mathrm{L}}}{\ell(\ell+1)}\left[\frac{\omega^{(1)}_{\mathrm{ng}}(2\pi\nu)}{\bar{\omega}_{\mathrm{gw,in}}(2\pi\nu)} + \frac{3}{5}\left(6 - n_{\mathrm{gw}}(\nu)\right)\right]^2$$



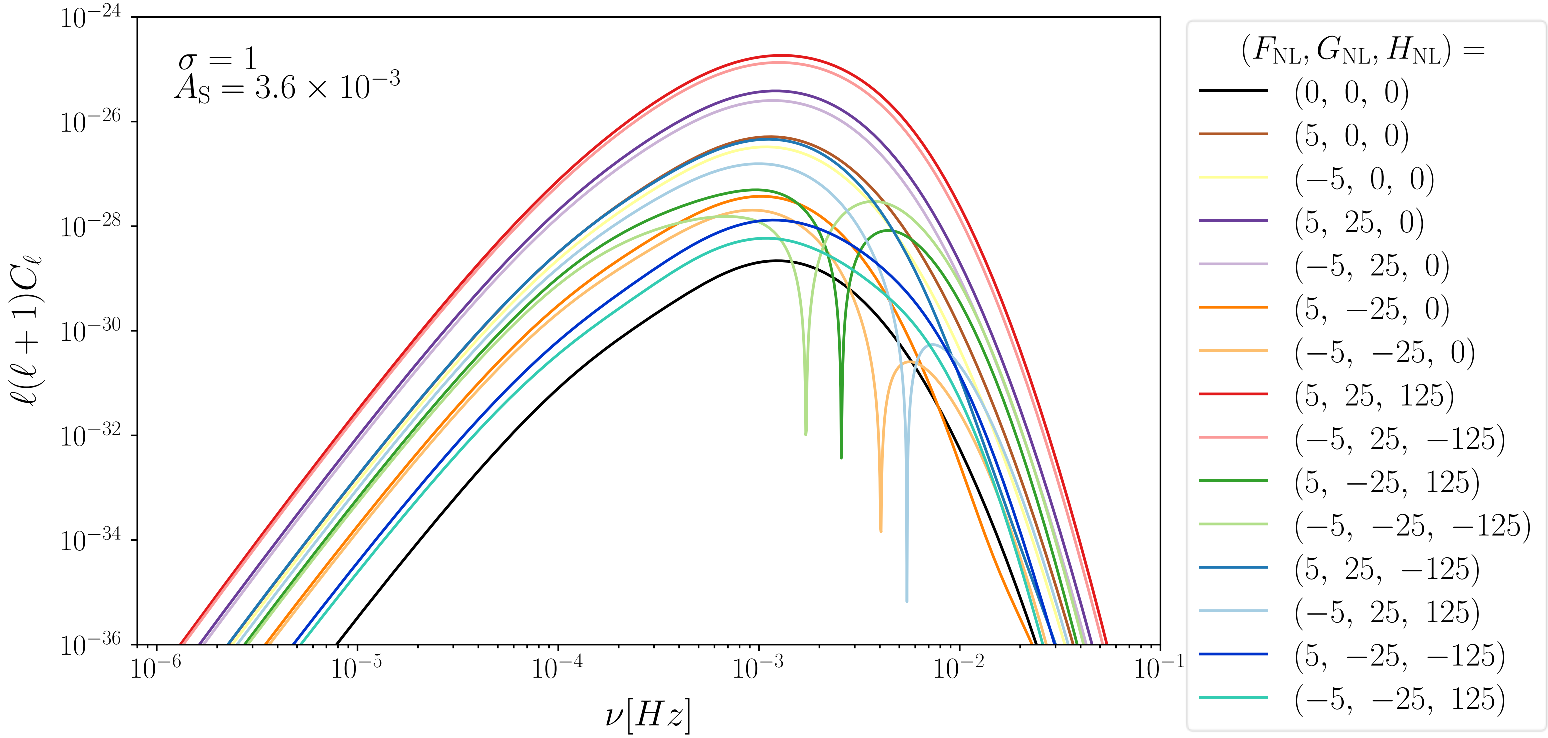
Dominated by PNG-induced inhomogeneity



Angular Power Spectrum

$$C_\ell(\nu) = \left[\frac{\bar{\Omega}_{\mathrm{gw},0}(\nu)}{4\pi}\right]^2 \tilde{C}_\ell(\nu)$$

JL, S. Wang, Z.-C. Zhao, and K. Kohri, arXiv:2309.07792



Parameter degeneracy breaking

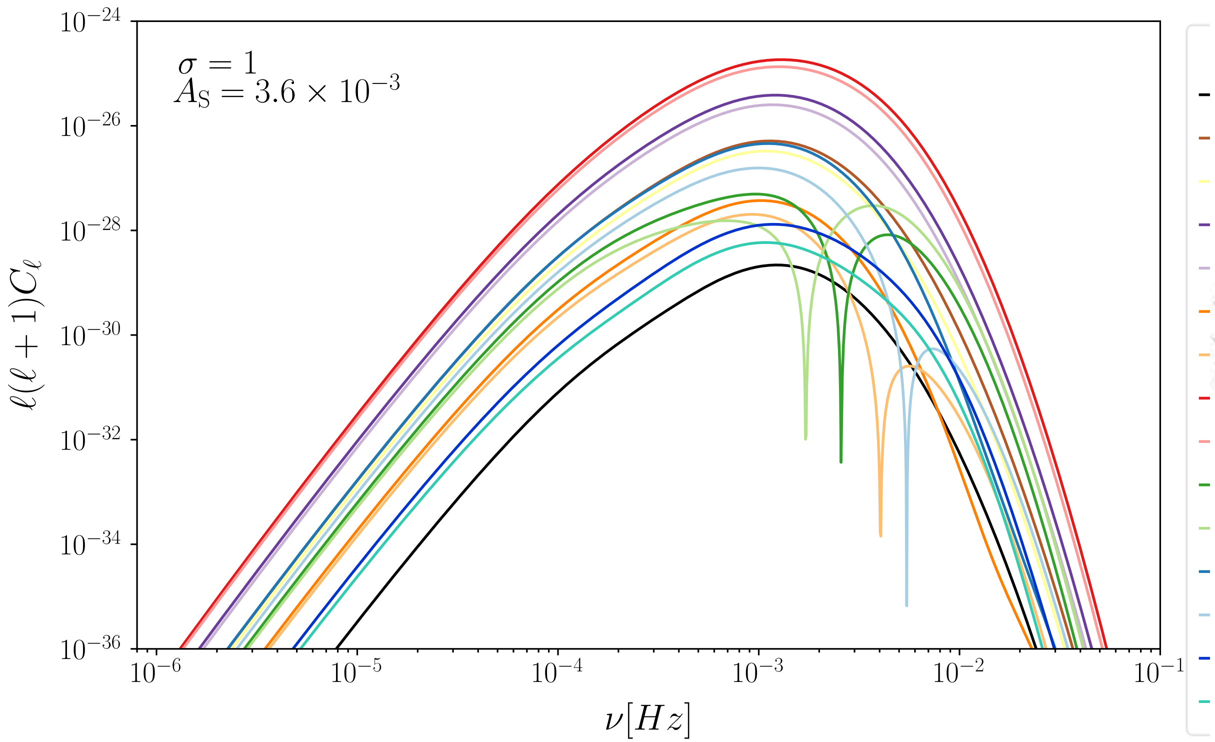
SIGWs: $C_\ell(\nu) \propto [\ell(\ell+1)]^{-1}$

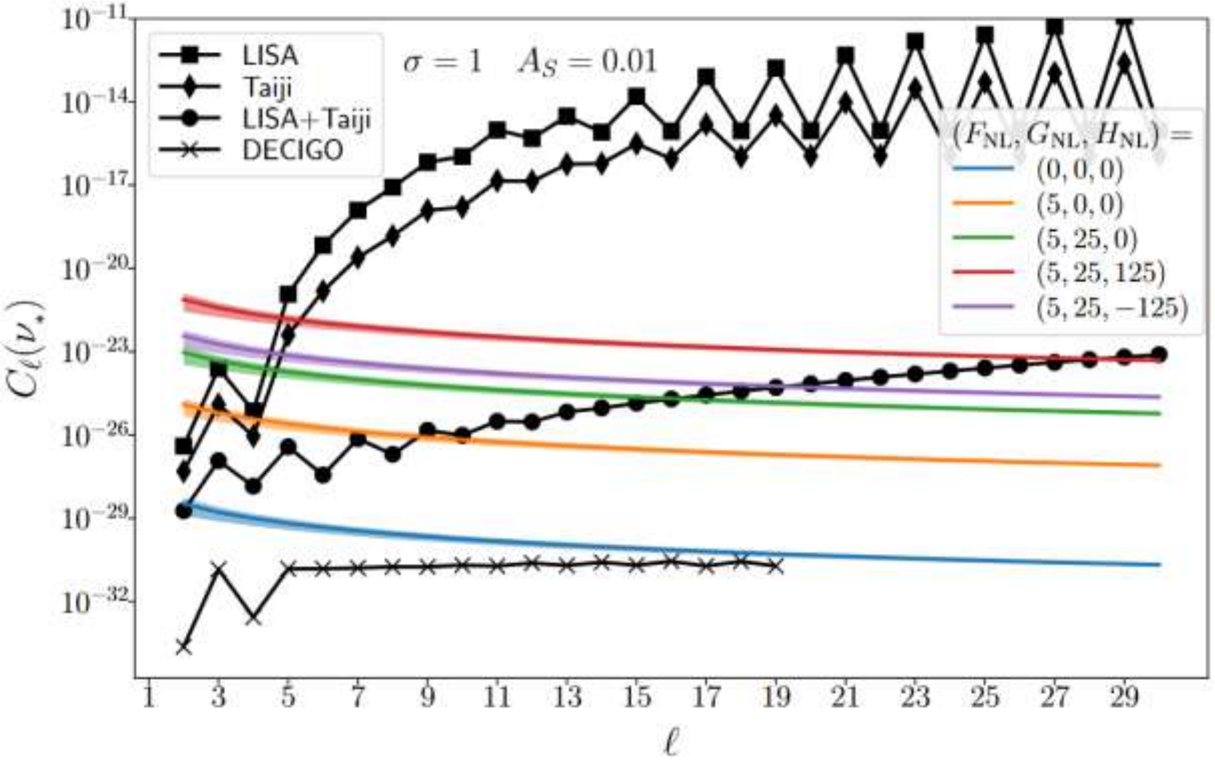
Astrophysical GWs: $C_\ell(\nu) \propto (\ell+1/2)^{-1}$

Angular Power Spectrum

$$C_\ell(\nu) = \left[\frac{\bar{\Omega}_{\mathrm{gw},0}(\nu)}{4\pi}\right]^2 \tilde{C}_\ell(\nu)$$

JL, S. Wang, Z.-C. Zhao, and K. Kohri, arXiv:2309.07792





Parameter degeneracy breaking

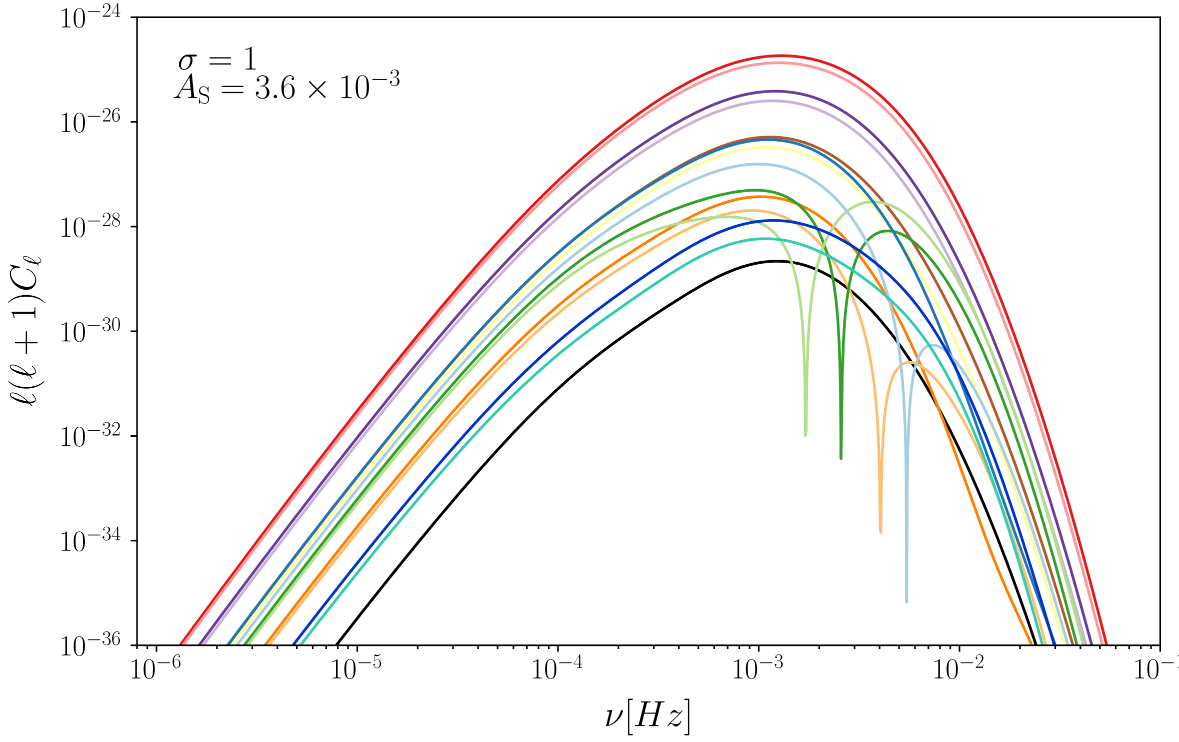
SIGWs: $C_\ell(\nu) \propto [\ell(\ell+1)]^{-1}$

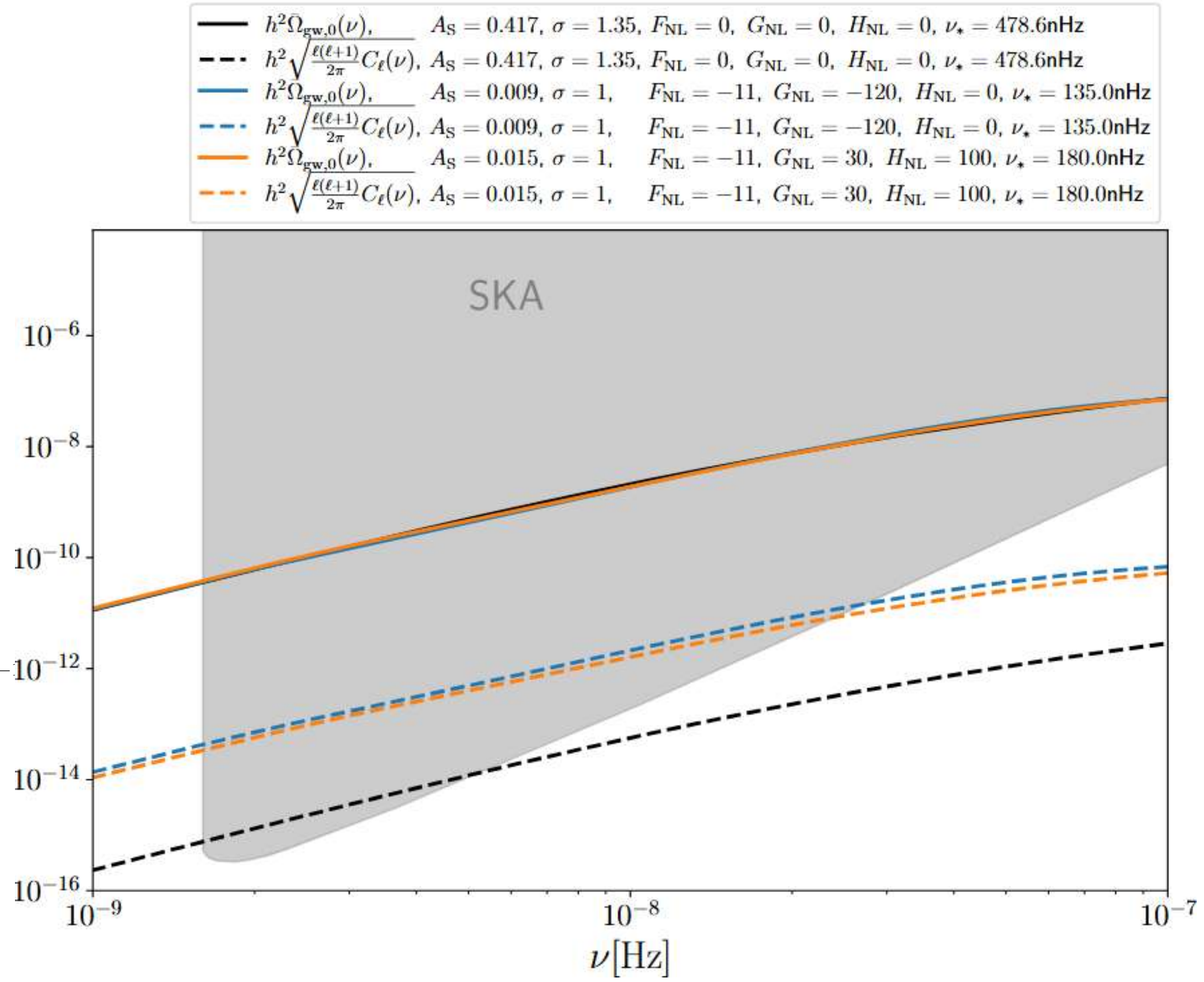
Astrophysical GWs: $C_\ell(\nu) \propto (\ell+1/2)^{-1}$

Angular Power Spectrum

$$C_\ell(\nu) = \left[\frac{\bar{\Omega}_{\mathrm{gw},0}(\nu)}{4\pi}\right]^2 \tilde{C}_\ell(\nu)$$

JL, S. Wang, Z.-C. Zhao, and K. Kohri, arXiv:2309.07792





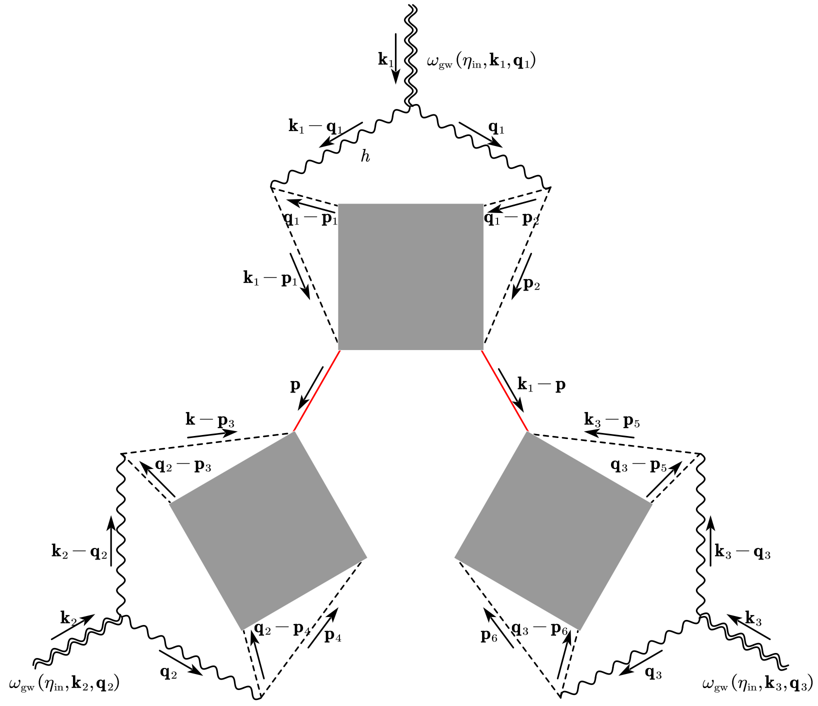
Parameter degeneracy breaking

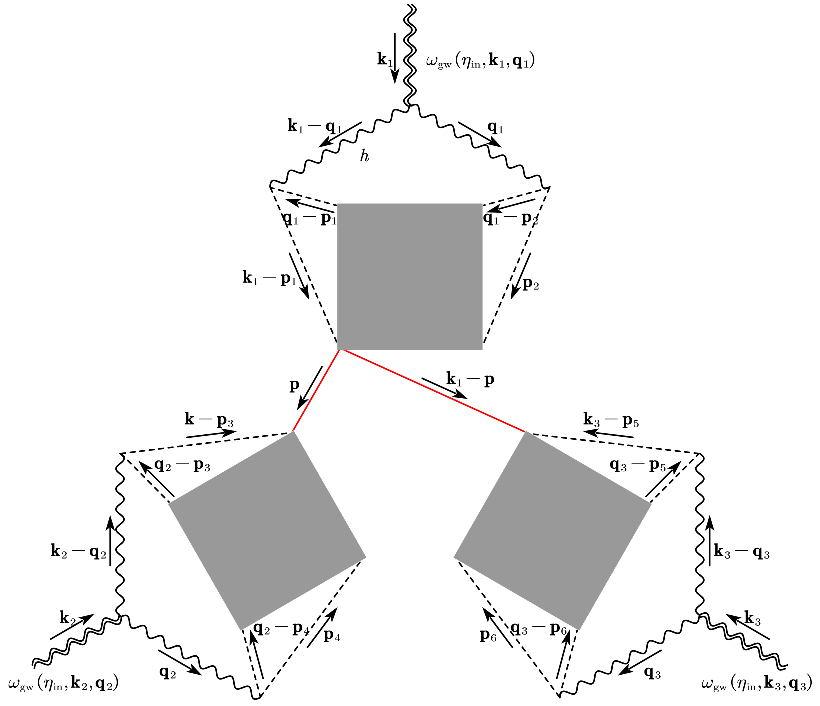
SIGWs: $C_\ell(\nu) \propto [\ell(\ell+1)]^{-1}$

Astrophysical GWs: $C_\ell(\nu) \propto (\ell + 1/2)^{-1}$

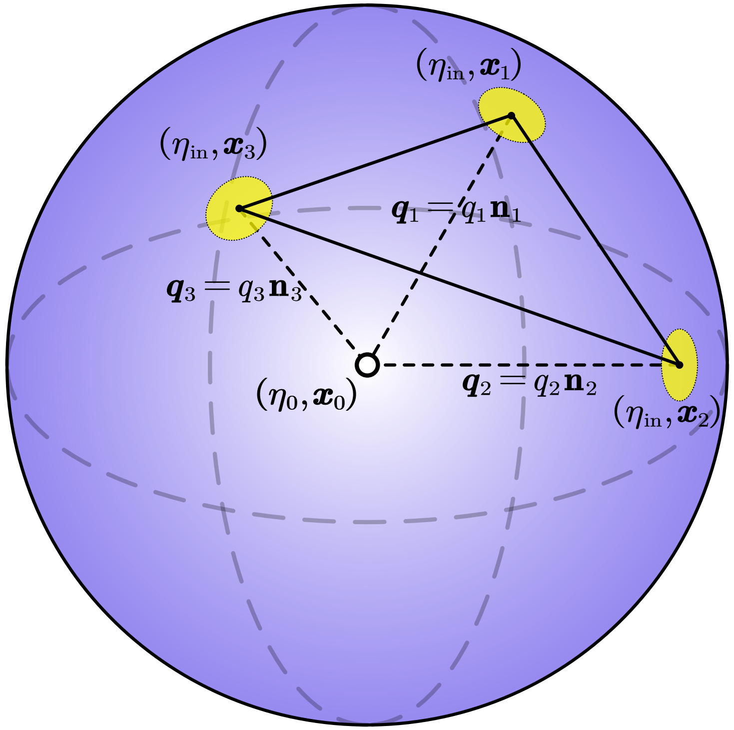
Three-point Angular Correlator

JL, S. Wang, Z.-C. Zho, and K. Kohri, JCAP 05 (2024), 109





$$\left\langle \prod_{i=1}^{3} \omega_{\rm gw}(\eta_{\rm in},\mathbf{k}_i,q) \right\rangle$$



$$\omega_{\rm gw,in}(q) = \bar{\omega}_{\rm gw,in}(q) + \omega^{(1)}_{\rm gw,in}(q) + \omega^{(2)}_{\rm gw,in}(q)$$

$$\omega^{(2)}_{\rm gw,in}(q) = \omega^{(2)}_{\rm ng}(q)\int \frac{{\rm d}^3\mathbf{k}\,{\rm d}^3\mathbf{p}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}_{\rm in}}\zeta_{\rm gL}(\mathbf{p})\,\zeta_{\rm gL}(\mathbf{k}-\mathbf{p})$$

Angular Bispectrum

$$B_{\ell_1\ell_2\ell_3}(\nu) = b(\nu)\, h_{\ell_1\ell_2\ell_3}\left[\frac{1}{\ell_1\ell_2(\ell_1+1)(\ell_2+1)} + (\ell_1 \leftrightarrow \ell_3) + (\ell_2 \leftrightarrow \ell_3)\right]$$

$$b(\nu) = 2(2\pi A_{\rm L})^2\left[\frac{\omega_{\rm ng}^{(1)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}(6 - n_{\rm gw}(\nu))\right]^2\left[\frac{\omega_{\rm ng}^{(2)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}F_{\rm NL}(6 - n_{\rm gw}(\nu))\right]$$

PNG-induced Inhomogeneity SW effect & IIC

$$\omega_{\rm ng}^{(2)}(q) = \sum_{c=0}^{4}\sum_{b=0}^{\lfloor 4-c\rfloor}\sum_{a=0}^{\lfloor 4-b-c\rfloor}\bar{\omega}_{\rm gw,in}^{(a,b,c)}(q)\,(\mathscr{T}_1 + \mathscr{T}_2)$$

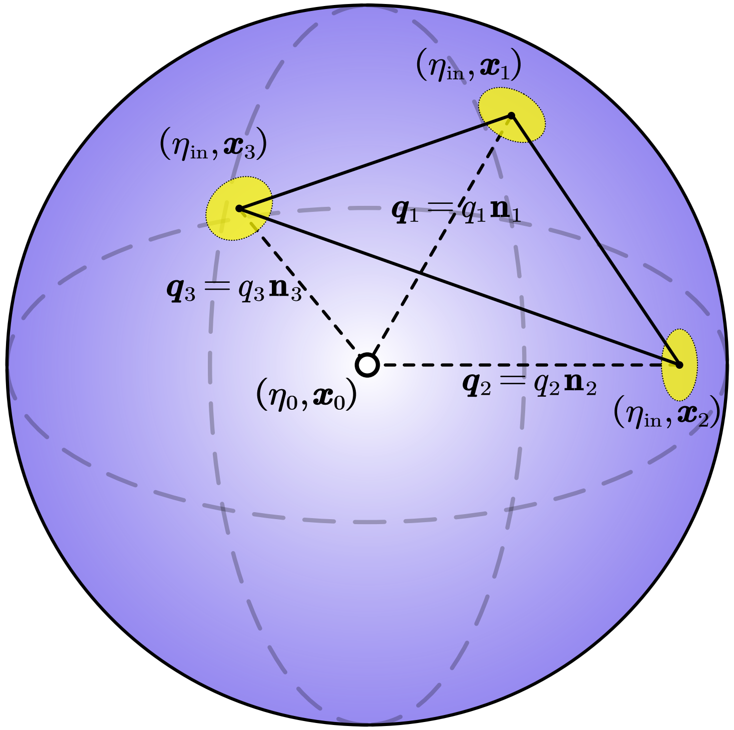
$$\mathscr{T}_1 = \sum_{l=1}^{2}\sum_{n=1}^{2}\sum_{i=1}^{o-1}\left(\frac{(i+l)!}{i!}\frac{V_1^{[i+1]}}{V_0^{[i]}}\right)^n\binom{N_i}{n}\delta_{ln,2}$$

$$\mathscr{T}_2 = \sum_{i=1}^{3}\sum_{j=1}^{\min(i-1,o-1)}\sum_{i=1}^{o-1}\frac{N_iN_j(i+1)(j+1)V_1^{[i+1]}V_1^{[j+1]}}{V_0^{[i]}V_0^{[j]}}$$

$$\bar{\omega}_{\rm gw,in}^{(a,b,c)} \propto F_{\rm NL}^{a}G_{\rm NL}^{b}H_{\rm NL}^{c}A^{(a+2b+3c)/2}$$

$$N_1 = 4 - a - b - c$$

$$N_2 = a \quad N_3 = b \quad N_4 = c$$



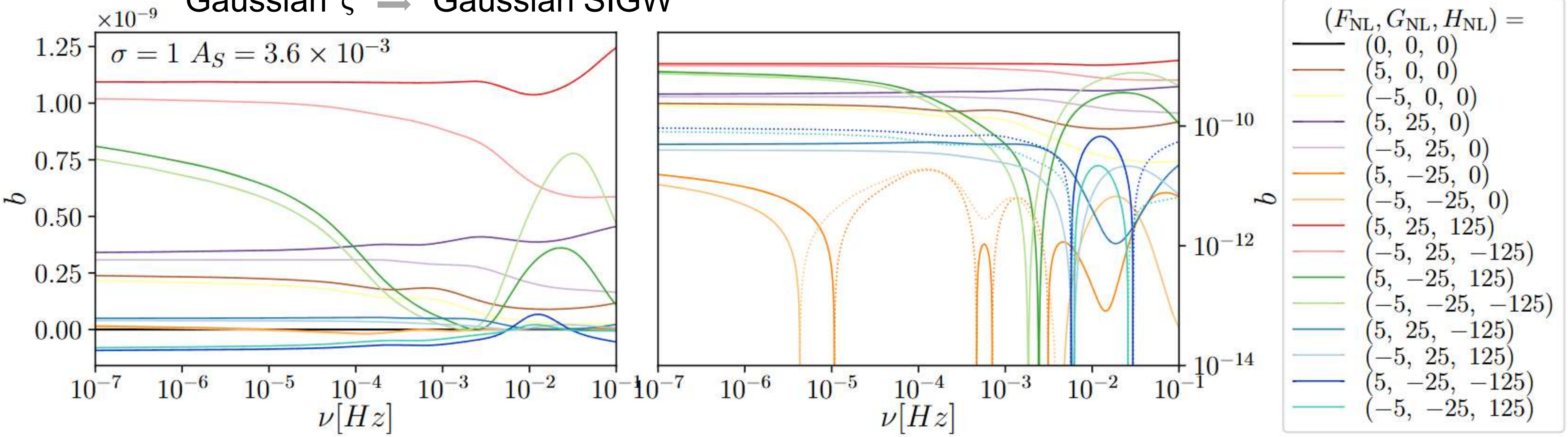
Angular Bispectrum

$$B_{\ell_1\ell_2\ell_3}(\nu) = b(\nu) h_{\ell_1\ell_2\ell_3}\left[\frac{1}{\ell_1\ell_2(\ell_1+1)(\ell_2+1)} + (\ell_1 \leftrightarrow \ell_3) + (\ell_2 \leftrightarrow \ell_3)\right]$$

$$b(\nu) = 2(2\pi A_{\rm L})^2\left[\frac{\omega_{\rm ng}^{(1)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}(6 - n_{\rm gw}(\nu))\right]^2\left[\frac{\omega_{\rm ng}^{(2)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}F_{\rm NL}(6 - n_{\rm gw}(\nu))\right]$$

PNG-induced Inhomogeneity   SW effect & IIC

Gaussian ζ ➡ Gaussian SIGW



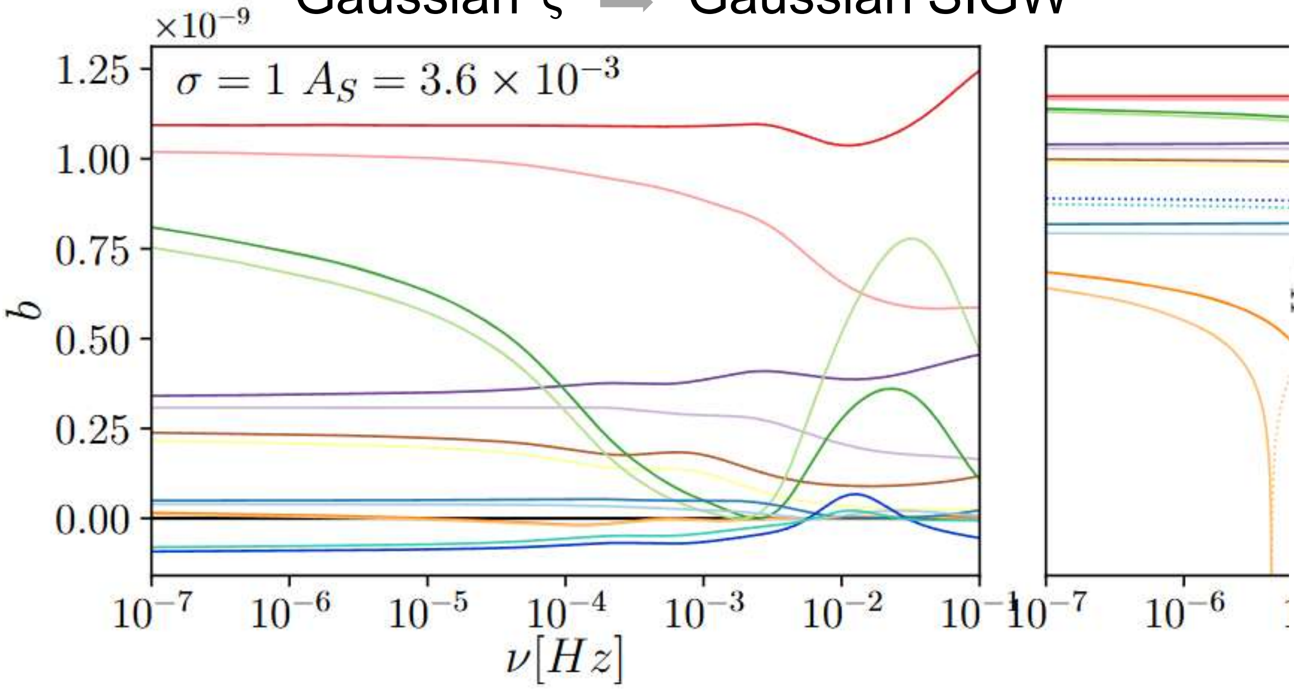
Angular Bispectrum

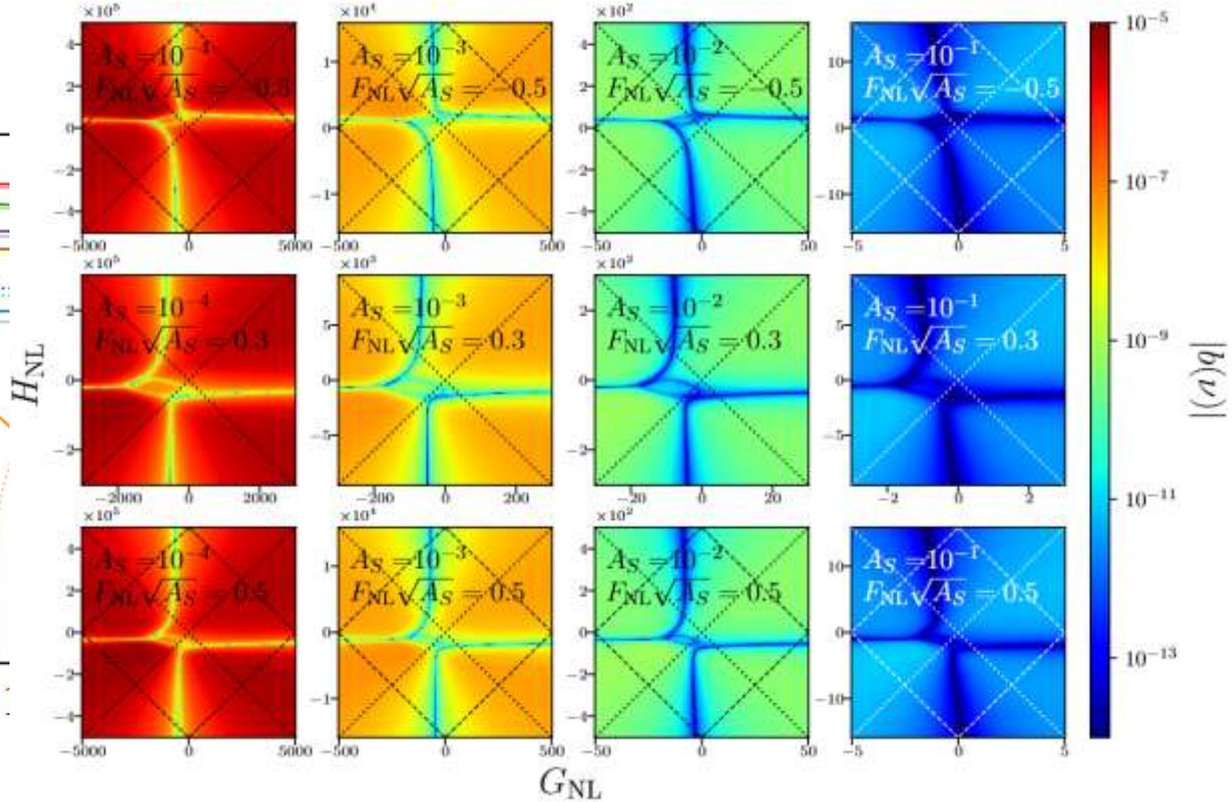
$$B_{\ell_1\ell_2\ell_3}(\nu) = b(\nu)\, h_{\ell_1\ell_2\ell_3}\left[\frac{1}{\ell_1\ell_2(\ell_1+1)(\ell_2+1)} + (\ell_1 \leftrightarrow \ell_3) + (\ell_2 \leftrightarrow \ell_3)\right]$$

$$b(\nu) = 2(2\pi A_{\rm L})^2\left[\frac{\omega_{\rm ng}^{(1)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}(6-n_{\rm gw}(\nu))\right]^2\left[\frac{\omega_{\rm ng}^{(2)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}F_{\rm NL}(6-n_{\rm gw}(\nu))\right]$$

PNG-induced Inhomogeneity SW effect & IIC

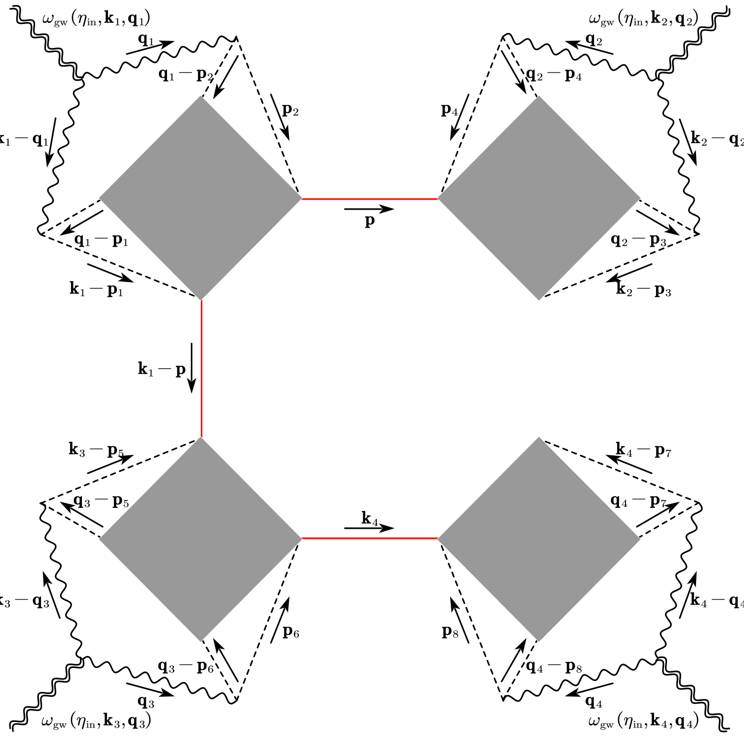
Gaussian ζ ➡ Gaussian SIGW

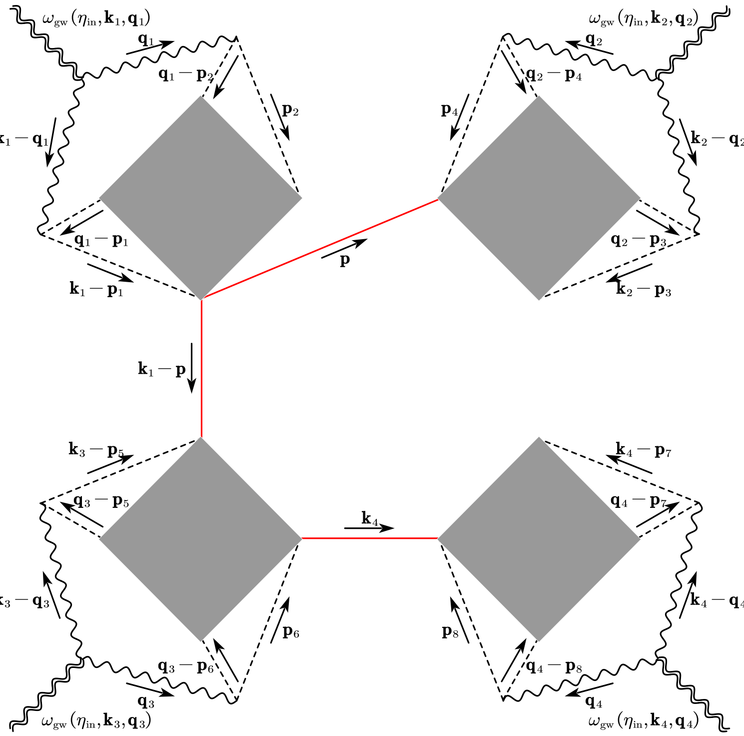




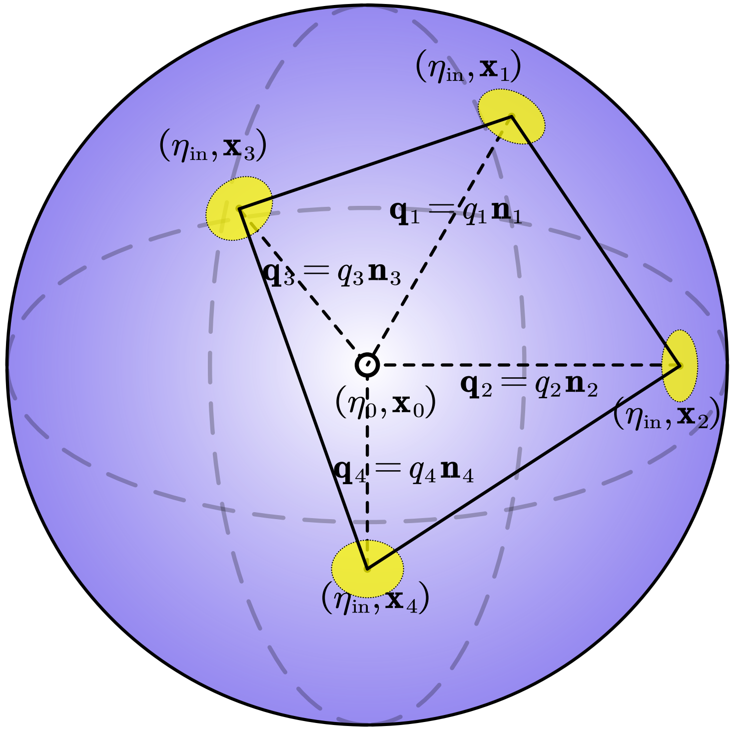
Four-point Angular Correlator

JL, S. Wang, Z.-C. Zho, and K. Kohri, JCAP 05 (2024), 109





$$\left\langle \prod_{i=1}^{4} \omega_{\mathrm{gw}}\left(\eta_{\mathrm{in}}, \mathbf{k}_i, q\right) \right\rangle$$

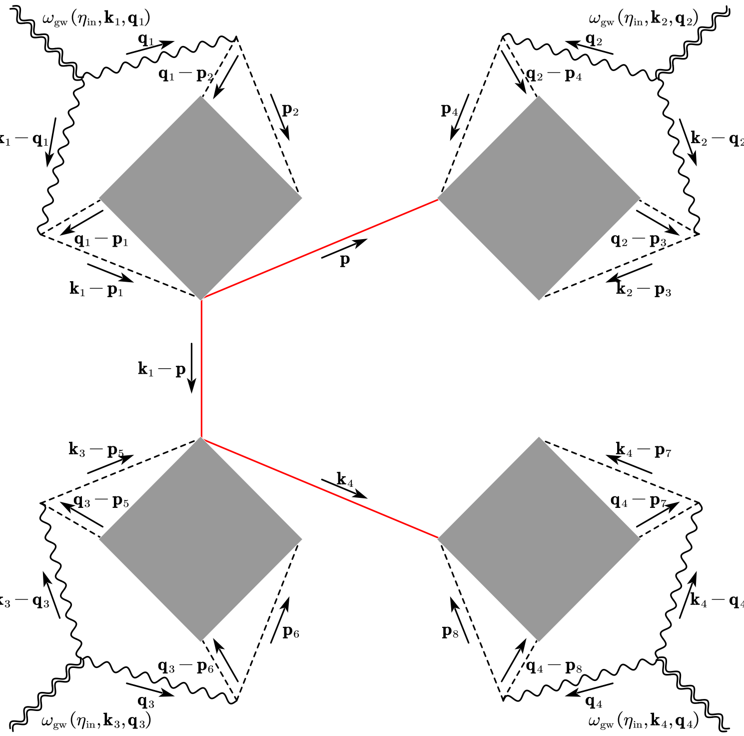


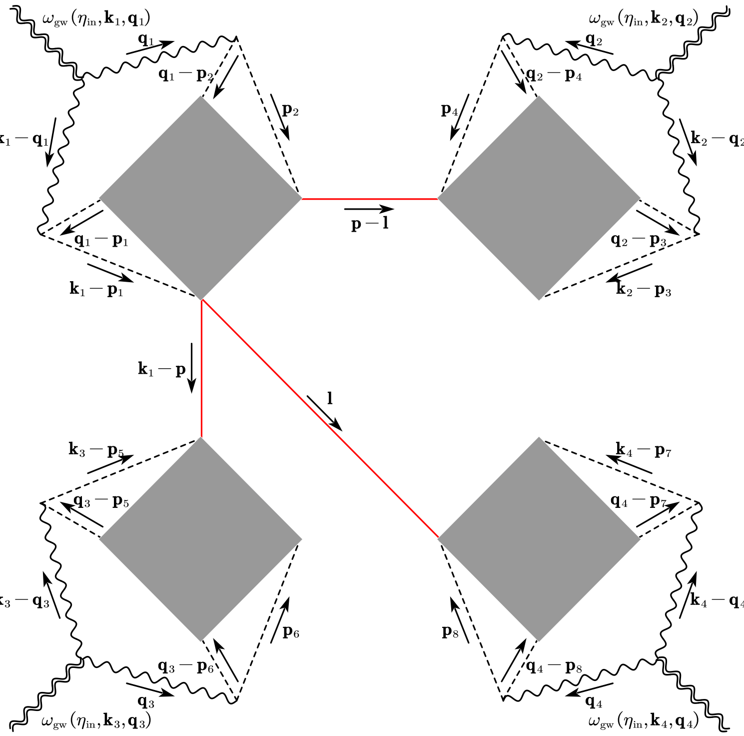
$$\omega_{\mathrm{gw,in}}(q) = \bar{\omega}_{\mathrm{gw,in}}(q) + \omega^{(1)}_{\mathrm{gw,in}}(q) + \omega^{(2)}_{\mathrm{gw,in}}(q)$$

$$\omega^{(2)}_{\mathrm{gw,in}}(q) = \omega^{(2)}_{\mathrm{ng}}(q) \int \frac{\mathrm{d}^3\mathbf{k}\,\mathrm{d}^3\mathbf{p}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}_{\mathrm{in}}} \zeta_{\mathrm{gL}}(\mathbf{p}) \zeta_{\mathrm{gL}}(\mathbf{k}-\mathbf{p})$$

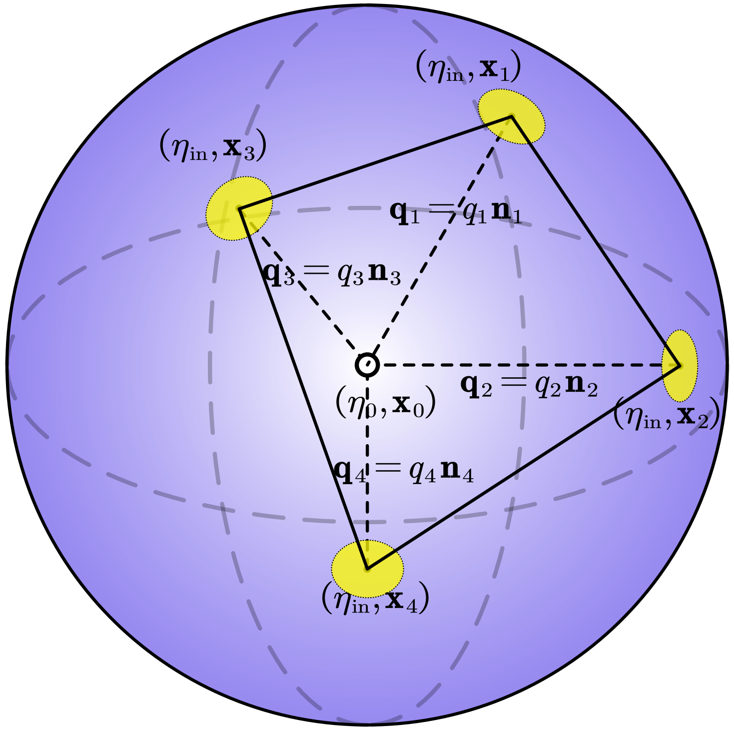
Four-point Angular Correlator

JL, S. Wang, Z.-C. Zho, and K. Kohri, JCAP 05 (2024), 109





$$\left\langle \prod_{i=1}^{4} \omega_{\rm gw}(\eta_{\rm in}, \mathbf{k}_i, q) \right\rangle$$



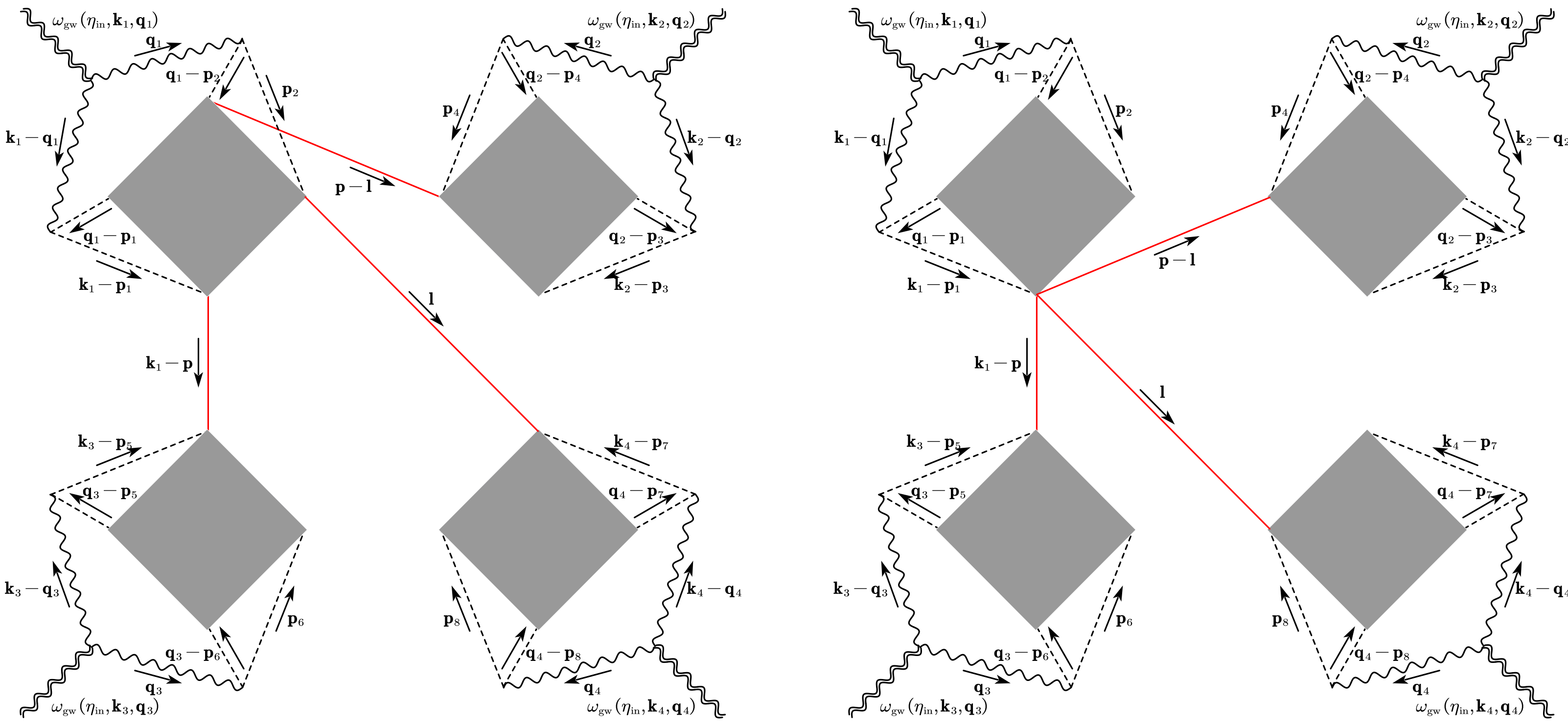
$$\omega_{\rm gw,in}(q) = \bar{\omega}_{\rm gw,in}(q) + \omega^{(1)}_{\rm gw,in}(q) + \omega^{(2)}_{\rm gw,in}(q) + \omega^{(3)}_{\rm gw,in}(q)$$

$$\omega^{(2)}_{\rm gw,in}(q) = \omega^{(2)}_{\rm ng}(q) \int \frac{d^3\mathbf{k}\, d^3\mathbf{p}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}_{\rm in}} \zeta_{\rm gL}(\mathbf{p})\zeta_{\rm gL}(\mathbf{k}-\mathbf{p})$$

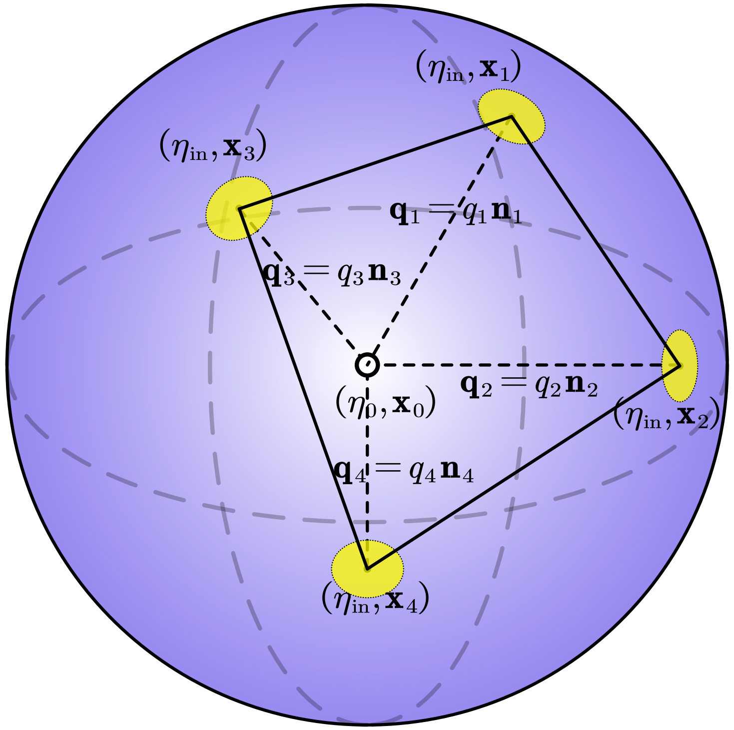
$$\omega^{(3)}_{\rm gw,in}(q) = \omega^{(3)}_{\rm ng}(q) \int \frac{d^3\mathbf{k}\, d^3\mathbf{p}\, d^3\mathbf{l}}{(2\pi)^{9/2}} e^{i\mathbf{k}\cdot\mathbf{x}_{\rm in}} \zeta_{\rm gL}(\mathbf{l})\zeta_{\rm gL}(\mathbf{p}-\mathbf{l})\zeta_{\rm gL}(\mathbf{k}-\mathbf{p})$$

Four-point Angular Correlator

JL, S. Wang, Z.-C. Zho, and K. Kohri, JCAP 05 (2024), 109



$$\left\langle \prod_{i=1}^{4} \omega_{\rm gw}(\eta_{\rm in}, \mathbf{k}_i, q) \right\rangle$$



$$\omega_{\rm gw,in}(q) = \bar{\omega}_{\rm gw,in}(q) + \omega^{(1)}_{\rm gw,in}(q) + \omega^{(2)}_{\rm gw,in}(q) + \omega^{(3)}_{\rm gw,in}(q)$$

$$\omega^{(2)}_{\rm gw,in}(q) = \omega^{(2)}_{\rm ng}(q) \int \frac{{\rm d}^3\mathbf{k}\, {\rm d}^3\mathbf{p}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}_{\rm in}} \zeta_{\rm gL}(\mathbf{p})\, \zeta_{\rm gL}(\mathbf{k}-\mathbf{p})$$

$$\omega^{(3)}_{\rm gw,in}(q) = \omega^{(3)}_{\rm ng}(q) \int \frac{{\rm d}^3\mathbf{k}\, {\rm d}^3\mathbf{p}\, {\rm d}^3\mathbf{l}}{(2\pi)^{9/2}} e^{i\mathbf{k}\cdot\mathbf{x}_{\rm in}} \zeta_{\rm gL}(\mathbf{l})\, \zeta_{\rm gL}(\mathbf{p}-\mathbf{l})\, \zeta_{\rm gL}(\mathbf{k}-\mathbf{p})$$

Angular Trispectrum

$$t^{\ell_1\ell_2}_{\ell_3\ell_4}(L,\nu)=\left[\frac{t_1(\nu)}{L(L+1)}+\frac{t_2(\nu)}{\ell_1(\ell_1+1)}+\frac{t_2(\nu)}{\ell_3(\ell_3+1)}\right]\frac{h_{\ell_1\ell_2L}}{\ell_2(\ell_2+1)}\frac{h_{\ell_3\ell_4L}}{\ell_4(\ell_4+1)}$$

$$t_1(\nu)=4(2\pi A_{\rm L})^3\left[\frac{\omega^{(1)}_{\rm ng}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)}+\frac{3}{5}(6-n_{\rm gw}(\nu))\right]^2\left[\frac{\omega^{(2)}_{\rm ng}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)}+\frac{3}{5}F_{\rm NL}(6-n_{\rm gw}(\nu))\right]^2$$

$$t_2(\nu)=\frac{(2\pi A_{\rm L})^3}{2\pi}\left[\frac{\omega^{(1)}_{\rm ng}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)}+\frac{3}{5}(6-n_{\rm gw}(\nu))\right]^3\left[\frac{\omega^{(3)}_{\rm ng}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)}+\frac{3}{5}G_{\rm NL}(6-n_{\rm gw}(\nu))\right]$$

PNG-induced Inhomogeneity SW effect & IIC

$$t_1(\nu)=\frac{b^2(\nu)}{\ell(\ell+1)\tilde{C}_\ell(\nu)}$$

Different multipole combinations ➞ $t_1(\nu)$ and $t_1(\nu)$ are distinguishable

Angular Trispectrum

$$t_2(\nu) = \frac{(2\pi A_{\rm L})^3}{2\pi}\left[\frac{\omega_{\rm ng}^{(1)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}(6 - n_{\rm gw}(\nu))\right]^3 \left[\frac{\omega_{\rm ng}^{(3)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}G_{\rm NL}(6 - n_{\rm gw}(\nu))\right]$$

$$\omega_{\rm ng}^{(2)}(q) = \sum_{c=0}^{4}\sum_{b=0}^{\lfloor 4-c\rfloor}\sum_{a=0}^{\lfloor 4-b-c\rfloor}\bar{\omega}_{\rm gw,in}^{(a,b,c)}(q)\left(\mathscr{T}_1 + \mathscr{T}_2 + \mathscr{T}_3 + \mathscr{T}_4\right)$$

$$\mathscr{T}_1 = \sum_{l=1}^{3}\sum_{n=1}^{3}\sum_{i=1}^{o-1}\left(\frac{(i+l)!}{i!}\frac{V_1^{[i+1]}}{V_0^{[i]}}\right)^n\binom{N_i}{n}\delta_{ln,3}$$

$$\mathscr{T}_2 = \sum_{i=1}^{3}\sum_{j=1}^{\min(i-1,o-1)}\sum_{i=1}^{o-1}N_i(N_i-1)(i+2)(i+1)^2\frac{V_1^{[i+2]}V_1^{[i+1]}}{(V_0^{[i]})^2}$$

$$\mathscr{T}_3 = \sum_{l_1=1}^{3}\sum_{n_1=1}^{3}\sum_{l_2=1}^{3}\sum_{n_2=1}^{o-l_1}\sum_{j=1}^{\min(i-1,o-l_2)}\left(\frac{(i+l_1)!}{i!}\frac{V_1^{[i+l_1]}}{V_0^{[i]}}\right)^{n_1}\binom{N_i}{n_1}\left(\frac{(j+l_2)!}{j!}\frac{V_1^{[i+l_2]}}{V_0^{[j]}}\right)^{n_2}\binom{N_j}{n_2}\delta_{l_1n_1+l_2n_2,3}$$

$$\mathscr{T}_4 = \sum_{i=1}^{3}\sum_{j=1}^{\min(i-1,o-1)}\sum_{k=1}^{\min(j-1,o-1)}N_iN_jN_k(i+1)(j+1)(k+1)\frac{V_1^{[i+1]}V_1^{[j+1]}V_1^{[k+1]}}{V_0^{[i]}V_0^{[j]}V_0^{[k]}}$$

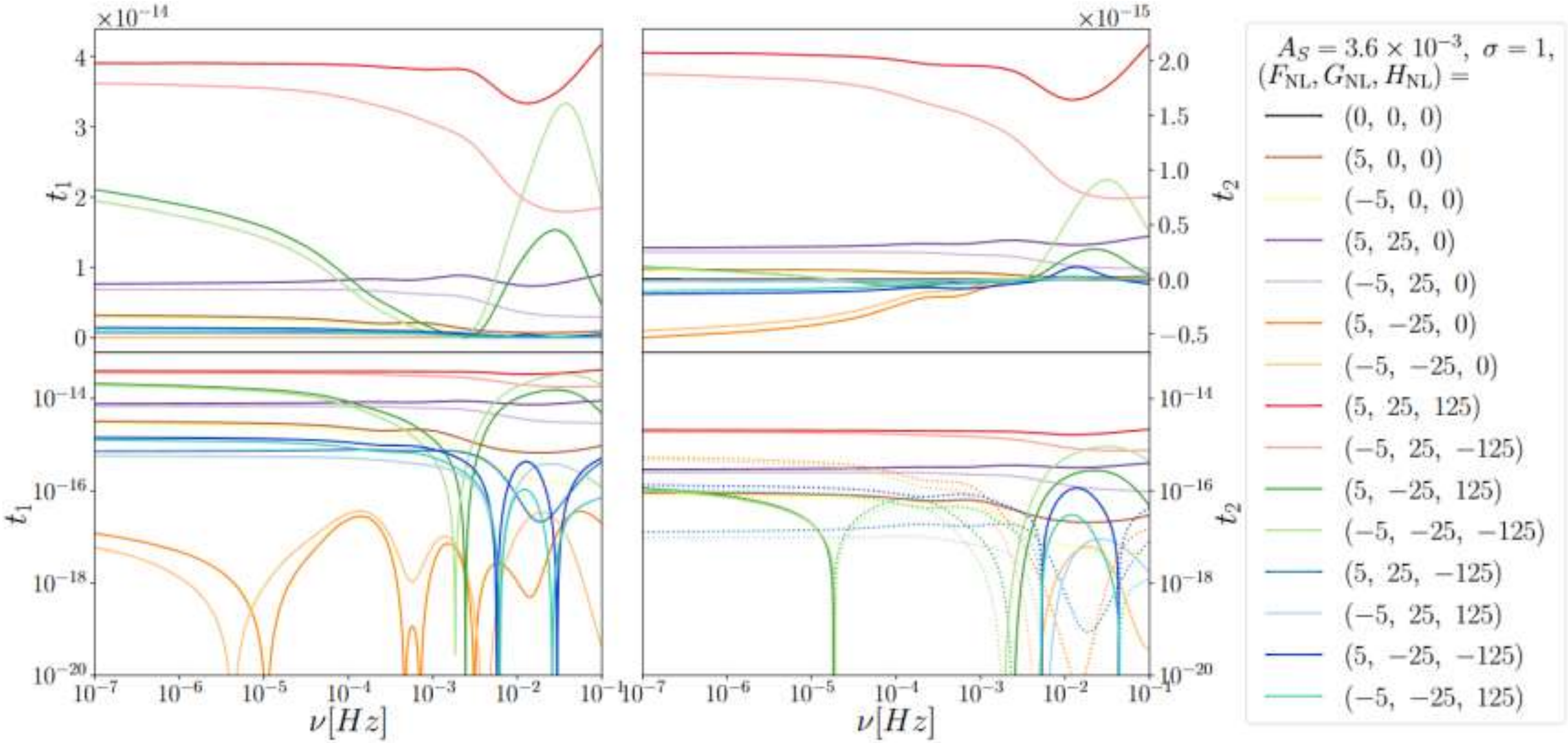
Angular Trispectrum

$$t_1(\nu) = 4(2\pi A_{\rm L})^3 \left[\frac{\omega_{\rm ng}^{(1)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}(6 - n_{\rm gw}(\nu))\right]^2 \left[\frac{\omega_{\rm ng}^{(2)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}F_{\rm NL}(6 - n_{\rm gw}(\nu))\right]^2$$

$$t_2(\nu) = \frac{(2\pi A_{\rm L})^3}{2\pi} \left[\frac{\omega_{\rm ng}^{(1)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}(6 - n_{\rm gw}(\nu))\right]^3 \left[\frac{\omega_{\rm ng}^{(3)}(2\pi\nu)}{\bar{\omega}_{\rm gw,in}(2\pi\nu)} + \frac{3}{5}G_{\rm NL}(6 - n_{\rm gw}(\nu))\right]$$

PNG-induced Inhomogeneity

SW effect & IIC



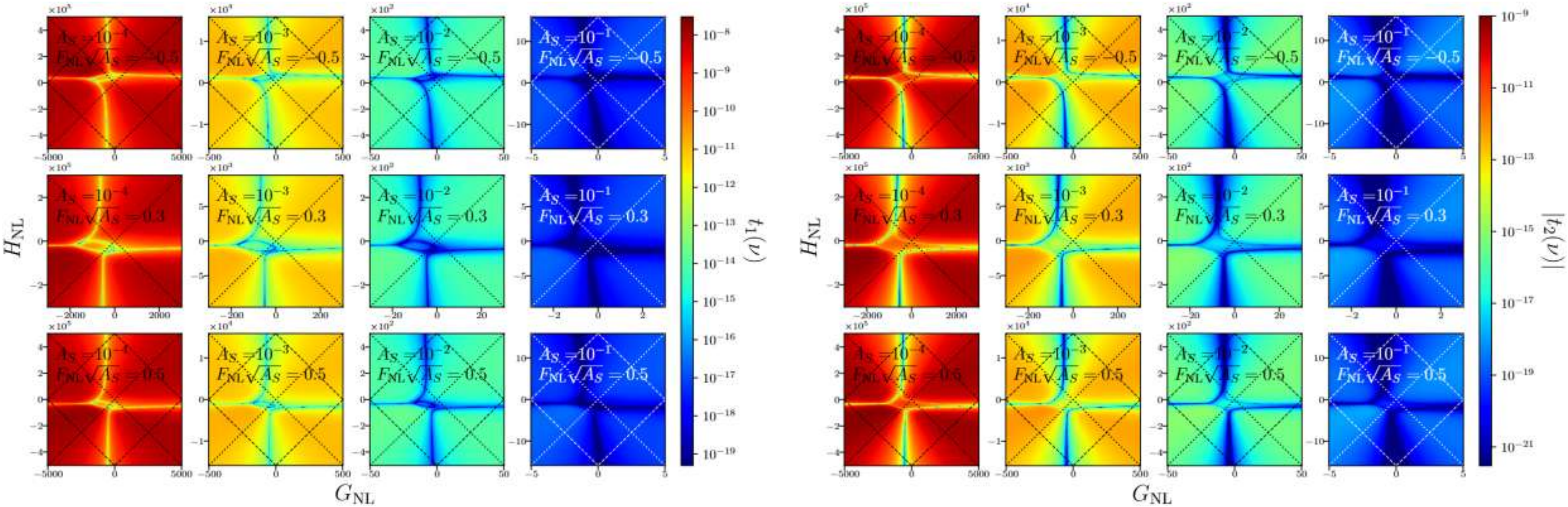
Angular Trispectrum

$$t_1(\nu) = 4(2\pi A_{\mathrm{L}})^3 \left[\frac{\omega_{\mathrm{ng}}^{(1)}(2\pi\nu)}{\bar{\omega}_{\mathrm{gw,in}}(2\pi\nu)} + \frac{3}{5}(6 - n_{\mathrm{gw}}(\nu)) \right]^2 \left[\frac{\omega_{\mathrm{ng}}^{(2)}(2\pi\nu)}{\bar{\omega}_{\mathrm{gw,in}}(2\pi\nu)} + \frac{3}{5} F_{\mathrm{NL}} (6 - n_{\mathrm{gw}}(\nu)) \right]^2$$

$$t_2(\nu) = \frac{(2\pi A_{\mathrm{L}})^3}{2\pi} \left[\frac{\omega_{\mathrm{ng}}^{(1)}(2\pi\nu)}{\bar{\omega}_{\mathrm{gw,in}}(2\pi\nu)} + \frac{3}{5}(6 - n_{\mathrm{gw}}(\nu)) \right]^3 \left[\frac{\omega_{\mathrm{ng}}^{(3)}(2\pi\nu)}{\bar{\omega}_{\mathrm{gw,in}}(2\pi\nu)} + \frac{3}{5} G_{\mathrm{NL}} (6 - n_{\mathrm{gw}}(\nu)) \right]$$

PNG-induced Inhomogeneity

SW effect & IIC



Summary

Summary

Achievements & Key Results

- Developed a framework to compute the energy-density fraction spectrum, angular power spectrum, bispectrum, and trispectrum of SIGWs for primordial non-Gaussianity up to arbitrary order.
- SIGW energy density is sensitive to the primordial trispectrum in the ultraviolet regime;
- Local-type primordial non-Gaussianity can induce both large anisotropies and non-Gaussianity on the SIGW background;
- …

Scientific Implications & Future Prospects

- A supplementary probe for physics in the early universe that surpasses the reach of measurements from CMB and LSS;
- A promising tool for the search for PBHs;
- Extraction of SIGW signals from the stochastic backgrounds and inference of model parameters;
- …

Thanks!