Memory and soft modes in GW signals

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[De Luca, Khoury & **Wong**, 2412.01910]
[De Luca, Khoury & **Wong**, 2412.12273]
& work in progress

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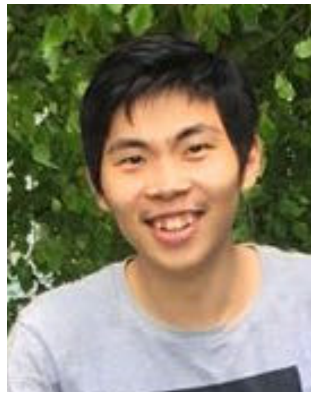

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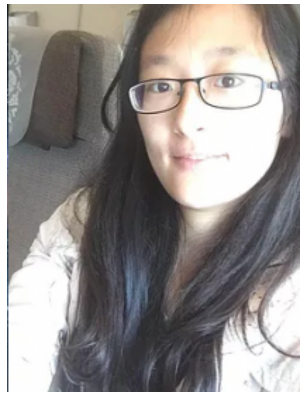

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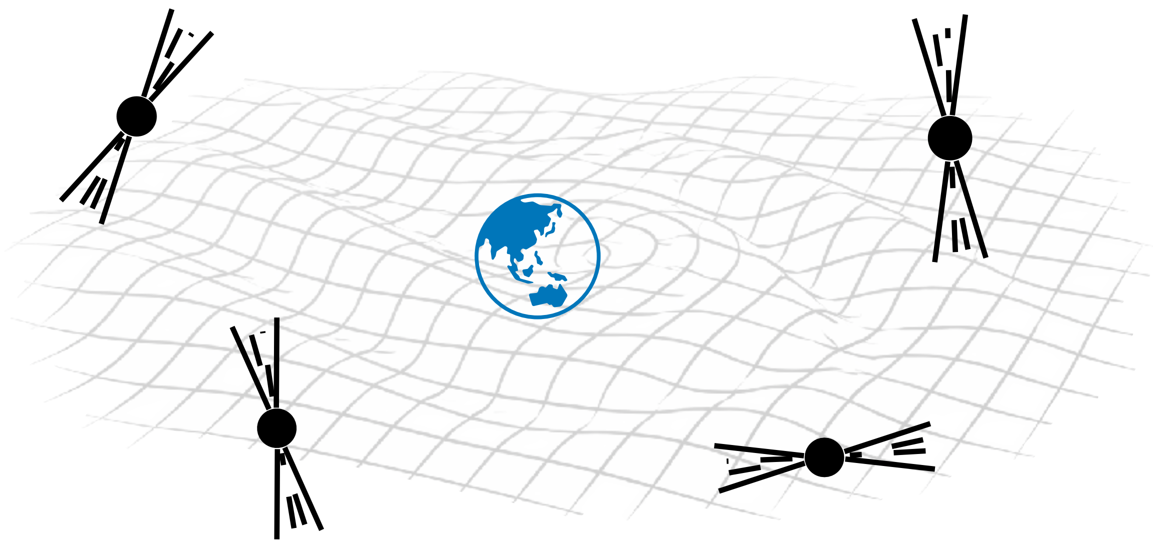
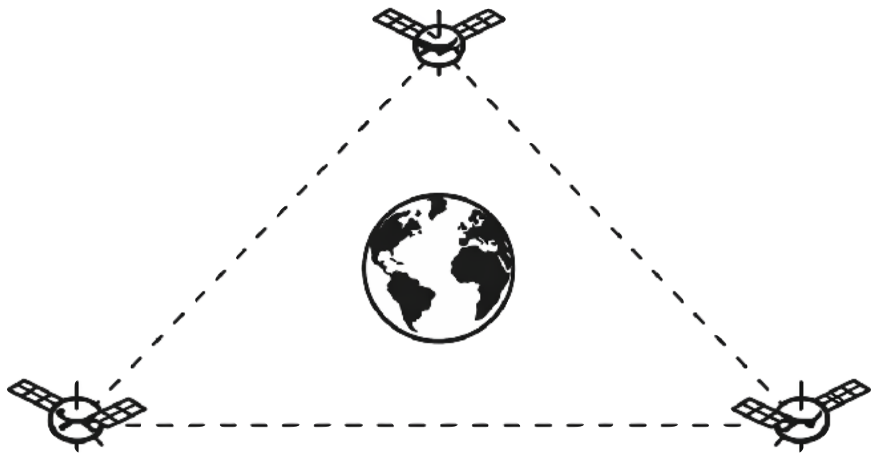
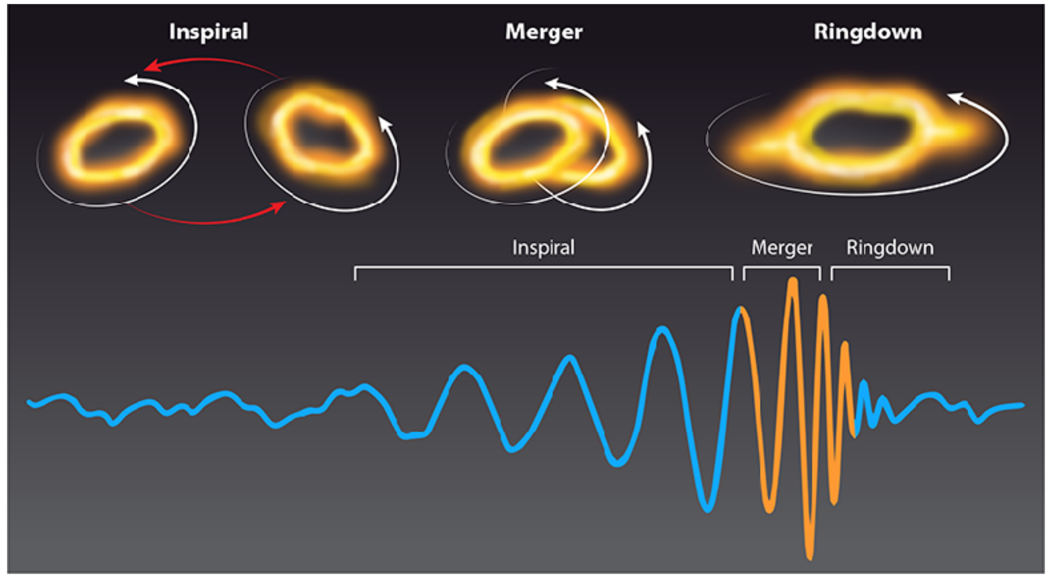
[Liang, Lin, Trodden & **Wong**, 2309.16666]





Two types of GW signals

Single event signal

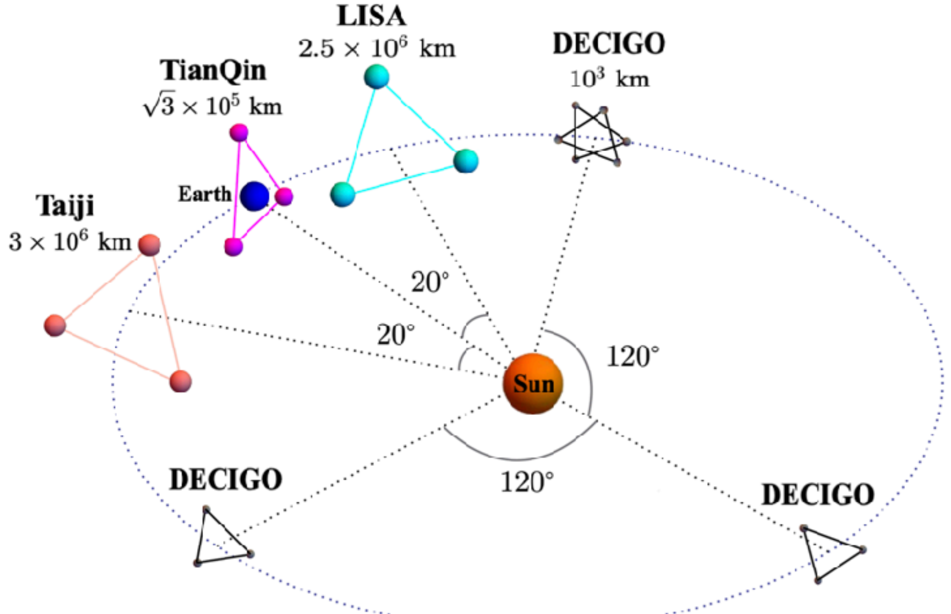


Credit: https://physics.aps.org/articles/v16/29

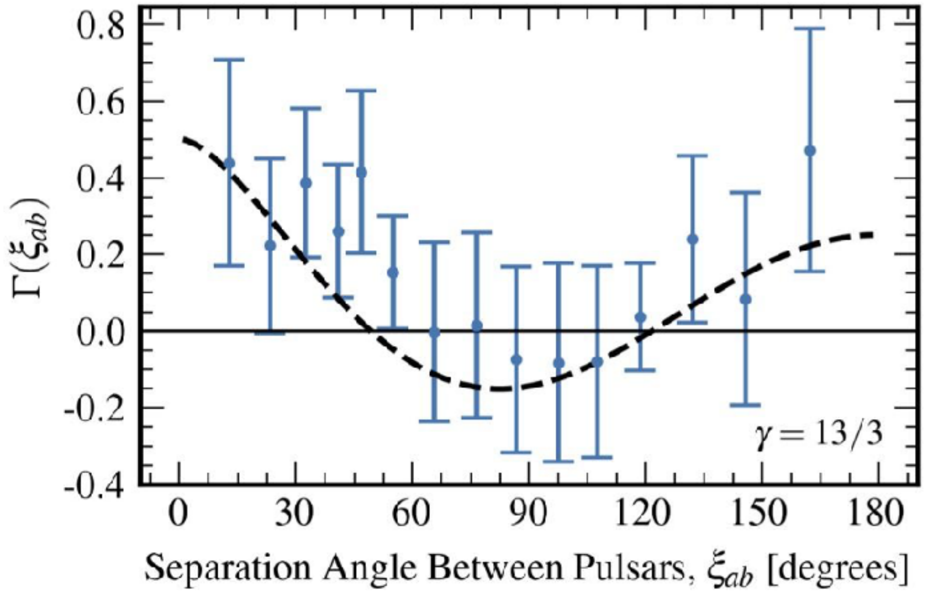
LIGO-VIRGO-KAGRA

Future: (Ground based) Cosmic Explorer, Einstein telescope

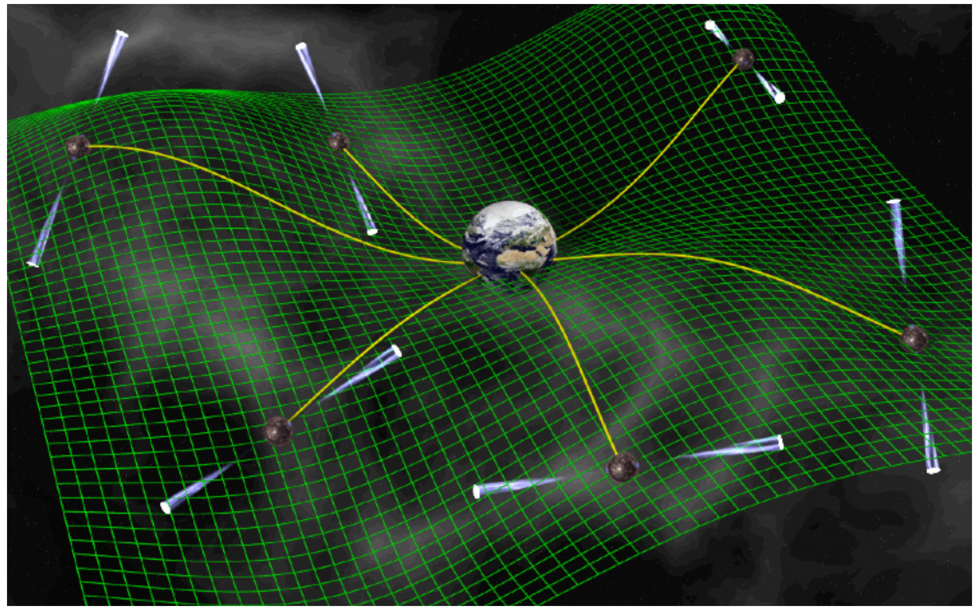
(Space based) LISA, Taiji/Tianqin, DECIGO



Stochastic



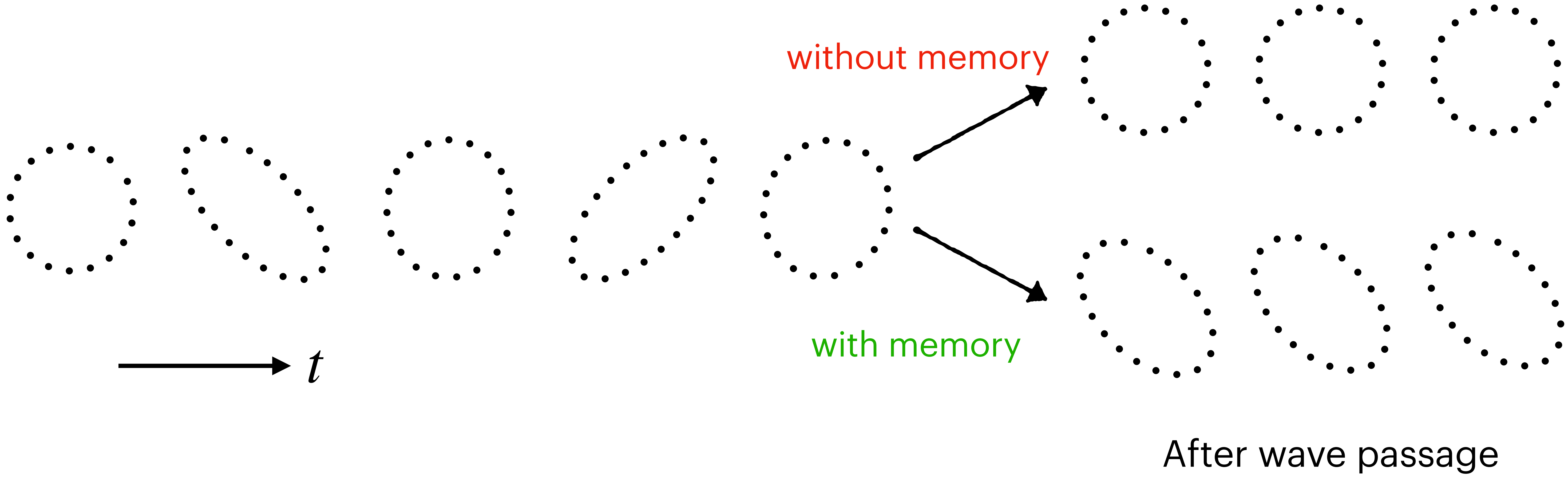
[NANOGrav 15-yr, 2306.16213]



Gravitational memory

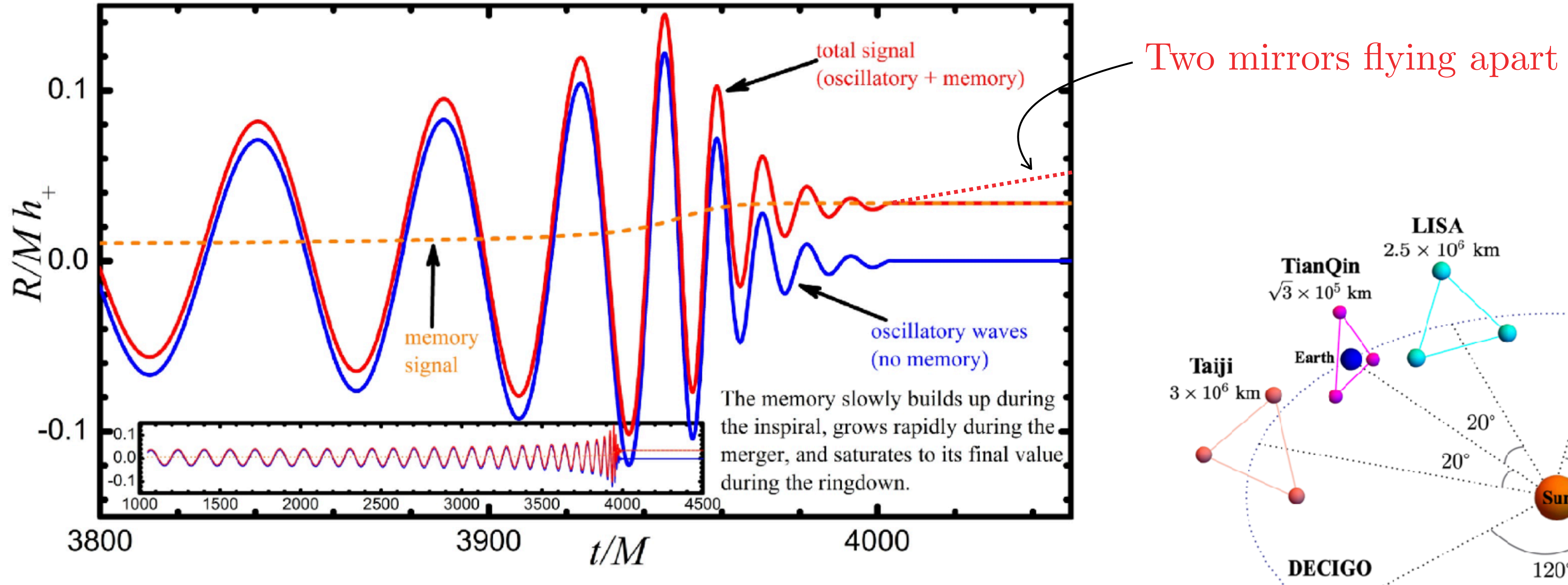
Gravitational meomry (soft modes in single event signal)

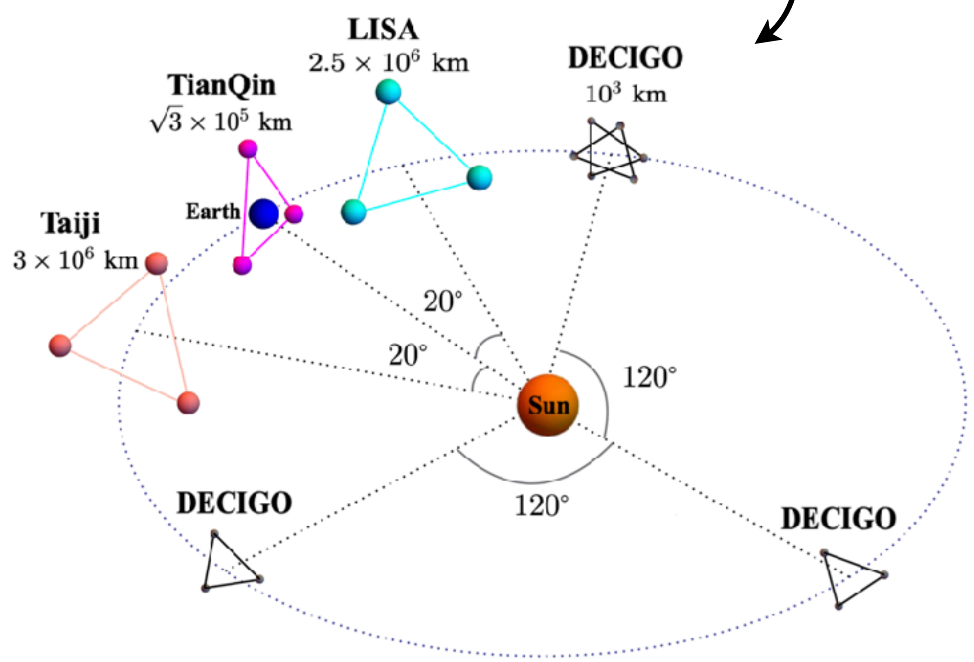
- Permanent change of spacetime after wave trains have passed through
- Linear memory (caused by explicit matter source), nonlinear memory (gravity itself)
- For a pair of inertial detectors, the relative distance, relative velocity can be altered permanently
- Inertial detectors: Lisa, Tianqin/Taiji, DECIGO

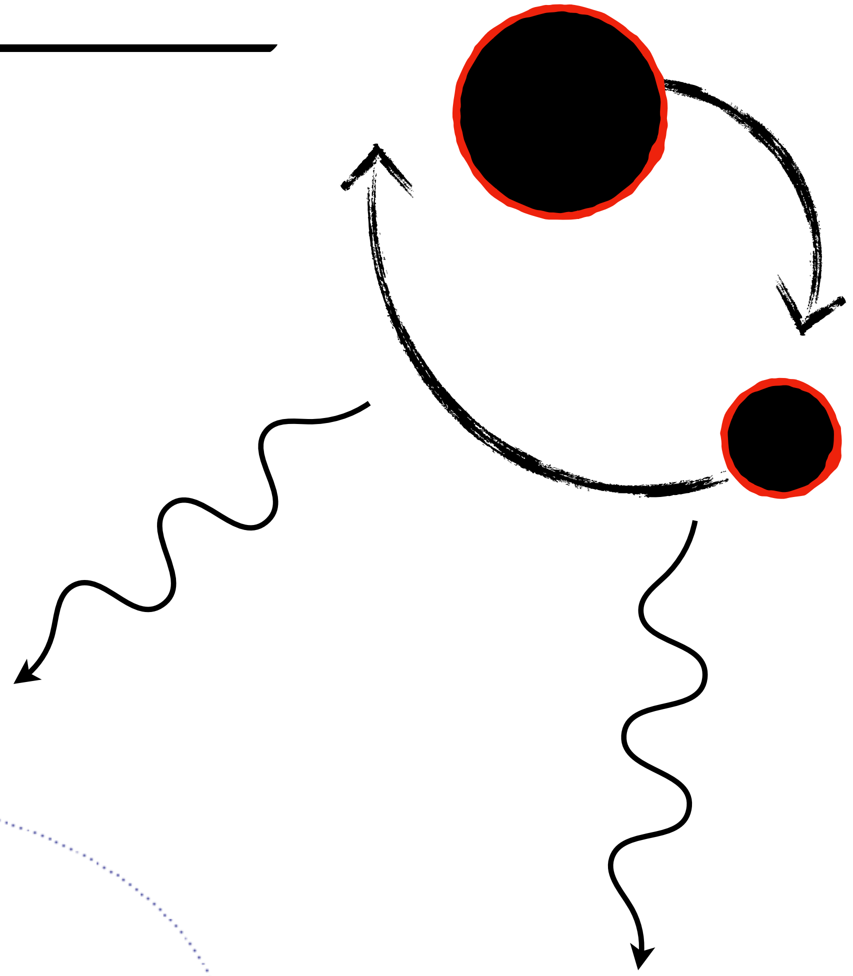


Gravitational meomry

- Examples of sources: black hole merger





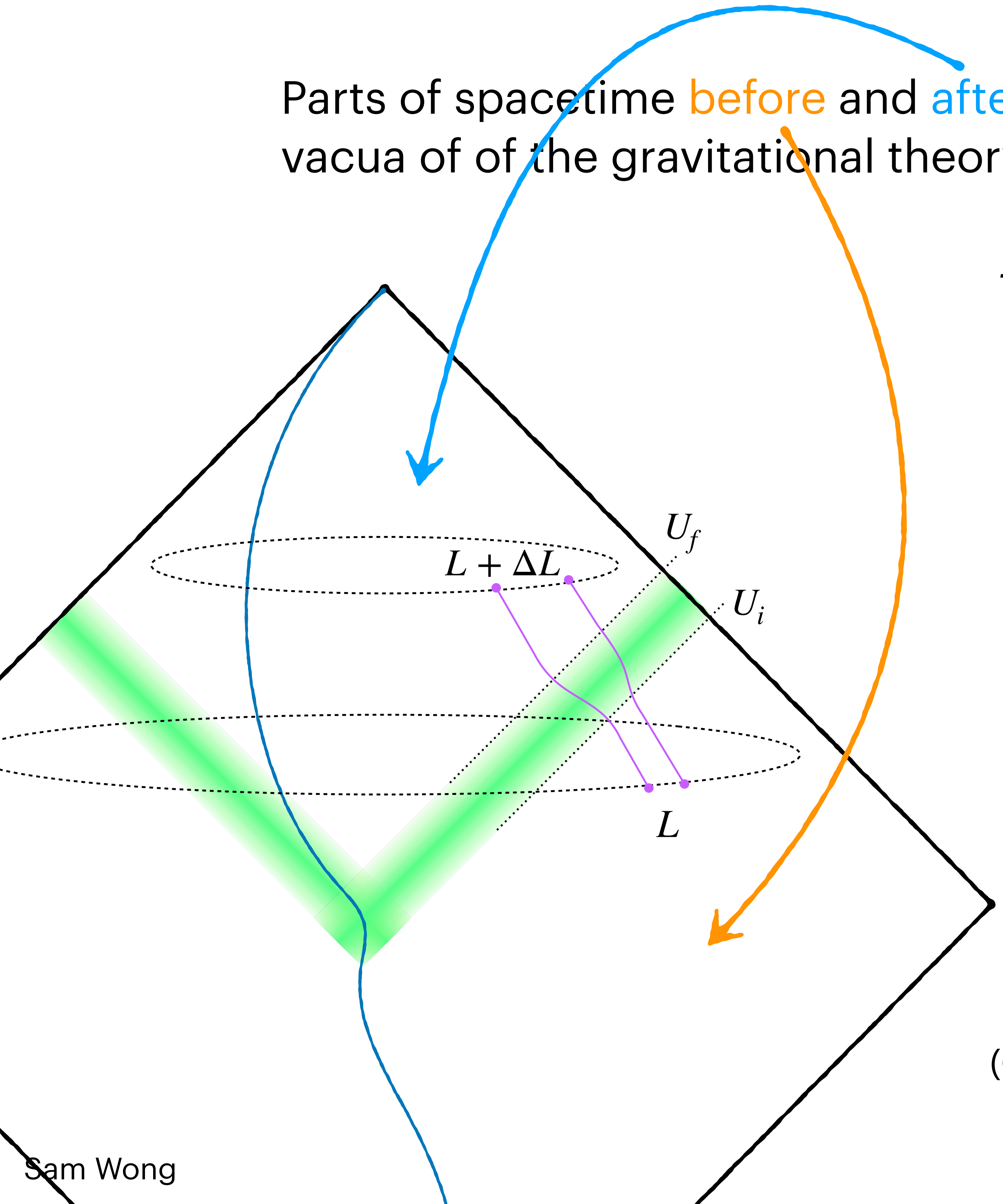


- Other known examples in simulations:

  Supernova simulations and GRBs (due to ejection of mass, linear memory)

Soft modes and meomry

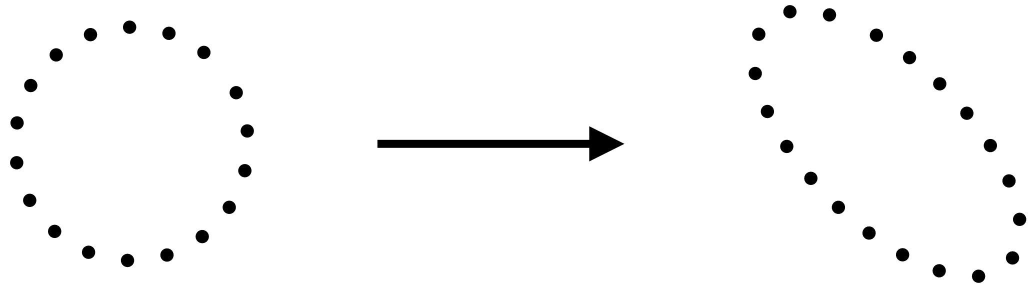
Parts of spacetime before and after the passage of wave are described by different vacua of of the gravitational theory.



These vacua are related to each other by diffs.

For instance: boosted black hole

Can be thought as “Rescaling by long wavelength GWs”



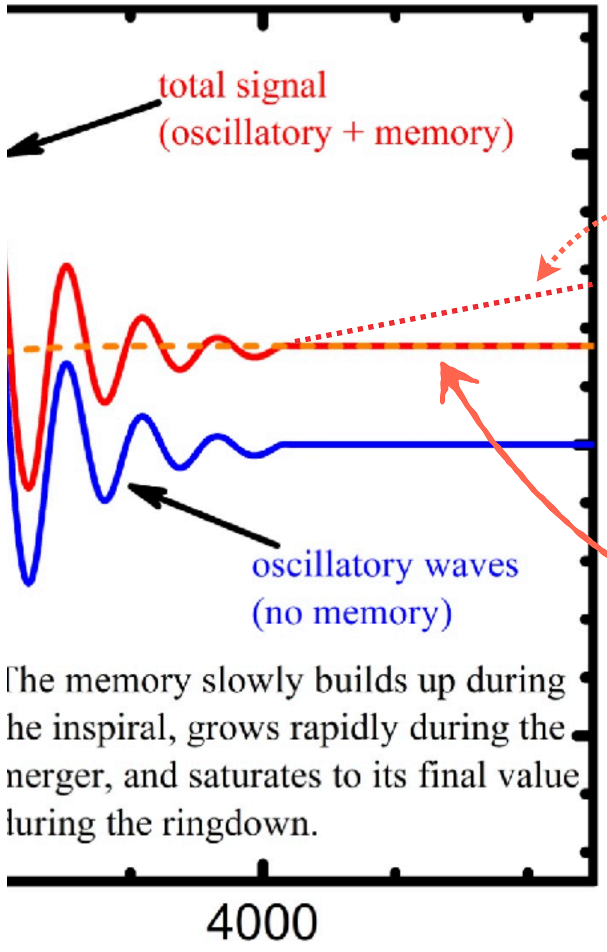
Memories can be chracterized by symmetries!

(Caution: the part of the spacetime with the wave train cannot be described by diff)

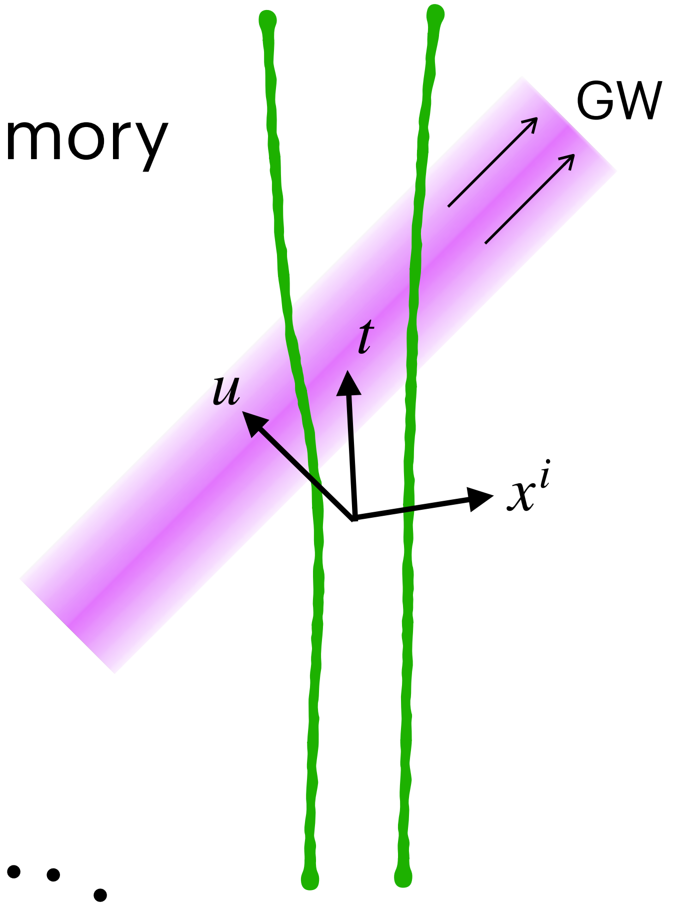
Soft modes and meomry

Local TT frame residual diffeomorphism $x^i \to x^i + \xi^i(x)$

The multipoles exactly describe the *long wavelength modes* of those memory terms:



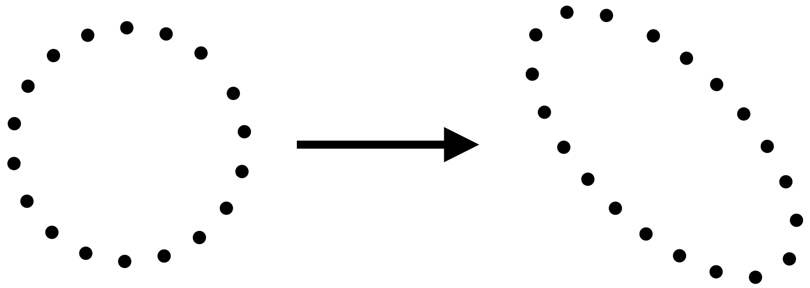
$$\xi_i = -\left(A_{ij} + B_{ij}u\right)x^j - \frac{1}{2}B_{jk}\bar{N}_i x^j x^k + \ldots$$



Time dependent
(mirrors flying apart)

$$H_{ij}^{\mathrm{TT}}(U, N) = A_{ij}(\bar{N}) + B_{ij}(\bar{N})u + \ldots$$

time independent part
(constant displacement)



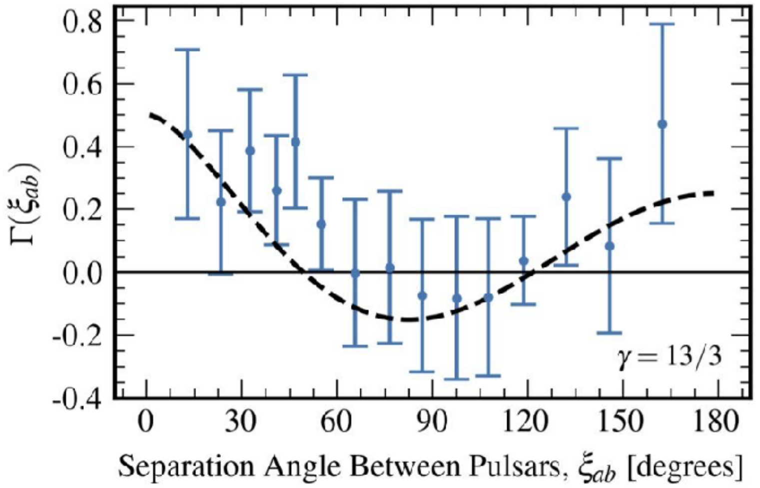
Anisotropic rescaling

(The higher multipoles characterizes all possible memories)

[De Luca, Khoury & **Wong**, 2412.01910]
[De Luca, Khoury & **Wong**, 2412.12273]

Stochastic signals

SGWB (PTA & Astrometry)



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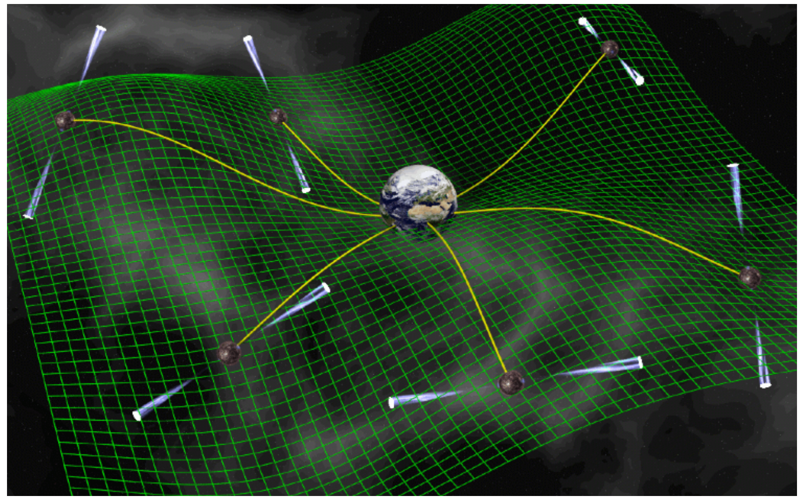


Photon geodesics:

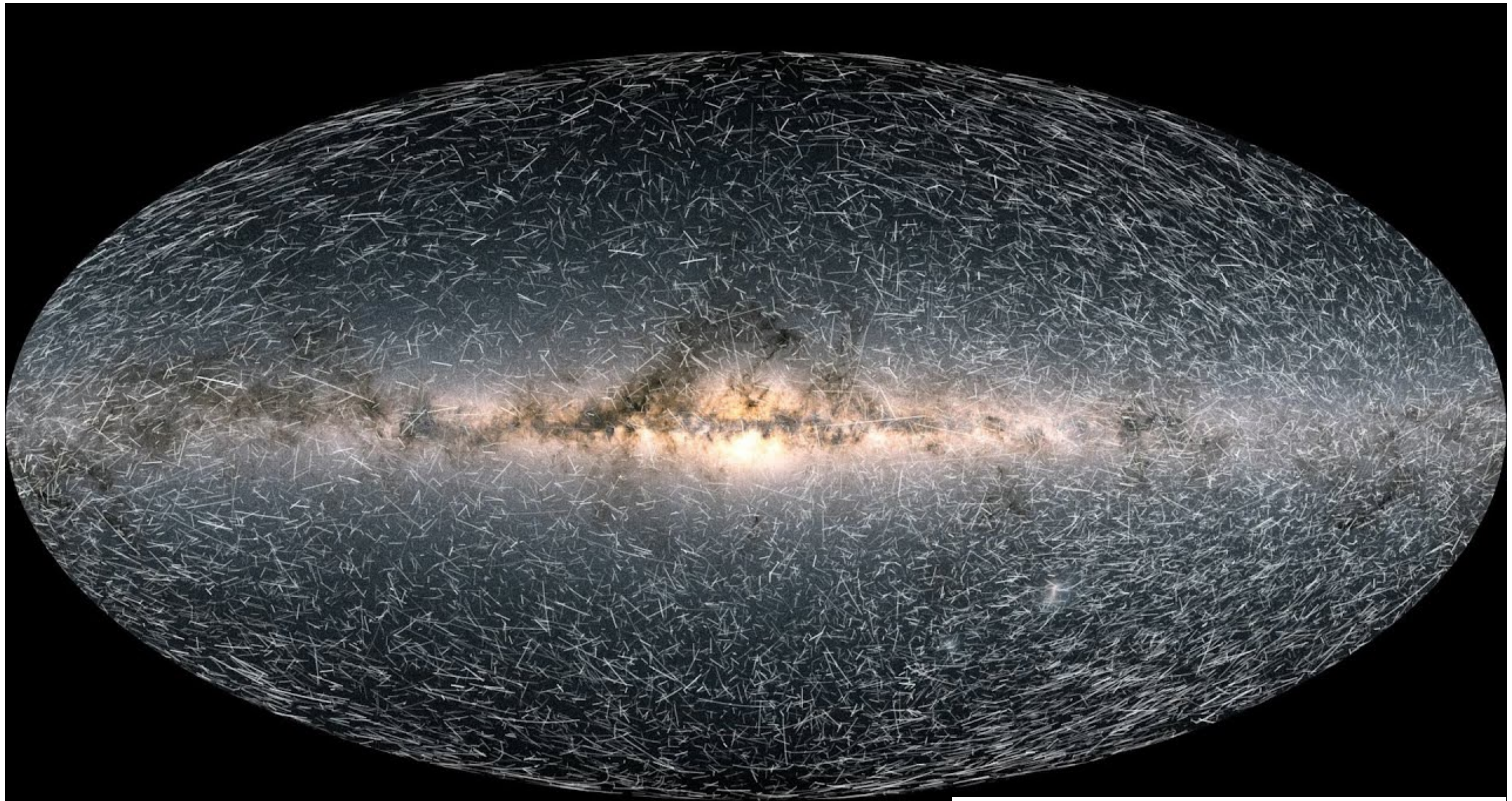
$$x^\mu(\lambda) = \Big(x^0(\lambda),\ x^i(\lambda)\Big)$$

PTA

Astrometry

Defecltion of star positions

GAIA: billions of stars, 10^6 quasars (for SGWB), ~10-100μas

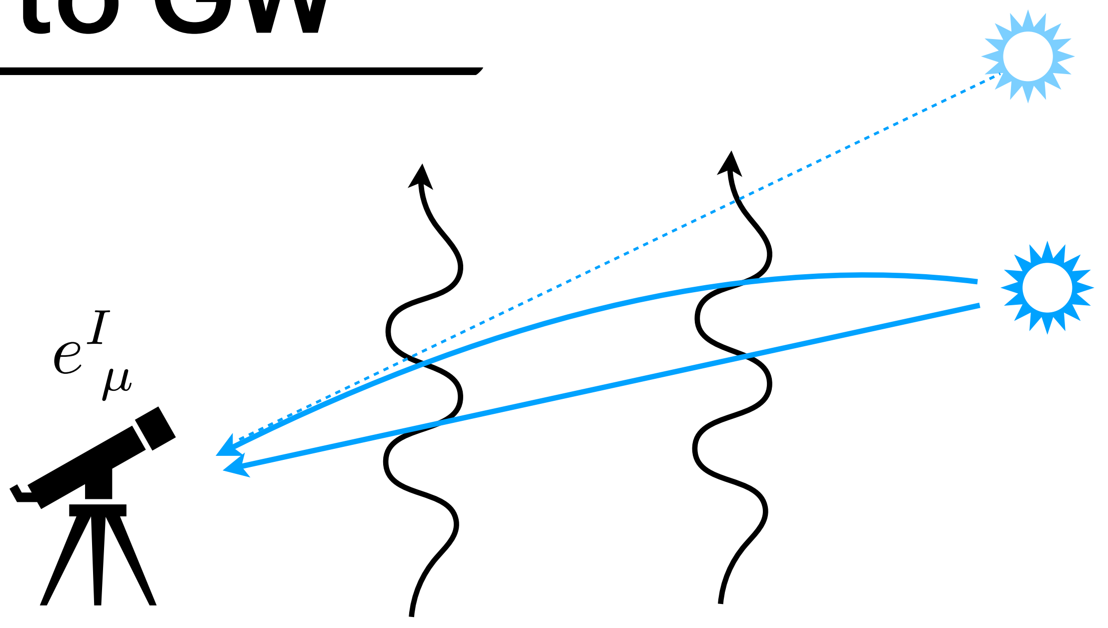

(Image credit: ESA/Gaia/DPAC)

Deflection of star-lights due to GW

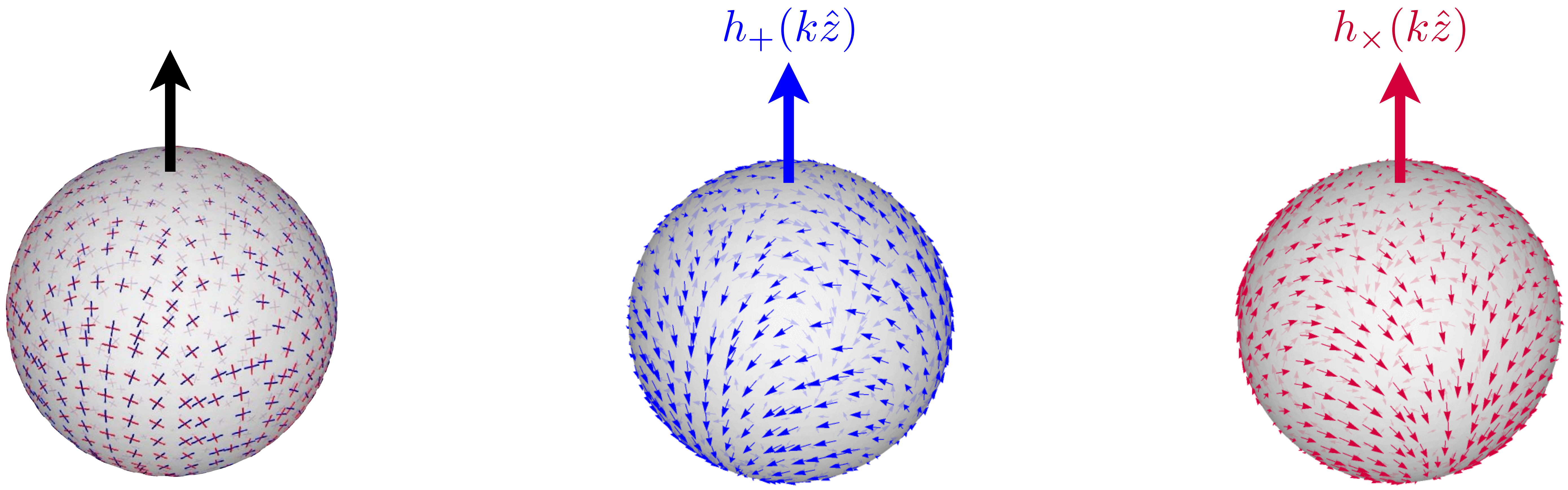
$$x^\mu(\lambda) = \left(x^0(\lambda),\ x^i(\lambda)\right)$$

Defelction: $$\delta n^I(t,\hat{n}) = e^I{}_\mu \delta n^\mu(t,\hat{n}) = \mathcal{R}^{IJK}(\hat{n},\hat{k}) h_{JK}(\vec{k}) e^{-ikt}$$

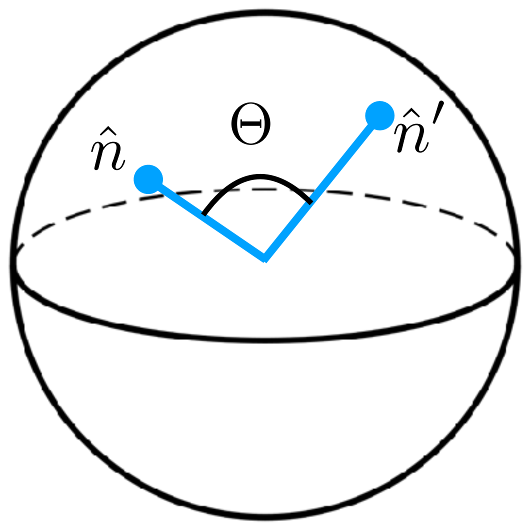
$$\mathcal{R}^{IJK}(\hat{n},\hat{k}) = \frac{\hat{n}^I + \hat{k}^I}{2(1+\hat{k}\cdot\hat{n})}\hat{n}^J\hat{n}^K - \frac{1}{2}\delta^{IJ}\hat{n}^K$$



[Book & Flanagan, 1009.4192]



~~Parity~~ signal in astrometry

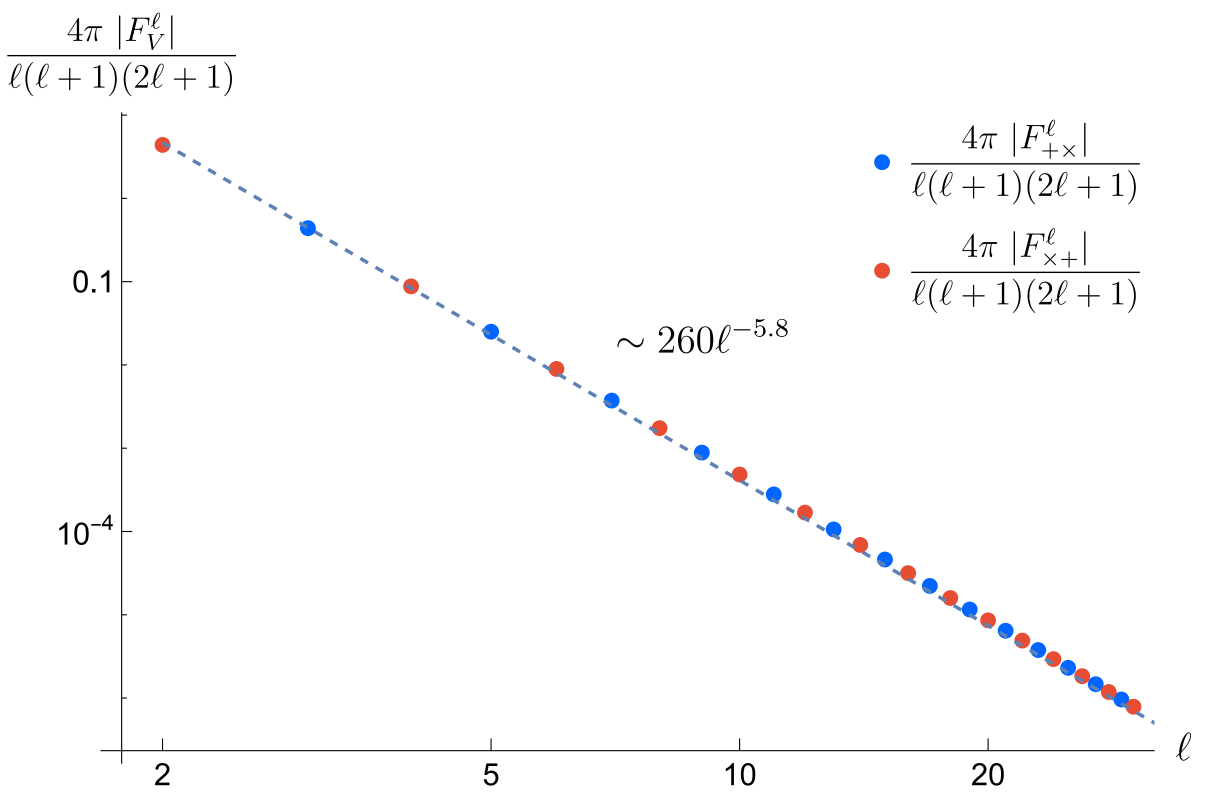


Vector spherical harmonics (E/B modes, cosmologist's favourite)

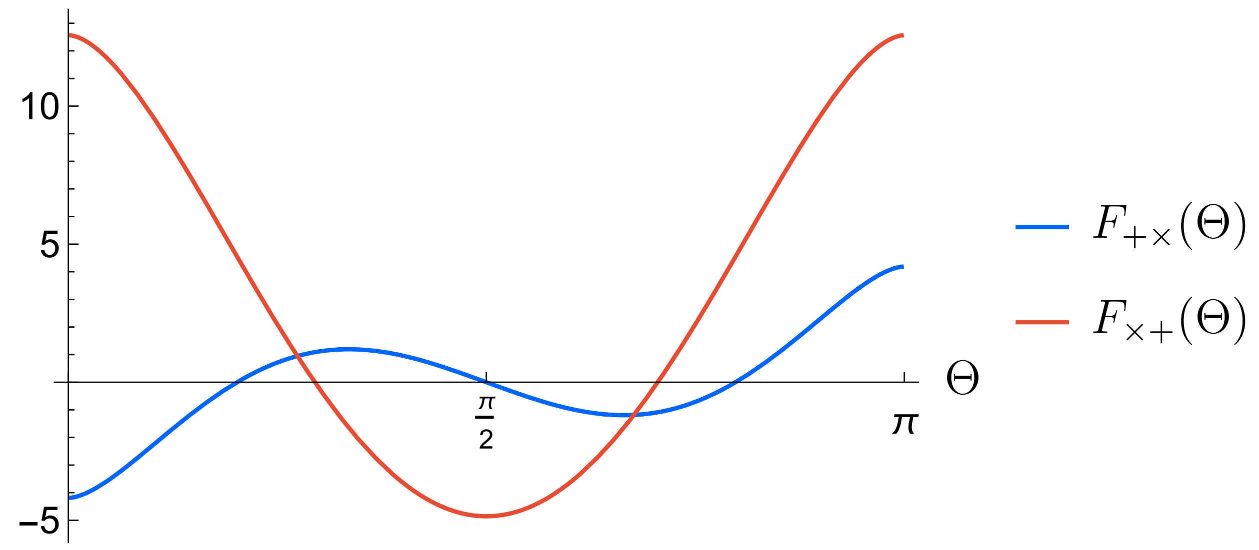
$$\delta n(t,\hat{n}) = \sum_{\ell m}\left[\delta n_{E\ell m}(t)\vec{Y}^{E}_{\ell m}(\hat{n}) + \delta n_{B\ell m}(t)\vec{Y}^{B}_{\ell m}(\hat{n})\right]$$

EB correlations:

$$\begin{aligned}\langle \delta n_{E\ell m}(t)\delta n_{B\ell' m'}(t)^*\rangle &= \int d^2\Omega_{\hat{n}} d^2\Omega_{\hat{n}'} Y^{E*}_{\ell m I}(\hat{n}) Y^{B}_{\ell' m' J}(\hat{n}') \left\langle \delta n^I(t,\hat{n})\delta n^J(t,\hat{n}')\right\rangle \\ &= \frac{1}{\ell(\ell+1)}\int d^2\Omega_{\hat{n}} d^2\Omega_{\hat{n}'} Y^{*}_{\ell m}(\hat{n}) Y_{\ell' m'}(\hat{n}')\, \epsilon_{JKL}\nabla_I \nabla'^L \left(\hat{n}'^K \left\langle \delta n^I(t,\hat{n})\delta n^J(t,\hat{n}')\right\rangle\right) \\ &= \frac{\delta_{\ell\ell'}\delta_{mm'}}{\ell(\ell+1)}\frac{4\pi}{2\ell+1}\left(\mathcal{A}_{+\times}F^{\ell}_{+\times} + \mathcal{A}_{\times+}F^{\ell}_{\times+}\right)\end{aligned}$$



$$\epsilon_{JKL}\nabla_I \nabla'^L \left(\hat{n}'^K \left\langle \delta n^I(t,\hat{n})\delta n^J(t,\hat{n}')\right\rangle\right) = \mathcal{A}_{+\times}F_{+\times}(\Theta) + \mathcal{A}_{\times+}F_{\times+}(\Theta)$$



[Liang, Lin, Trodden & **Wong**, 2309.16666]

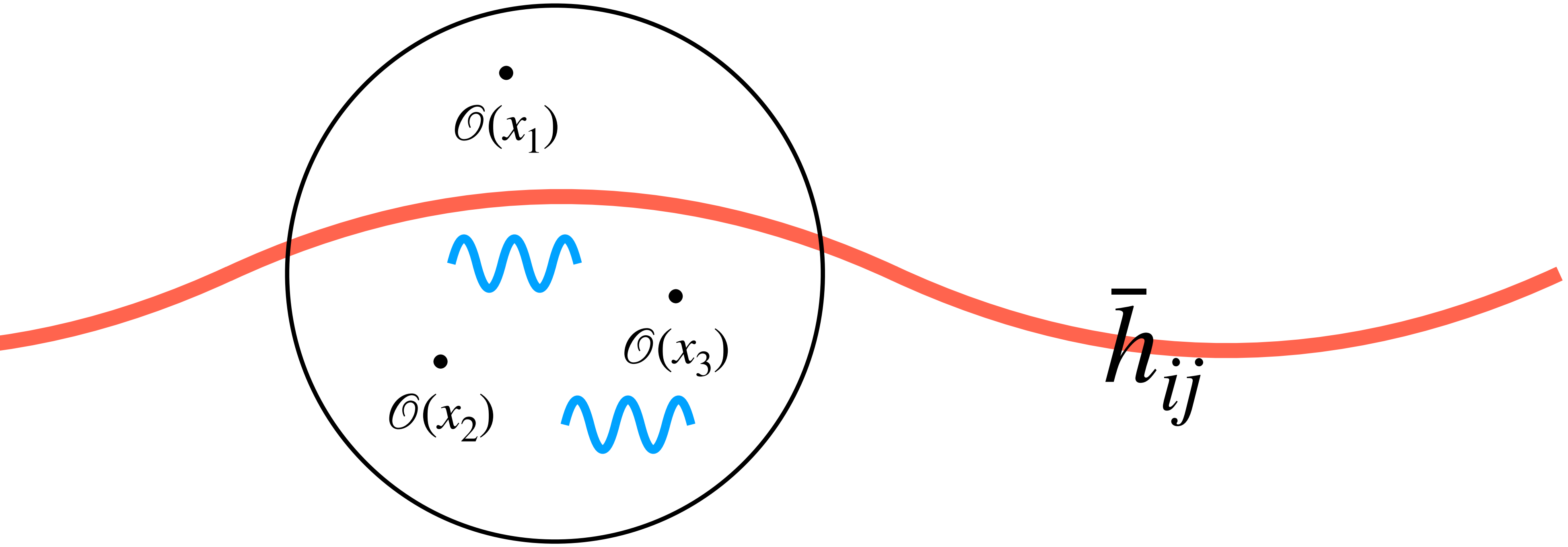
Soft modes in stochastic signals

Remember, the presence of a long wavelength gravitational wave can be thought as a diff in the background geometry

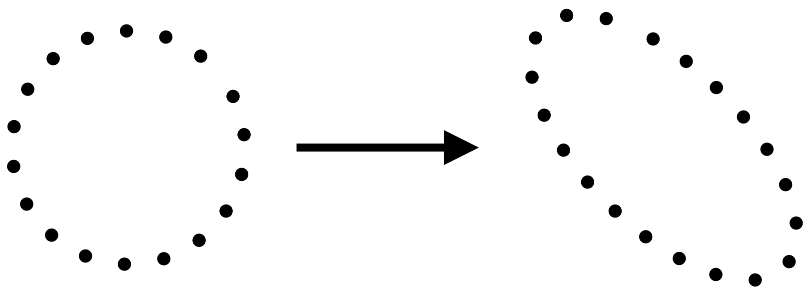
in the presence of a long mode $\quad \langle \mathcal{O}(x_1, \ldots, x_N) \rangle_{h \to \text{const.}} = \langle \mathcal{O}(\tilde{x}_1, \ldots, \tilde{x}_N) \rangle \quad$ in a rescaled coordinate

expanding in the long mode $\Rightarrow$

$$\langle \mathcal{O}(\tilde{x}_1, \ldots \tilde{x}_N) \rangle = \left(1 + \frac{1}{2} \bar{h}^k{}_\ell \sum_{m=1}^{N} x_m^\ell \frac{\partial}{\partial x_m^k} + \mathcal{O}\left(\bar{h}^2\right) \right) \langle \mathcal{O}(x_1, \ldots, x_N) \rangle$$



$$\tilde{x}^i = x^i + \frac{1}{2} \bar{h}^i{}_j x^j$$



Anisotropic rescaling

Soft modes in stochastic signals

in the presence of a long mode

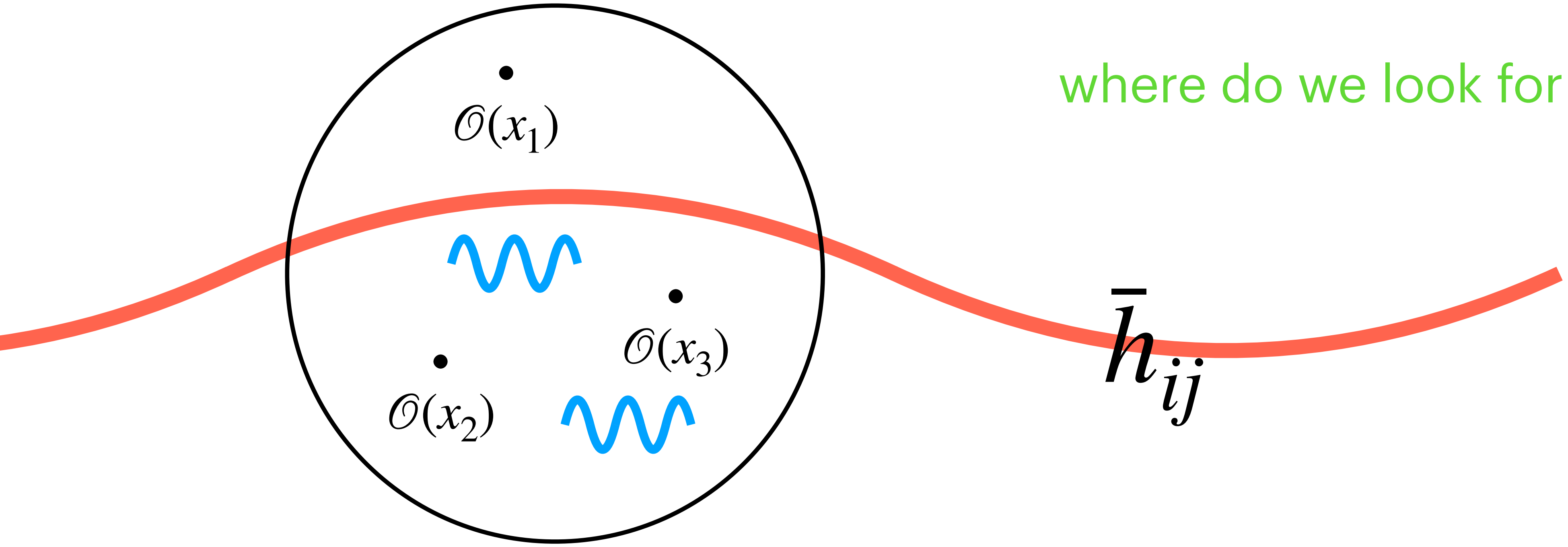
$$\langle \mathcal{O}(x_1, \ldots, x_N) \rangle_{h \to \text{const.}} = \langle \mathcal{O}(\tilde{x}_1, \ldots, \tilde{x}_N) \rangle$$

in a rescaled coordinate

expanding in the long mode

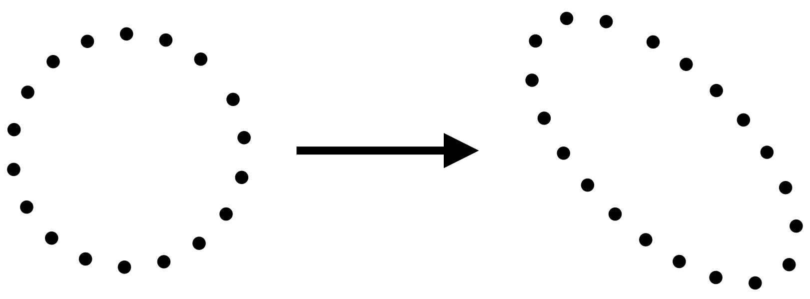
$$\Rightarrow \quad \langle \mathcal{O}(\tilde{x}_1, \ldots \tilde{x}_N) \rangle = \left(1 + \frac{1}{2} \bar{h}^k{}_\ell \sum_{m=1}^{N} x_m^\ell \frac{\partial}{\partial x_m^k} + \mathcal{O}\left(\bar{h}^2\right) \right) \langle \mathcal{O}(x_1, \ldots, x_N) \rangle$$

$$\lim_{h \to \text{const.}} \langle h_{ij} \mathcal{O}(x_1, \ldots, x_N) \rangle = \lim_{h \to \text{const.}} \frac{1}{2} \langle h_{ij} \bar{h}_{k\ell} \rangle \sum_{m=1}^{N} x_m^\ell \frac{\partial}{\partial x_m^k} \langle \mathcal{O}(x_1, \ldots, x_N) \rangle$$



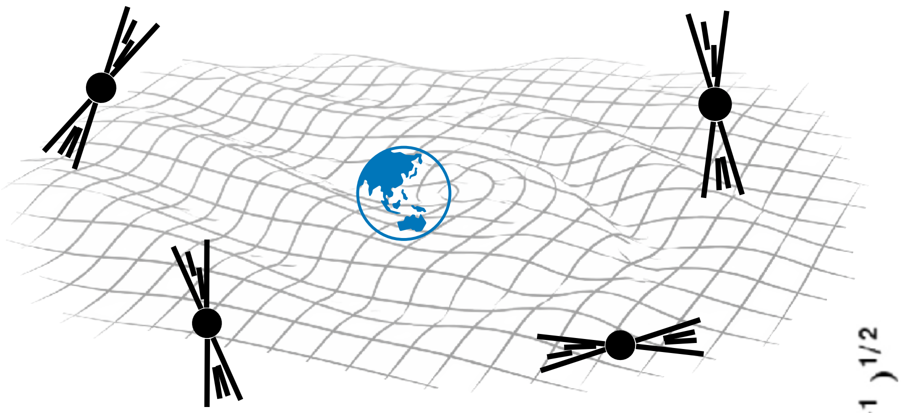
where do we look for this?

$$\tilde{x}^i = x^i + \frac{1}{2} \bar{h}^i{}_j x^j$$



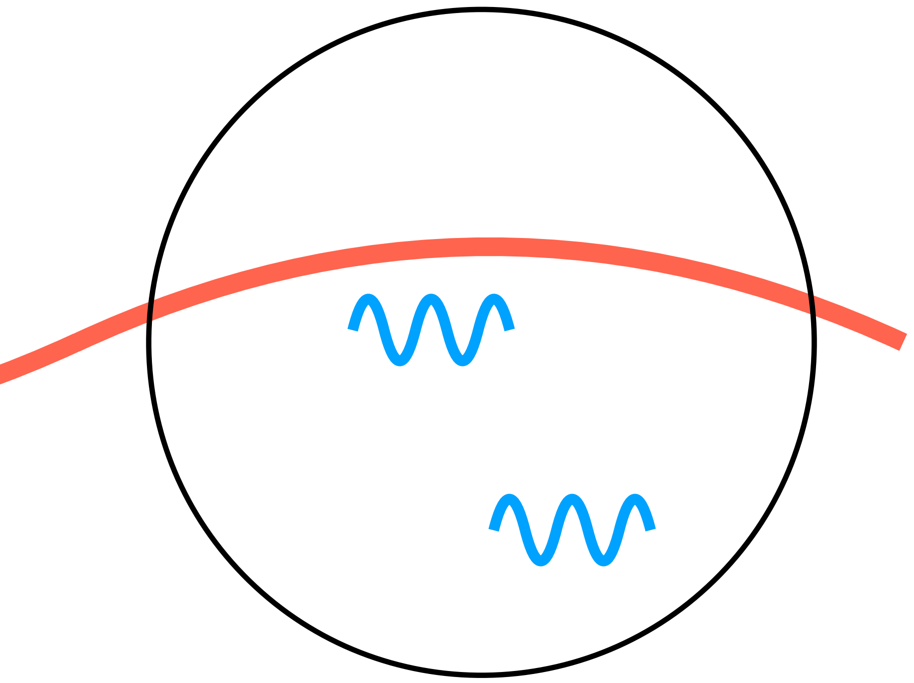
Anisotropic rescaling

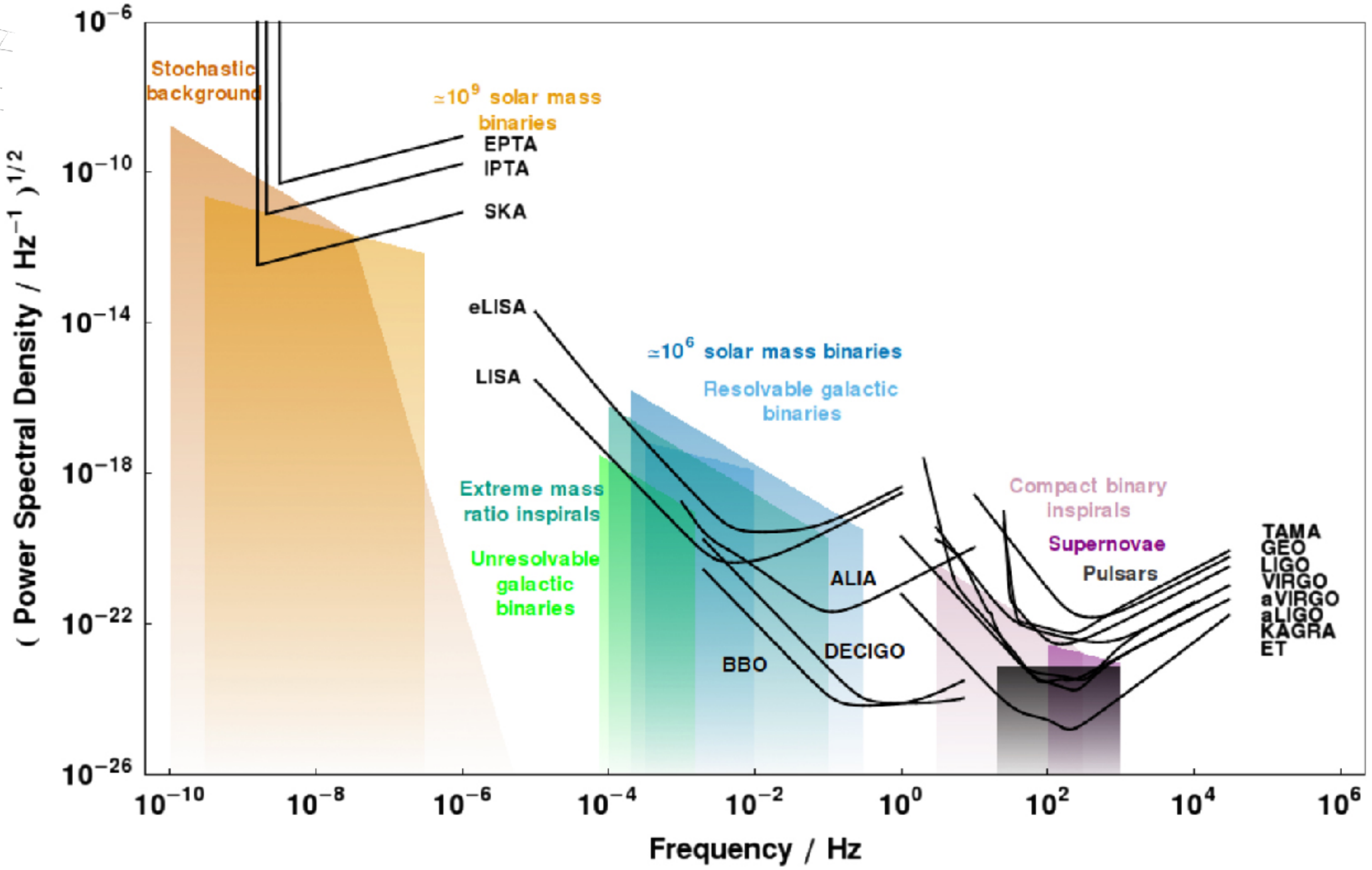
PTA - Interferometer correlations

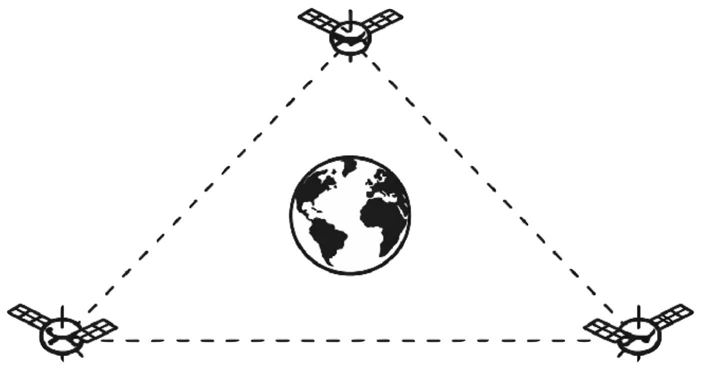


(PTA/astrometry)

Low frequency GWs, $\sim$10 nHz, signal: $z_a(t)$





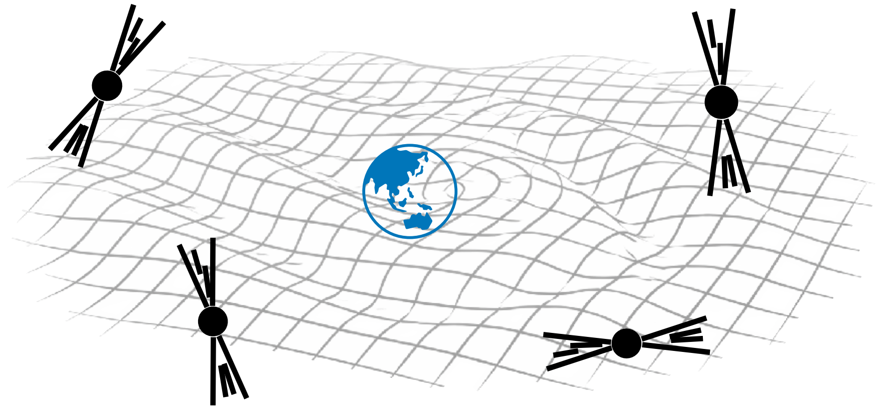


Higher frequency GWs, 10^{-4}~1 Hz, signal: $D(t)$

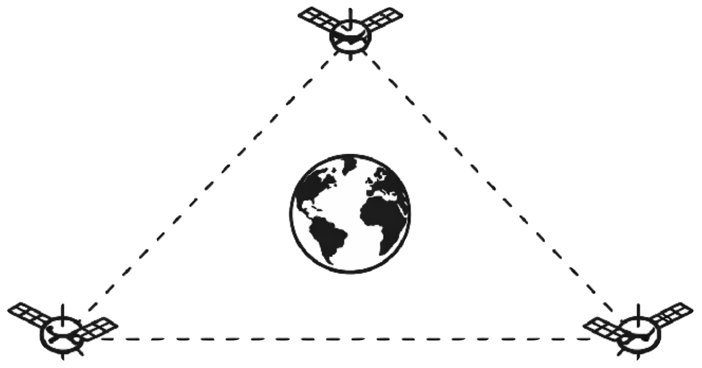
Schematically: $\langle z_a(t) D_A(t) D_B(t) \rangle \sim \langle h_{f_1} h_{f_2} h_{f_3} \rangle_{f_1 \ll f_2, f_3}$

PTA - Interferometer correlations

Relating three point functions to power spectrums at different frequency ranges

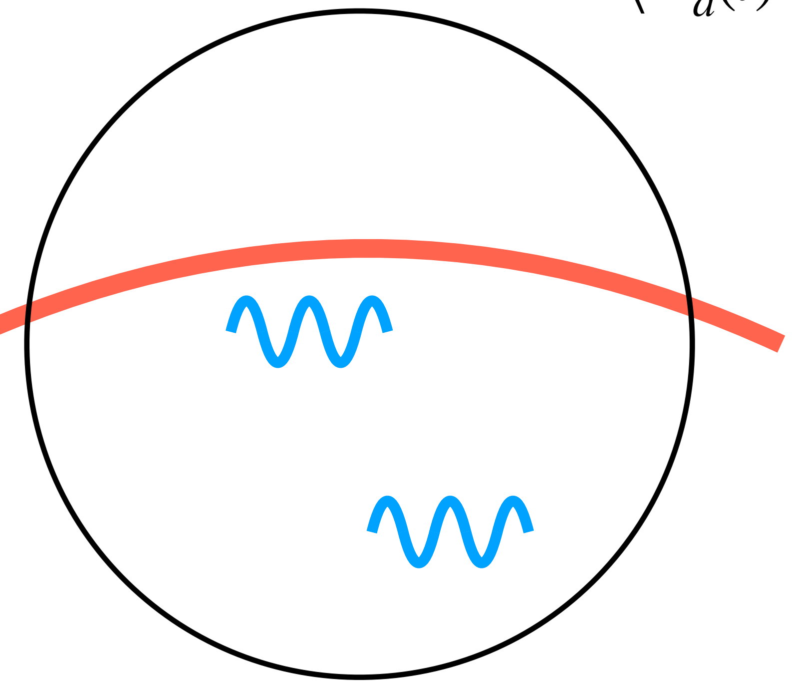


signal: $z_c(t)$, power spectrum at f_0



signal: $D_a(t)$

$$
\begin{aligned}
\langle D_a(t) D_b(t) z_c(t) \rangle = \frac{1}{8} \int \mathrm{d}f_0 S_h(f_0) \int \mathrm{d}f S_h(f) \Bigg\{ & \int \frac{\mathrm{d}^2\Omega}{4\pi} [1 - 3(n_c \cdot \Omega)^2] \sum_A F_a^A(\Omega) F_b^A(\Omega) \\
& + \int \frac{\mathrm{d}^2\Omega}{4\pi} \Bigg[\frac{1}{3} \Omega^j l_b^i (l_b \cdot \Omega) - \frac{1}{3} \Omega^j m_b^i (m_b \cdot \Omega) - l_b^i l_b^j (n_c \cdot l_b)^2 - m_b^i m_b^j (n_c \cdot m_b)^2 \\
& + n_c^i l_b^j \left((n_c \cdot l_b) - (n_c \cdot \Omega)(l_b \cdot \Omega) \right) + n_c^i m_b^j \left((n_c \cdot m_b) - (n_c \cdot \Omega)(m_b \cdot \Omega) \right) \Bigg] \sum_A F_a^A(\Omega) e_{ij}^A(\Omega) \\
& + (a \leftrightarrow b) \Bigg\}
\end{aligned}
$$

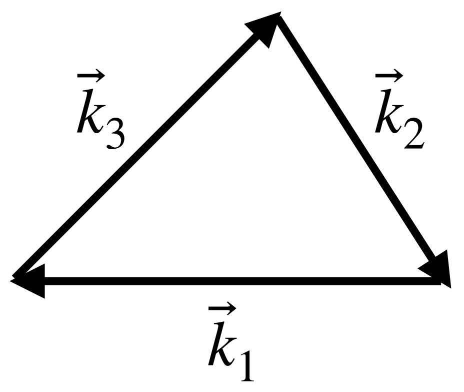


[De Luca, Khoury & **Wong**, work in progress]

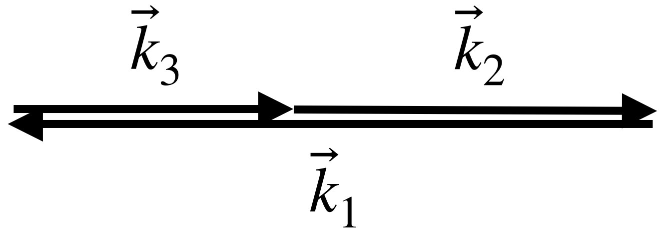
Non-gaussianities in the bispectrum

- The squeezed limit obstained from $\langle z_a(t) D_b(t) D_c(t) \rangle$ is a test of gravity (diff. inv.) and a signal of local non-gaussianity
- Correlations of different detectors, $\langle D_a(t) D_b(t) D_c(t) \rangle$, contain interesting information such as "time translational invariance" of the signal

wave vectors of the modes:



early universe signals

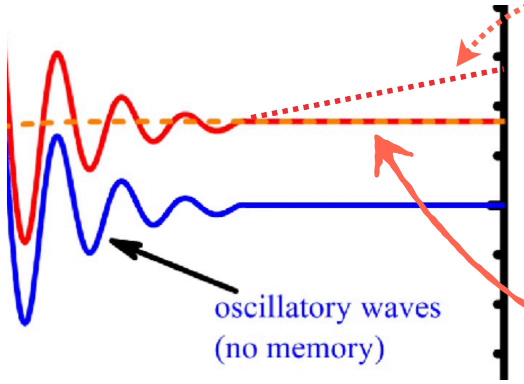


time translational invariance enforces $\delta(f_1 + f_2 + f_3)$

[De Luca, Khoury & **Wong**, work in progress]

Brief summary

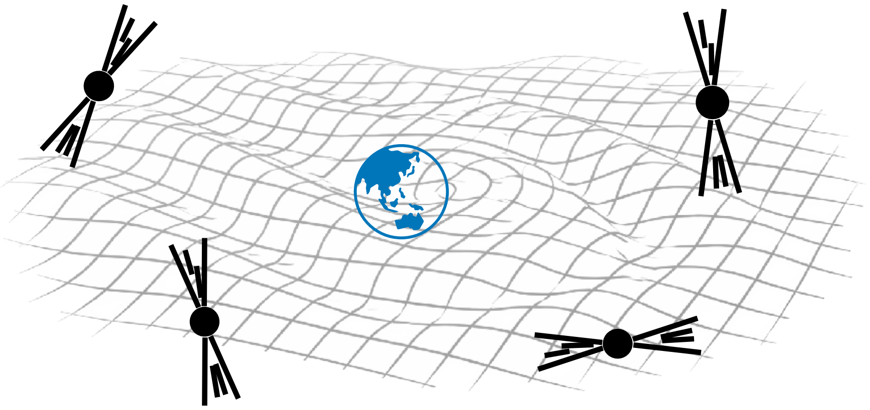
Single event signal: memories characterized by symmetry



$$H_{ij}^{\mathrm{TT}}(U, N) = A_{ij}(\bar{N}) + B_{ij}(\bar{N})u + \dots$$

Stochastic signal: symmetry dictates the squeezed limit, fixing local non-gaussianity

$$\langle z_a(t) D_b(t) D_c(t) \rangle \sim S(f_{\mathrm{low}}) S(f_{\mathrm{high}})$$



PTA/astrometry