14 Mar 2026

"Gravitational Waves and the Early Universe" @ Nagoya University

Gravitational waves from cosmic textures

～ ongoing work ～

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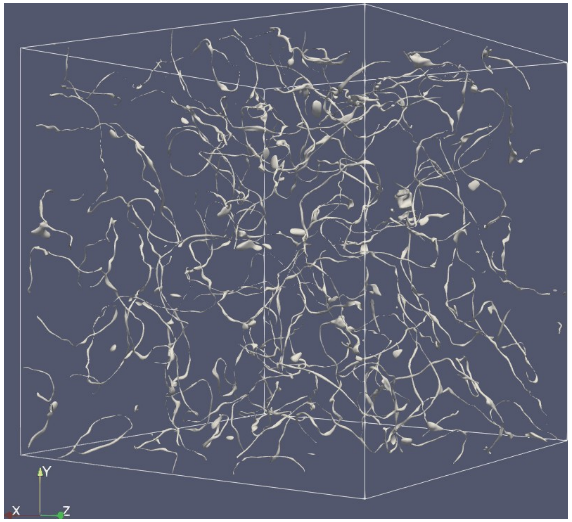
Topological defect

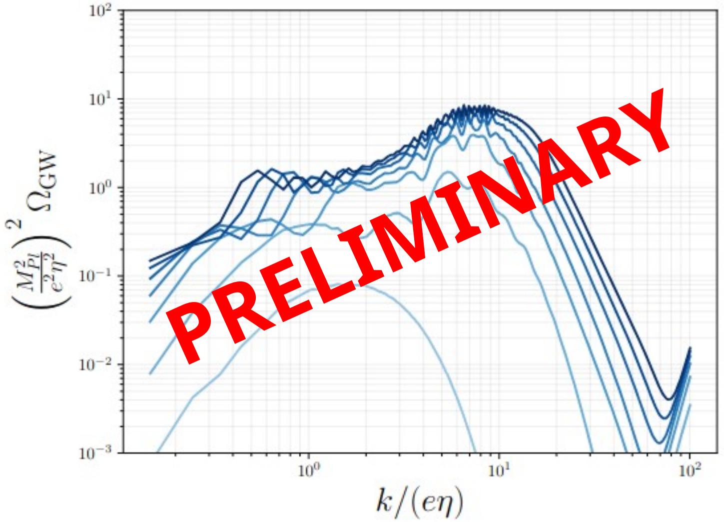
Spontaneous symmetry breaking $G \xrightarrow{\phi} H$



$|\phi| = v$ almost everywhere, but false vacuum left in a specific shape where $|\phi| \neq v$

CMB **GW**

…we can access ultra-high energy physics.



GWs from superconducting string network

Jhun, TH, in preparation

$\pi_0(G/H) \neq I$ **Domain wall**

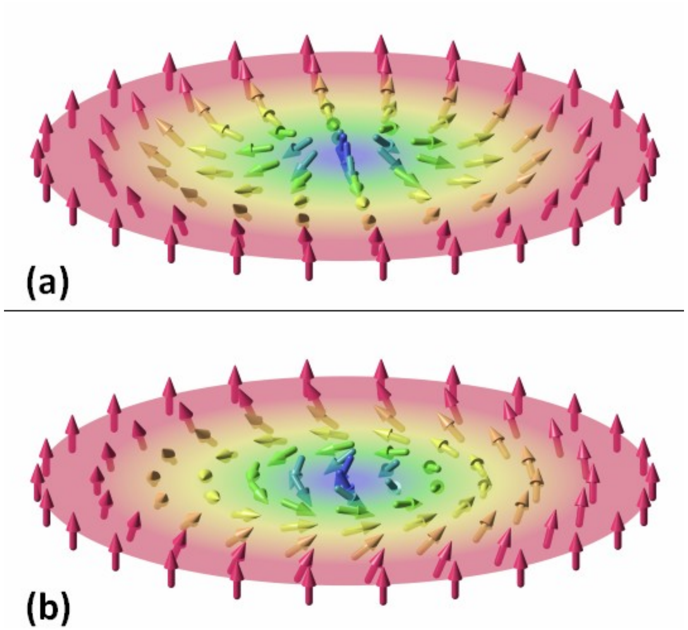
$\pi_1(G/H) \neq I$ **String**

$\pi_2(G/H) \neq I$ **Monopole**

Spontaneous symmetry breaking $G \xrightarrow{\phi} H$

$|\phi| = v$ everywhere

but the field configuration is non-trivial



Wikipedia : Magnetic skyrmion

$\pi_3(G/H) \neq I$ **Texture**

Gauged texture

Davis, Phys. Rev. D35 (1987) 3705

$SU(2) \longrightarrow I$

$$S = -\int d^4x\sqrt{-g}\left[D^\mu\Phi^\dagger D_\mu\Phi + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \lambda(\Phi^\dagger\Phi - v^2)^2\right]$$

In fundamental rep. of SU(2) $\quad \boldsymbol{\Phi} = U(\boldsymbol{x})\begin{pmatrix}0\\ v\end{pmatrix} \quad U(\boldsymbol{x}) \in SU(2)$

With the pure gauge, $A_\mu = U^{-1}(\boldsymbol{x})\partial_\mu U(\boldsymbol{x})$

$\longrightarrow$ $D_\mu\boldsymbol{\Phi} = 0,\ F_{\mu\nu} = 0$

$\longrightarrow$ $T_{\mu\nu} = 0$

No physical gauged texture in SU(2)

Gauged texture

Davis, Phys. Rev. D35 (1987) 3705

$$O(4) \longrightarrow O(3)$$

$$S = -\int d^4x\sqrt{-g}\left[\mathscr{D}^{\mu}\boldsymbol{\phi}\cdot\mathscr{D}_{\mu}\boldsymbol{\phi} + \frac{1}{4}\mathscr{F}^{\mu\nu}\mathscr{F}_{\mu\nu} + \lambda(\boldsymbol{\phi}\cdot\boldsymbol{\phi} - v^2)^2\right]$$

In fundamental rep. of O(4) $\quad \boldsymbol{\phi} = U(\boldsymbol{x})\begin{pmatrix}0\\0\\0\\v\end{pmatrix} \qquad U(\boldsymbol{x}) \in O(4)$

The gauge field such that $\mathscr{D}_{\mu}\boldsymbol{\phi} = (\partial_{\mu} - ie\mathscr{A}_{\mu})\boldsymbol{\phi} = 0$ yields

$$\longrightarrow \quad \mathscr{F}_{\mu\nu} \neq 0$$

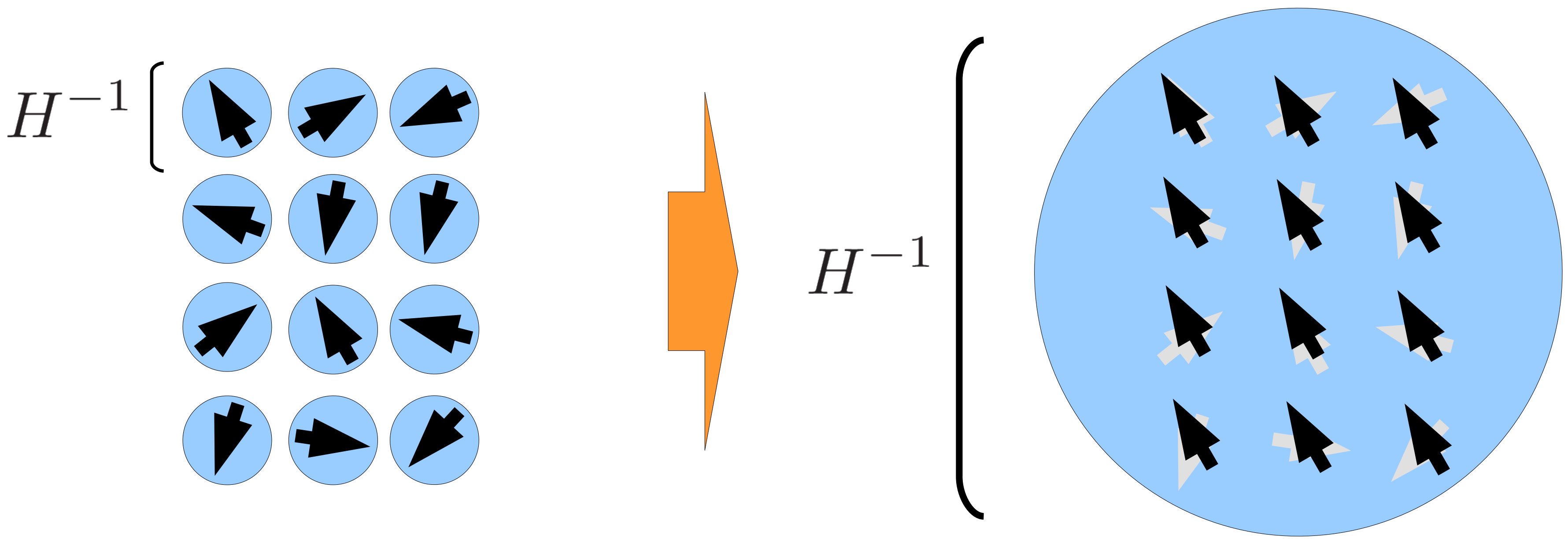
$$\longrightarrow \quad T_{\mu}{}^{\nu} = \frac{3v^4}{4g^2}\begin{pmatrix}-1 & 0 & 0 & 0\\ 0 & 1/3 & 0 & 0\\ 0 & 0 & 1/3 & 0\\ 0 & 0 & 0 & 1/3\end{pmatrix}$$

Physical object !

Field realignment

e.g., gravitational waves from
global $O(N \geq 4) \longrightarrow O(N-1)$

Krauss, Phys. Lett. B284 (1992) 229
Giblin, Prince, Siemens, Vlcek, JCAP 11 (2012) 006
Kuroyanagi, TH, Yokoyama, JCAP 02 (2016) 023

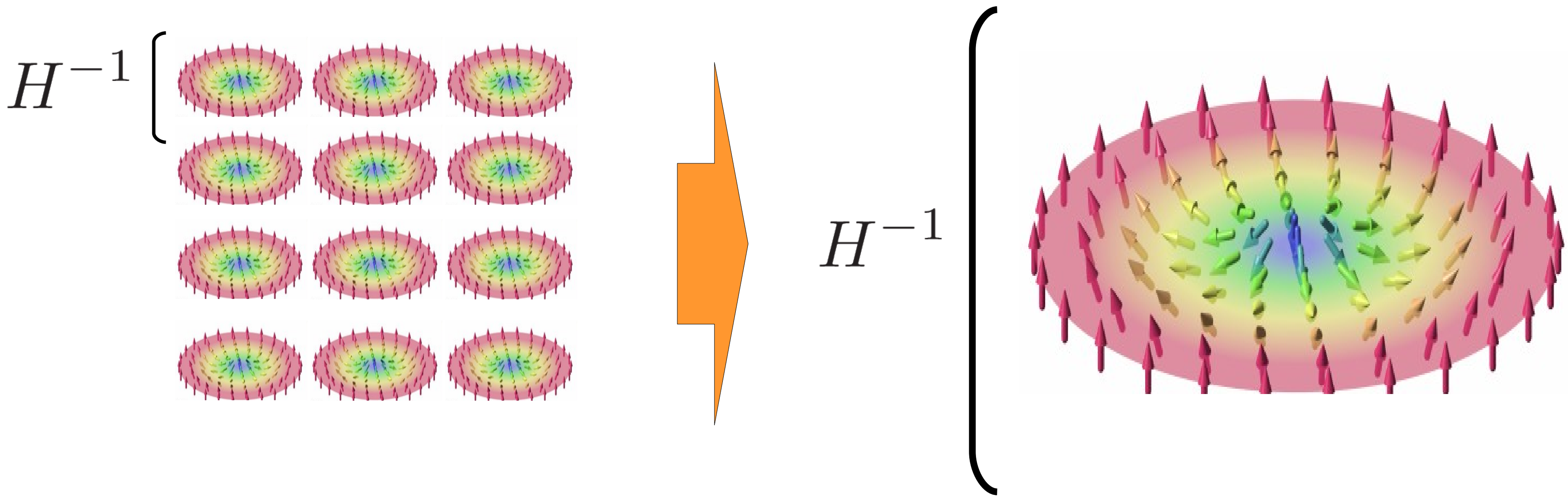


The field tends to be aligned to reduce the gradient energy inside a Hubble patch.

Gradient energy ⟶ Kinetic energy ⟶ Gravitational waves

Texture decay

Texture is not a stable object, expanding according to the cosmic expansion.



The field tends to be alinged with preserving the winding.

Gradient energy ⟶ Kinetic energy ⟶ Gravitational waves

Goal

- **Are there differences between the GW spectra from gauged SU(2) and O(4) textures?**

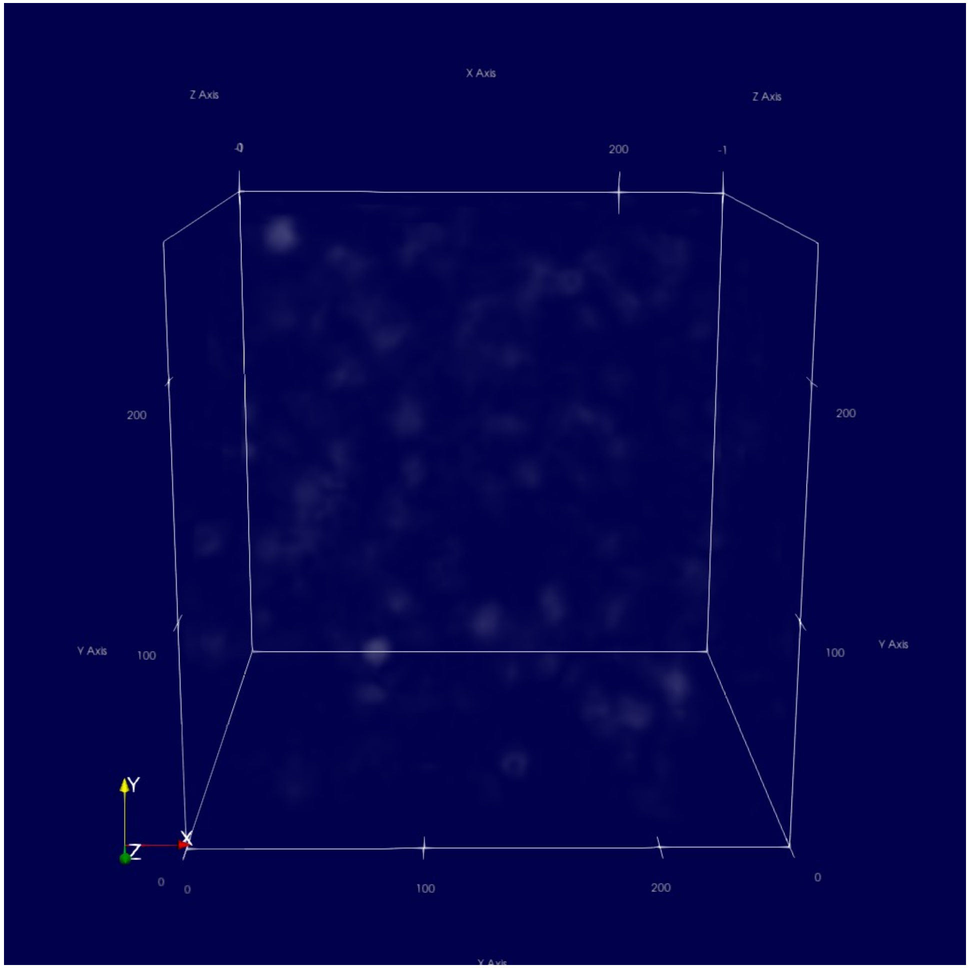
- **What about texture population?**

Simulation setup

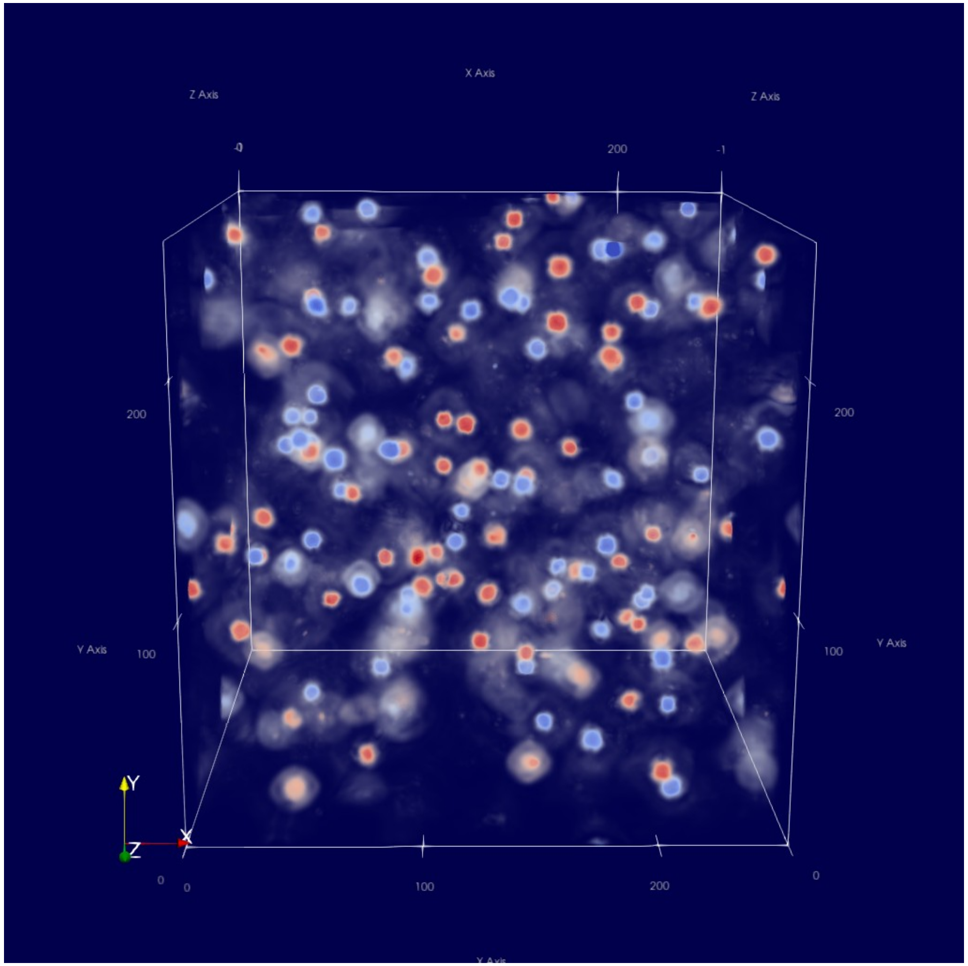
Grid size	$N = 512^3$
Comoving box size	$L_{\rm phys} = 24v^{-1} \quad a_{\rm f}/a_{\rm i} = 12$
Background	Radiation dominant Universe
Initial conditions	Flat spectrum (white noise) with $$P(k) = A^2 e^{-(k/k_{\rm cut})^2}$$ $$A = 10^{-2}\,,\; k_{\rm cut} = 10v$$ (A high-k cut-off is introduced to reduce numerical artifacts, confirmed no effects on the results.)

Result : Topological charge density

$$j^0 = \frac{1}{12\pi^2 v^4}\epsilon^{ijk}\epsilon^{abcd}\phi^a\partial_i\phi^b\partial_j\phi^c\partial_k\phi^d$$



$SU(2)$

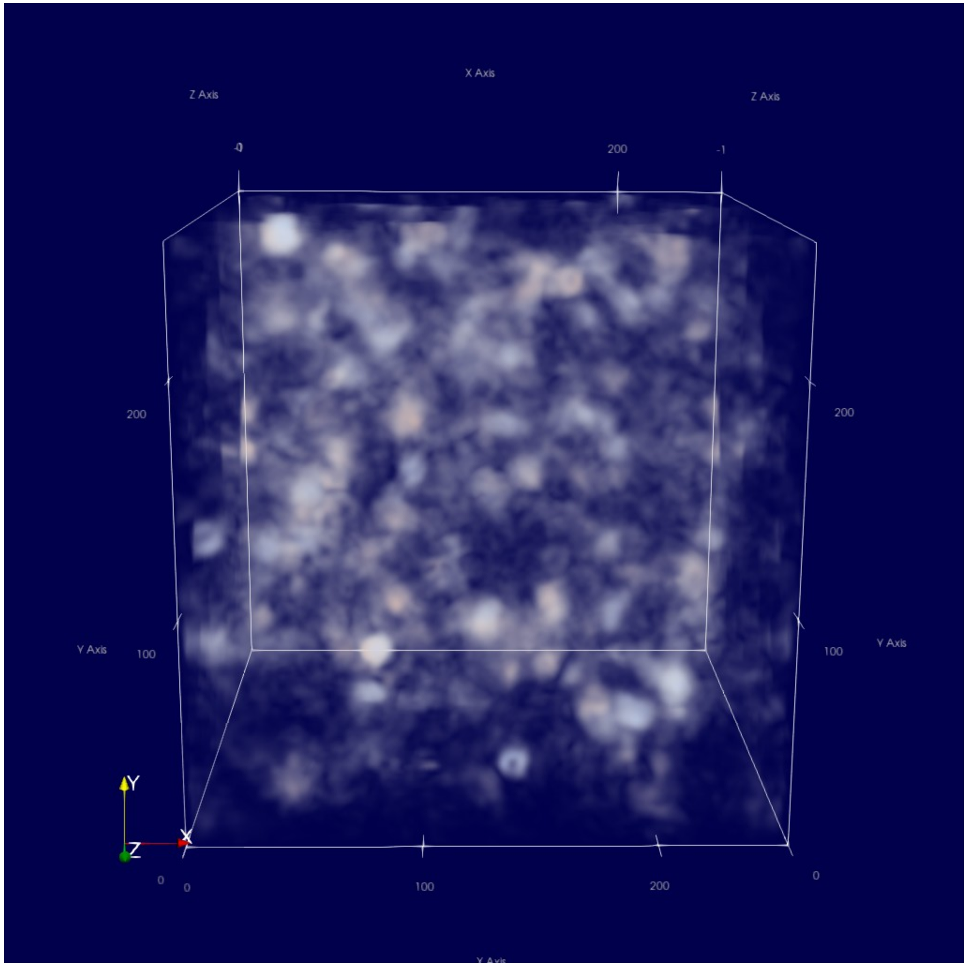


$O(4)$

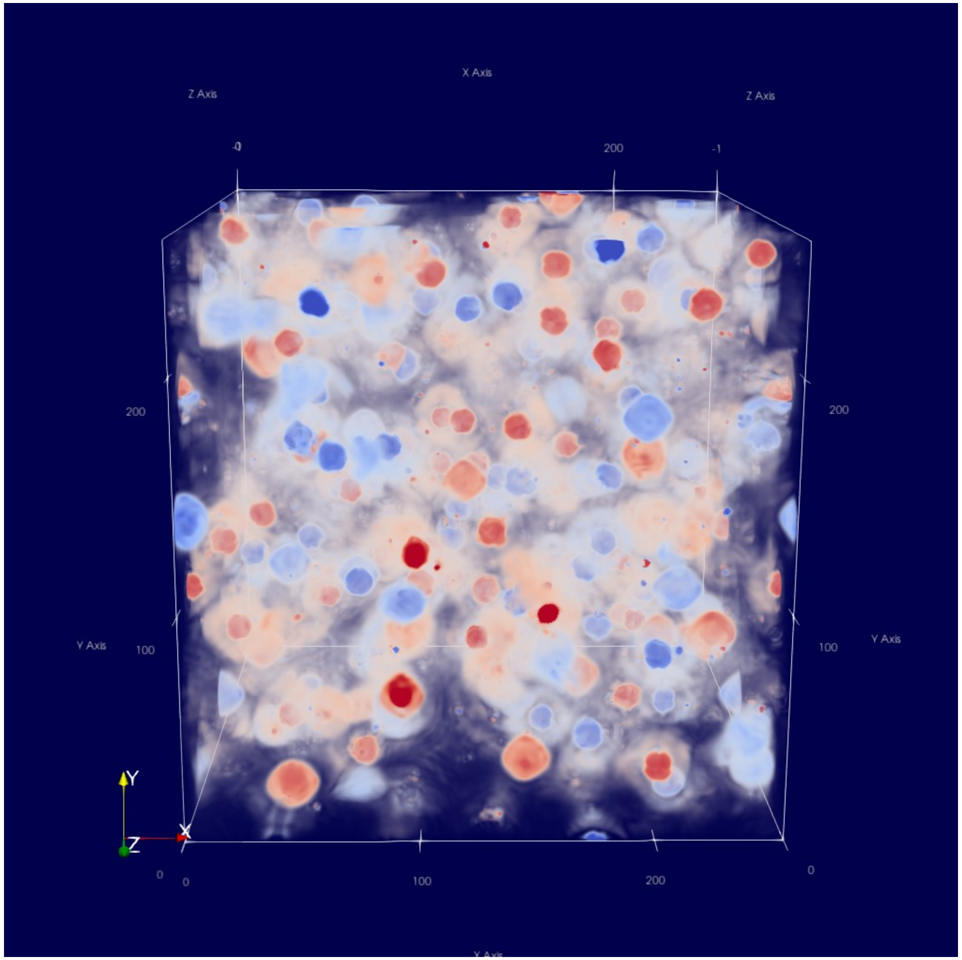
$$3\times 10^{-3}v < |j^0| < 3v$$

Result : Topological charge density

$$j^0 = \frac{1}{12\pi^2 v^4}\epsilon^{ijk}\epsilon^{abcd}\phi^a\partial_i\phi^b\partial_j\phi^c\partial_k\phi^d$$



$SU(2)$

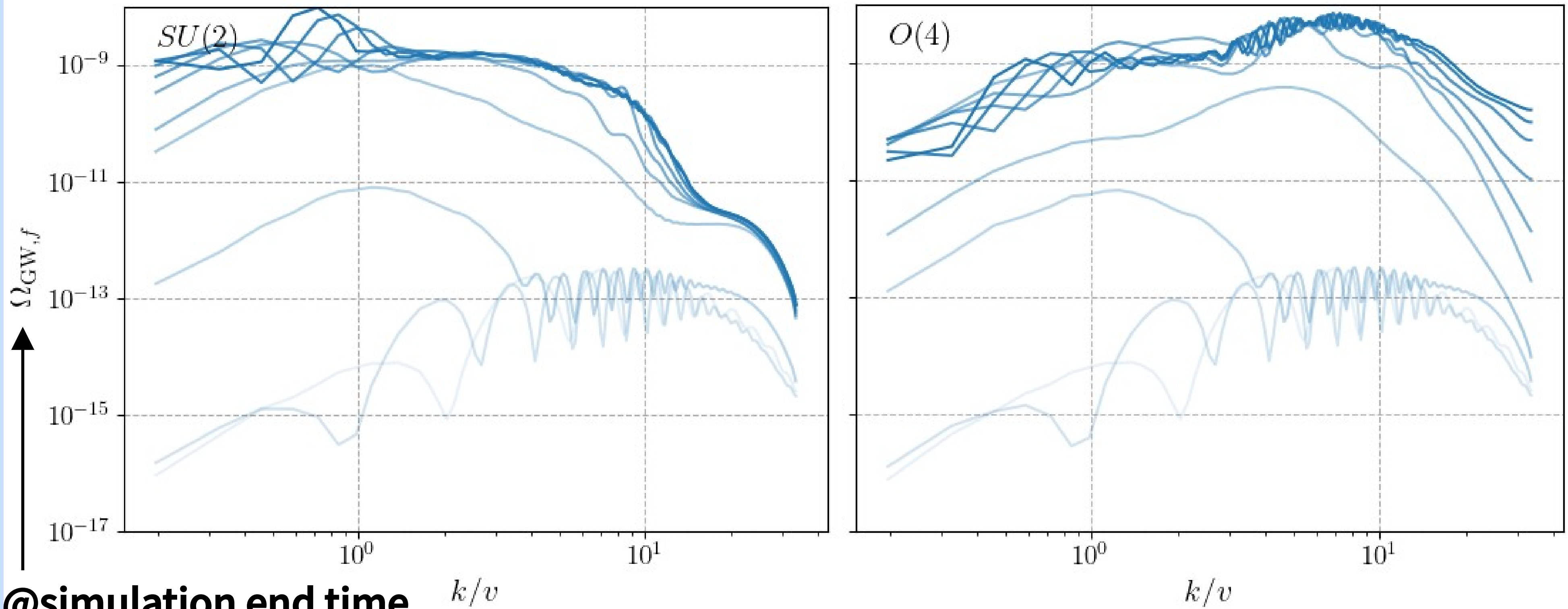


$O(4)$

$$3 \times 10^{-4} v < |j^0| < 0.3v$$

$$\Omega_{\rm GW}(k) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log k} \propto v^4$$

$$v/M_{\rm pl} = 10^{-2}$$

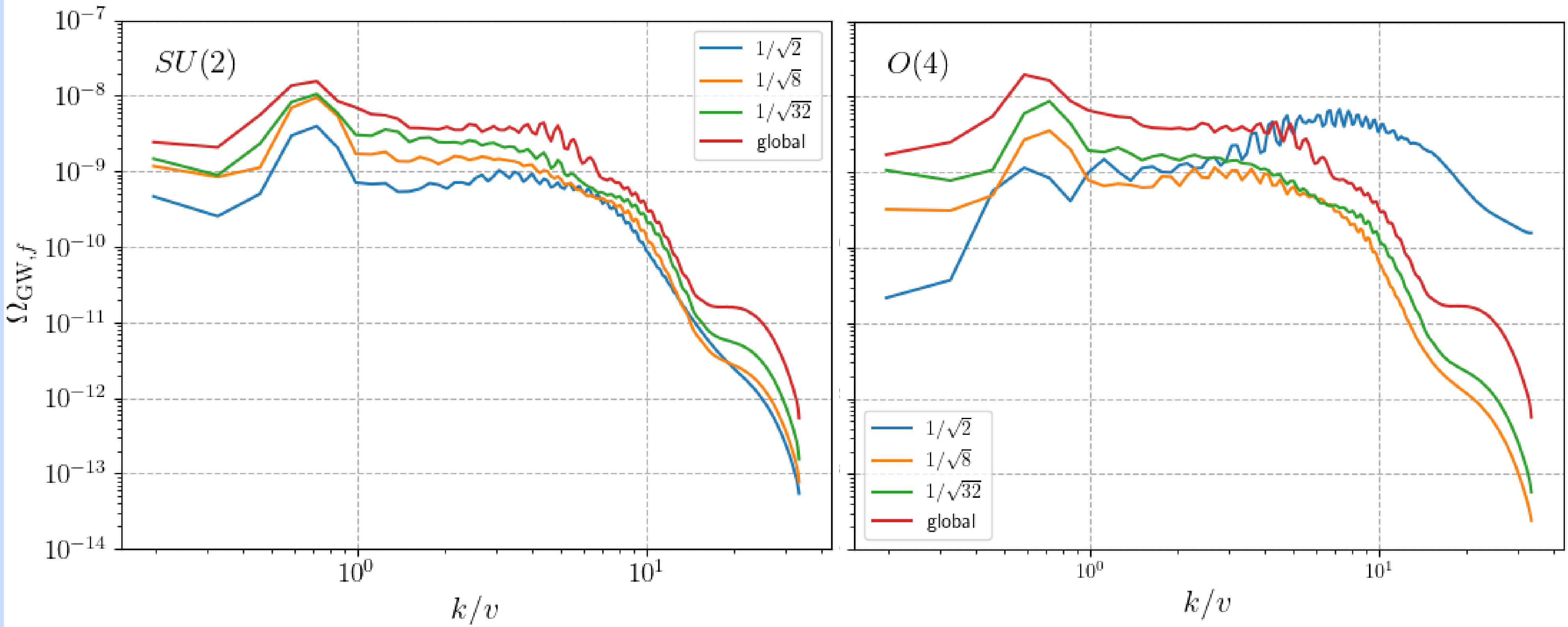


@simulation end time

In both models with the same ICs, the spectra converge as expected, since both $\rho_{\rm GW}$ and ρ_c scale as $\propto a^{-4}$.

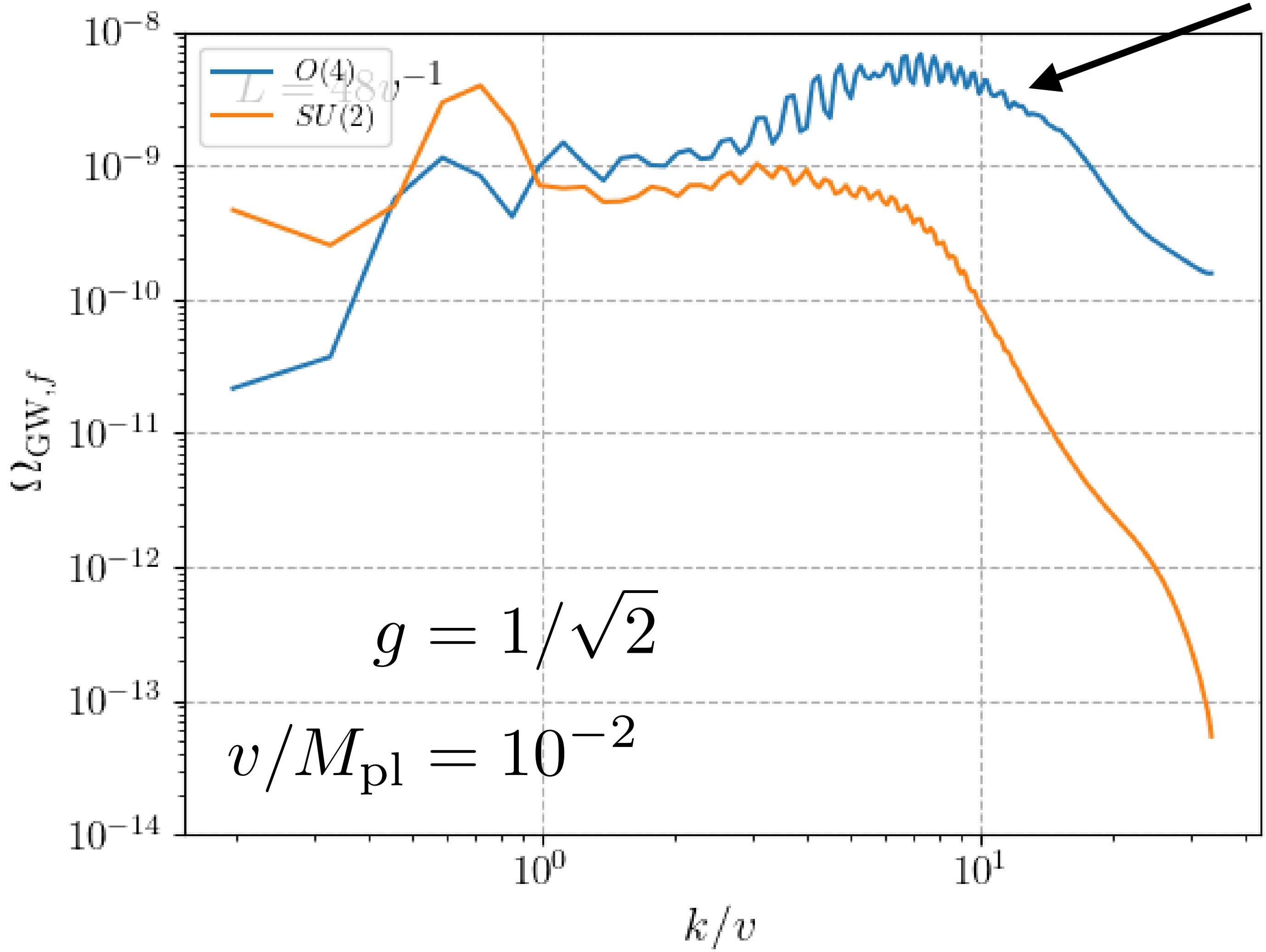
Result : Gauge-coupling constant dependence

$$g = \frac{1}{\sqrt{2}}, \ \frac{1}{\sqrt{8}}, \ \frac{1}{\sqrt{32}}, \ 0 \ \textbf{(global)}$$



With a large gauge-coupling constant, Ω_{GW} is suppressed.

Result : Gauge symmetry dependence



Enhanced by gauge field ?

From the large-scale amplitude, we can read $\Omega_{\mathrm{GW,f}} \sim 0.1(v/M_{\mathrm{pl}})^4$

Result : Observation

At the final time of the simulation

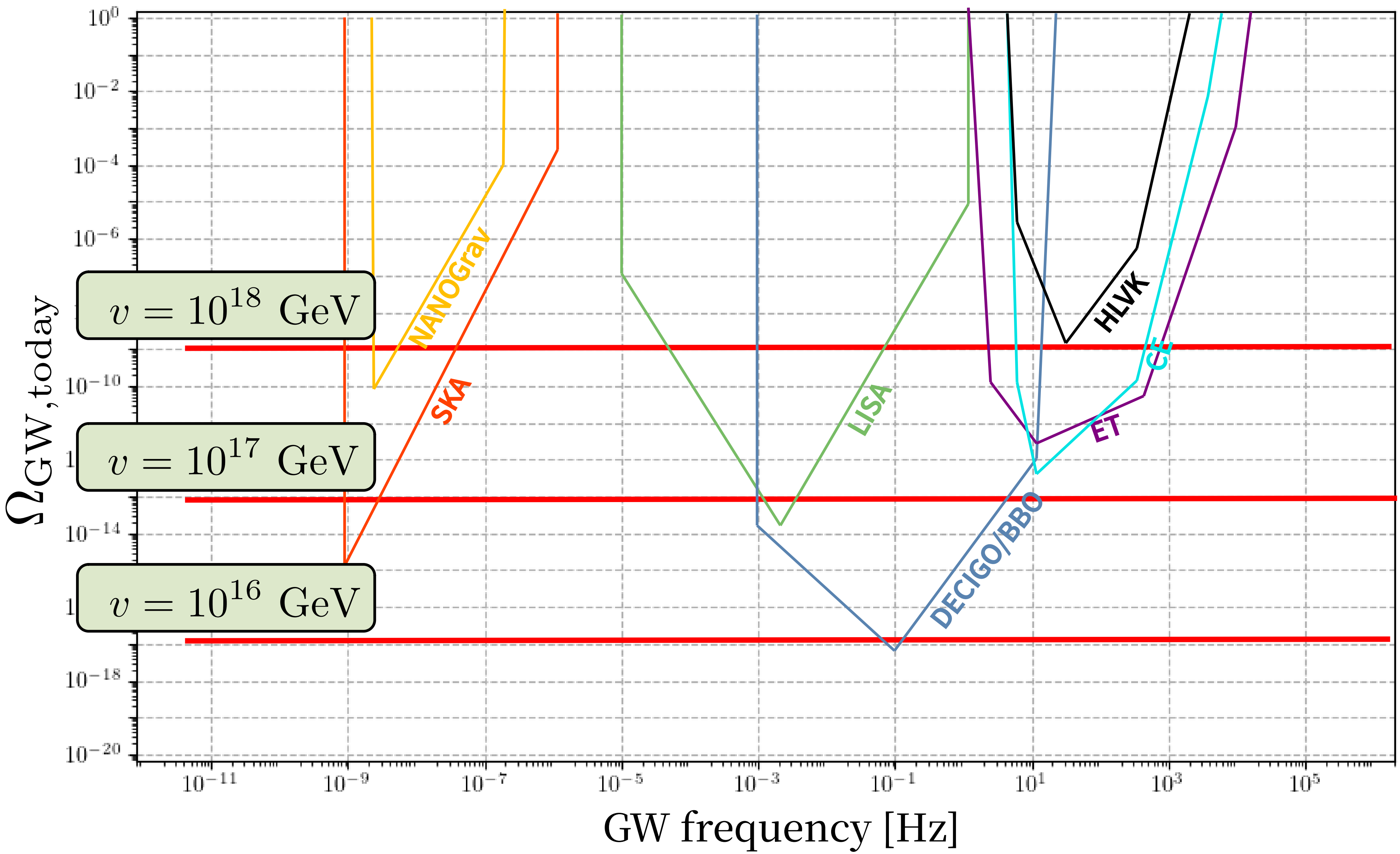
$$\Omega_{\mathrm{GW,f}} \sim 0.1 (v/M_{\mathrm{pl}})^4$$

At the present time

$$\Omega_{\mathrm{GW,today}} = \Omega_{\mathrm{R}} \Omega_{\mathrm{GW,f}} \sim 10^{-5} (v/M_{\mathrm{pl}})^4$$

Radiation fraction in present time

Result : Observation



CAUTION : Hand-writing sensitivity curves!

Summary

Are there differences between the GW spectra from gauged SU(2) and O(4) textures?

Almost identical, but slightly enhanced in O(4) case.

What about the texture population?

Apparently larger in O(4) case.