New ideas in semileptonic $b \to s\ell\ell$ and non-leptonic B decays to discover NP:

The fade of charming penguins, $b \to s\tau\tau$, inclusive/exclusive and the C_9 Dilemma

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Joaquim Matias

November, 2026

based on 2304.07330, 2301.10542, 2506.12478, 2605.06567, 2605.21291, 2609.40072

KMI2026: The 7th KMI International Symposium.

Diagnosis and Evolution of $b \to s\,\mu^+\mu^-$ Anomalies

$b \to s\,\mu^+\mu^-$ anomalies: P_5' anomaly reinforced+semileptonic branching ratios

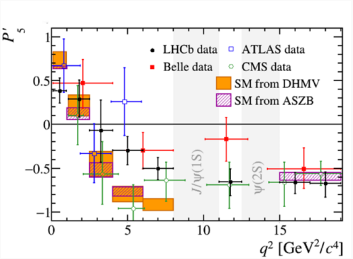
Angular observable P_5' in $B \to K^*\mu^+\mu^-$ (two anomaly bins $\sim 2\,\sigma$ each)

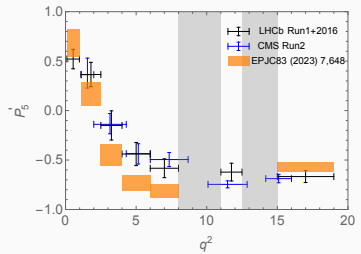
- Most tested anomaly LHCb, Belle, ATLAS measurements see a deviation from Standard model (SM) prediction, except for CMS.... till now.

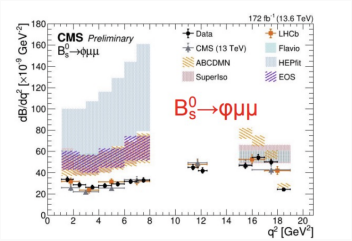
Several other LHCb & CMS measurements deviate from Standard model (SM) predictions:

- Branching ratios $B \to K\mu^+\mu^-$ (latt. more than 4σ), $B \to K^*\mu^+\mu^-$, and $B_s \to \phi\mu^+\mu^-$.
- Tension with SM above 2σ with ABCDMN and other determinations

$\to$ systematic deficit in muons in all measured channels







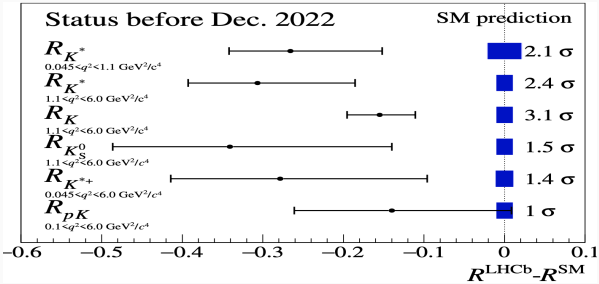
Hints for LFUV in $b \to s\,\ell^+\ell^-$ decays in first two generations: Picture before 2022

Measurements of lepton flavour universality (LFU) ratios $R_K^{[1.1,6]}$, $R_{K^*}^{[0.045,1.1]}$, $R_{K^*}^{[1.1,6]}$ **exhibited** deviations from SM by about 2-3σ each.

LHCb, arXiv:1705.05802
Belle, arXiv:1904.02440

- Cancellation of all uncertainties in SM (up to lepton masses) but strongly FF sensitive in presence of NP.

$$R_K = \frac{BR(B^+ \to K^+\mu^+\mu^-)}{BR(B^+ \to K^+e^+e^-)} \qquad R_{K^*} = \frac{BR(B \to K^*\mu^+\mu^-)}{BR(B \to K^*e^+e^-)}$$



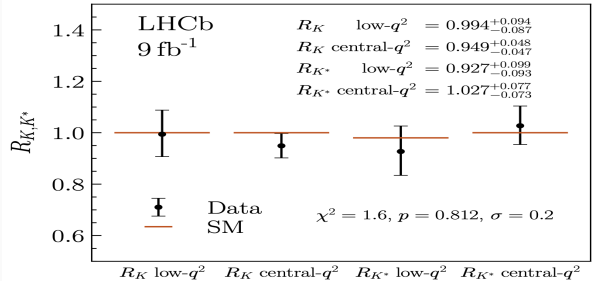
At that time electrons seemed more SM-like than muons.

Hints for LFUV in $b \to s\,\ell^+\ell^-$ decays: Picture after 2022, hints got diluted .. but $e^\pm$ are back!

Reanalysis of lepton flavour universality (LFU) ratios $R_K^{[1.1,6]}$, $R_{K^*}^{[0.045,1.1]}$, $R_{K^*}^{[1.1,6]}$ by LHCb due to PID problems with electrons.

LHCb, arXiv:1705.05802
Belle, arXiv:1904.02440

$$R_K = \frac{BR(B^+ \to K^+\mu^+\mu^-)}{BR(B^+ \to K^+e^+e^-)} \qquad R_{K^*} = \frac{BR(B \to K^*\mu^+\mu^-)}{BR(B \to K^*e^+e^-)}$$



Electrons and muons should deviate both in the same way

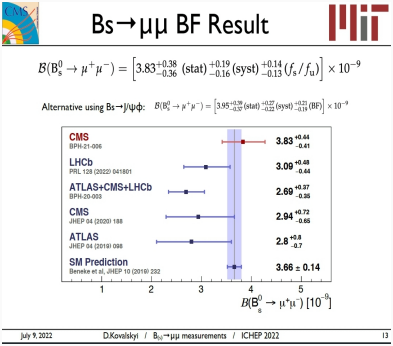
Also measurement of LFU observable $Q_i = P_i^{(\prime)\mu} - P_i^{(\prime)e}$ **by Belle** and later LHCb show no deviation from SM (except Q_{F_L} ($\sim 2\sigma$). [S. Wehle et al (Belle), PRL 118 (2017)]

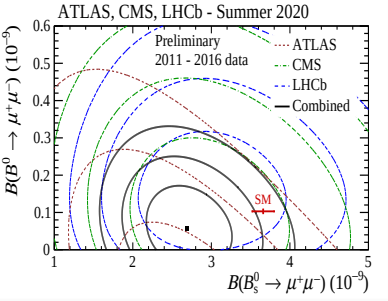
3rd anomaly?: Combination of $B_{s,d} \to \mu^+\mu^-$ measurements

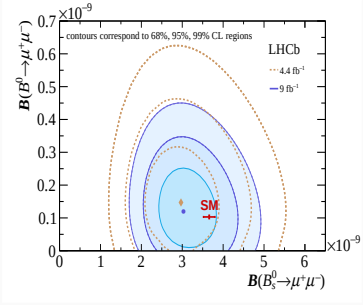
Measurements of BR($B_{s,d} \to \mu^+\mu^-$) by LHCb, CMS, and ATLAS **showed** a combined deviation from SM by about 2σ (with LHCb2021).

ATLAS, arXiv:1812.03017
CMS, arXiv:1910.12127
LHCb seminar 23 March 2021

(old combination) (new combination)





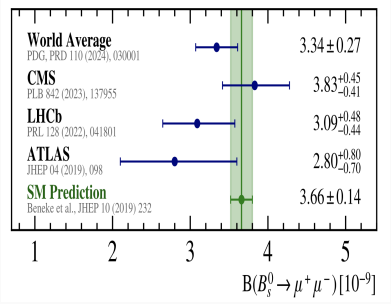


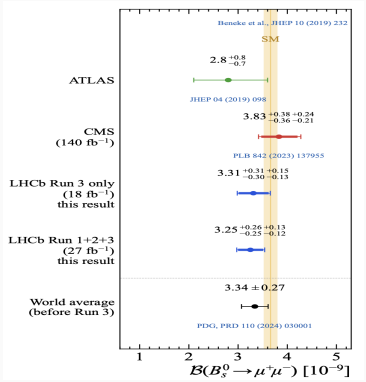
The new average of ATLAS, CMS, LHCb2021 is now closer to SM due to CMS.

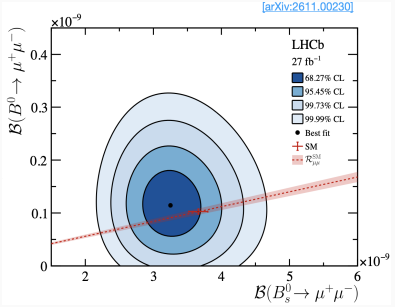
News from LHCb of past week

ATLAS - arXiv:1812.03017
CMS - arXiv:1910.12127
LHCb seminar, 23 March 2021

LHCb from Upgrade I (2024+2025) (18 fb^{-1})+ full Run 1 + 2 (9 fb^{-1}):
$\mathcal{B}_{B_s \to \mu^+\mu^-} = (3.25^{+0.26+0.13}_{-0.25-0.12}) \times 10^{-9}$





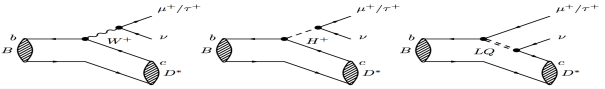


The new average of ATLAS, CMS, LHCb2021 is now closer to SM due to CMS.

Hints for LFU violation in $b \to c\,\tau\,\nu$ decays

Measurements of LFU ratios R_D and R_{D^*} by BaBar, Belle, and LHCb show combined deviation from SM by 3.8σ (excluding new Babar).

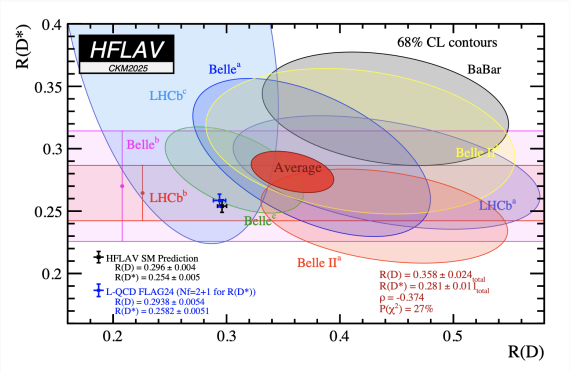
BaBar, arXiv:1205.5442, arXiv:1303.0571
LHCb, arXiv:1506.08614, arXiv:1708.08856
1507.03233, arXiv:1607.07923, arXiv:1612.00529, arXiv:1904.08794



SM NP

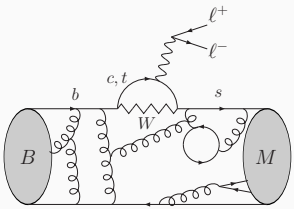
$$R_{D^{(*)}} = \frac{BR(B \to D^{(*)}\tau\nu)}{BR(B \to D^{(*)}\ell\nu)}$$

$$\ell \in \{e, \mu\}$$



How do we analyse all these $b \to s\ell\ell$ anomalies? An EFT approach

The framework: $b \to s\ell\ell$ effective Hamiltonian, Wilson Coefficients



$$b \to s\gamma(^*) : \mathcal{H}^{SM}_{\triangle F=1} \propto \sum V^*_{ts} V_{tb} \mathcal{C}_i \mathcal{O}_i + \dots$$

separate short and long distances ($\mu_b = m_b$)

- $\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}$
- $\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)$
- $\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$

$$\mathcal{C}_7^{\rm SM} = -0.29,\ \mathcal{C}_9^{\rm SM} = 4.1,\ \mathcal{C}_{10}^{\rm SM} = -4.3$$

NP changes short-distance $\mathcal{C}_i = \mathcal{C}_i^{\rm SM} + \mathcal{C}_i^{\rm NP}$ for SM or involve additional operators

- Chirally flipped ($W \to W_R$) $\quad \mathcal{O}_{7'} \propto (\bar{s}\sigma^{\mu\nu} P_L\, b) F_{\mu\nu},\ \mathcal{O}_{9'} \propto (\bar{s}\gamma_\mu P_R\, b)(\bar{\ell}\gamma^\mu \ell)$ ....
- (Pseudo)scalar ($W \to H^+$) $\quad \mathcal{O}_S \propto (\bar{s} P_R b)(\bar{\ell}\ell),\ \mathcal{O}_P \propto (\bar{s} P_R b)(\bar{\ell}\gamma_5 \ell)$
- Tensor operators ($\gamma \to T$) $\quad \mathcal{O}_T \propto \bar{s}\sigma_{\mu\nu}(1-\gamma_5) b\ \bar{\ell}\sigma_{\mu\nu}\ell$

Experimental inputs: Global analyses of $b \to s\ell\ell$ transitions

The latest most complete and updated analysis [M. Algueró et al. (2023)]

What's in the fits? ⇨ 254 obs (Global) + 24 obs (LFUV) from LHCb, Belle, ATLAS, CMS

- $B^0 \to K^{*0}\mu^+\mu^-$ $(P_{1,2,3}, P'_{4,5,6,8}, F_L)$ LHCb, $(P_1, P'_{4,5,6,8}, F_L)$ ATLAS, P_1, P'_5, F_L CMS
- $B^+ \to K^{*+}\mu^+\mu^-$ $(P_{1,2,3}, P'_{4,5,6,8}, F_L)$ LHCb
- $B^0 \to K^{*0}e^+e^-(P_{1,2}, P_3^{CP}, F_L)$ at low q^2 LHCb
- $B_s \to \phi\mu^+\mu^-$ $(P_1, P'_{4,6}, F_L, BR)$ LHCb
- $BR(B^{+,0} \to K^{+,0}\mu^+\mu^-), BR(B^{+,0} \to K^{*+,0}\mu^+\mu^-)$ LHCb + CMS + Belle
- $BR(B \to X_s\gamma)$ combination CLEO, Belle, Babar
- $BR(B \to X_s\mu^+\mu^-), BR(B \to X_se^+e^-)$ Babar
- $BR(B_s \to \mu^+\mu^-)$ Average LHCb, ATLAS, CMS
- $B^+ \to K^+\mu^+\mu^-(F_H, A_{FB}), B^+ \to K^{*+}\mu^+\mu^-(F_L, A_{FB})$ CMS
- **LFUV:** $R_{K^{*0}}, R_{K^+}, R_{K^{*+}}, R_{K_S}$ LHCb + Belle
- $P'^{\mu,e}_4, P'^{\mu,e}_5$ (and in combination **LFUV:** $Q_{4,5} = P'^{\mu}_{4,5} - P'^{e}_{4,5}$) Belle
- Radiative decays: $B^0 \to K^{*0}\gamma$ (A_I and $S_{K^*\gamma}$), $B^+ \to K^{*+}\gamma, B_s \to \phi\gamma$ PDG

Largest **tensions** with SM in some low-q^2 bins of P'^{μ}_5, in ~~R_K, R_{K^*}~~ and some bins of $BR(B_s \to \phi\mu\mu)$

$$\chi^2(C_i) = [O_{\text{exp}} - O_{\text{th}}(C_i^{NP})]_j \, [Cov^{-1}]_{jk} \, [O_{\text{exp}} - O_{\text{th}}(C_i^{NP})]_k$$

- $\mathbf{Cov} = \mathbf{Cov}^{\text{exp}} + \mathbf{Cov}^{\text{th}}$.
- Calculate Cov^{th}: correlated multigaussian scan over all nuisance parameters
- Cov^{th} depends on C_i^{NP}: Must check this dependence

For the Fit:

- Minimise $\chi^2 \to \chi^2_{\text{min}} = \chi^2(C_i^{NP\,0})$ (Best Fit Point $= C_i^{NP\,0}$)
- Confidence level regions: $\chi^2(C_i^{NP}) - \chi^2_{\text{min}} < \Delta\chi_{\sigma,n}$

Definition of Pull$_{SM}$

Pull$_{SM}$ tells you how much in a model defined by a set of Wilson coefficients the value of C_9 preferred by data $C_i^{NP\,0}$ is in tension with $C_9 = C_9^{SM}$.

Summary of dominant scenarios before and after 2022

$$\mathcal{C}^{\rm eff}_{9\mu} \to \mathcal{C}^{\rm eff}_{9\mu\, j} = \mathcal{C}^{\rm SM}_{9\mu\,{\rm pert}} + \mathcal{C}^{\rm NP}_{9\mu} + \mathcal{C}^{c\bar{c}\,B\to K^*}_{9\mu\, j} \qquad \mathcal{C}^{\rm NP}_{9\mu} = \mathcal{C}^{\mathbf{V}}_{9\mu} + \mathcal{C}^{\mathbf{U}}_{9}$$

Picture before December 2022:

Hypotheses	Param.	$\text{Pull}_{\rm SM}$
$\mathcal{C}^{\rm NP}_{9\mu} = -1.01$	1	**7.0**
$[\mathcal{C}^{\rm V}_{9\mu} = -\mathcal{C}^{\rm V}_{10\mu}, \mathcal{C}^{\rm U}_{9}]^* = [-0.34, -0.82]$	2	7.2
$[\mathcal{C}^{\rm NP}_{9\mu}, \mathcal{C}_{9'\mu} = -\mathcal{C}_{10'\mu}] = [-1.15, +0.17]$	2	7.1

With only 1 or 2 parameters one gets excellent fit to data. Scenario (*) moreover link with $b \to c\tau\nu$ anomaly and naturally generates LFU in C_9+imply large $b \to s\tau\tau$. Emerging RHC scenario.

After December 2022:

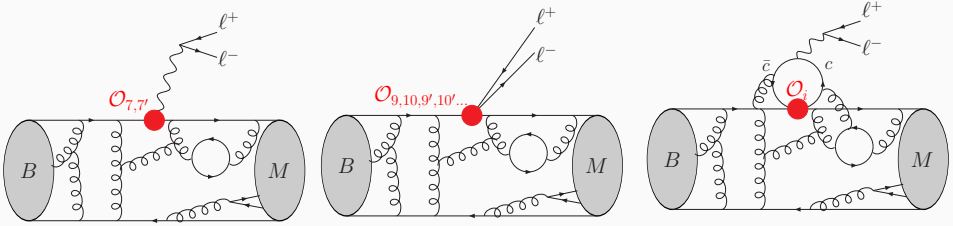
Hypotheses	Param.	$\text{Pull}_{\rm SM}$
$\mathcal{C}^{\rm NP}_{9\mu} = \mathcal{C}^{\rm NP}_{9e} = \mathcal{C}^{\rm U}_{9} = -1.2$	1	**5.8**
$[\mathcal{C}^{\rm V}_{9\mu} = -\mathcal{C}^{\rm V}_{10\mu}, \mathcal{C}^{\rm U}_{9}]^* = [-0.08, -1.10]$	2	5.6
$[\mathcal{C}^{\rm NP}_{9\mu}, \mathcal{C}_{9'\mu} = -\mathcal{C}_{10'\mu}] = [-0.78, +0.08]$	2	4.3

New Physics is now preferably Lepton Flavour Universal. Reduction of 2σ of significance, but still $> 5\sigma$.

We have been providing our SM predictions to new data and NOW we are updating our global fit results.

Two sources of hadronic uncertainties for exclusive decays

$$A(B \to M\ell\ell) = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* [(A_\mu + T_\mu)\bar{u}_\ell \gamma^\mu v_\ell + B_\mu \bar{u}_\ell \gamma^\mu \gamma_5 v_\ell]$$



Form factors (local) Form factors (local) Charm loop (non-local)

- Local contributions (more terms if NP in non-SM $\mathcal{C}_i$): form factors

$$\begin{aligned} A_\mu &= -\frac{2m_b q^\nu}{q^2} \mathcal{C}_7 \langle M|\bar{s}\sigma_{\mu\nu} P_R b|B\rangle + \mathcal{C}_9 \langle M|\bar{s}\gamma_\mu P_L b|B\rangle \\ B_\mu &= \mathcal{C}_{10} \langle M|\bar{s}\gamma_\mu P_L b|B\rangle \end{aligned}$$

- Non-local contributions (charm loops): hadronic contribs.

T_μ contributes like $\mathcal{O}_{7,9}$, but depends on q^2 and external states

What do we learn?

In summary...

- **Global Fits:** Consistently prefer a universal shift in $C_{9\mu}^{\text{NP}} = C_{9e}^{\text{NP}} = C_9^{\text{U}}$ ($C_9^{\text{NP}} \approx -1.2$).
- **The Core Question:**

  Is this New Physics or an EXTRA hypothetical constant, universal hadronic contribution mimicking New Physics?.

$\Rightarrow$ **Objective:** I will discuss **two different paths to discover New Physics** and at the same time **dismiss the relevance of a constant hadronic contribution or charming penguins** (beyond those included using LCSR).

$\hookrightarrow$ And Belle-II is a major player in both paths.

* **Confusion and overselling in the literature about ("charming penguins") ...**

The result of a search checking for
LATTICE charming penguins **computations/results** for
$B \to K^* \mu\mu$ in the literature is...

At the moment there is only a "promise" of a future attempt of a lattice computation that will need to be scrutinized IN DETAIL if ever finished. **But how one can claim that this explain B anomalies if there is no lattice computation to probe it??**

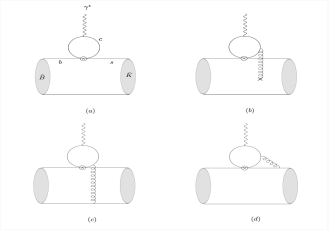
On the contrary,

a REAL computation of "charming penguins"

using LCSR with one soft gluon exchange EXISTS

Exclusive issues: LCSR REAL non-local power corrections

Non-local matrix elements $< B|TJ_\mu^{em} O_2|B >$:
with $\mathcal{O}_i$=$O_{1,2}$



Non-local charm contribution:

$$\mathcal{H}_\lambda^{B\to M} = -\frac{1}{16\pi^2}\left(\frac{q^2}{2M_B^2}\Delta C_9\,\mathcal{F}_\lambda^{B\to M} + \frac{m_b}{M_B}\Delta C_7\,\mathcal{F}_{\lambda,T}^{B\to M}\right) + 2\,Q_c\left(C_2 - \frac{C_1}{2N_C}\right)\tilde{\mathcal{V}}_\lambda^{B\to M} + \dots\,,$$

perturbative charm loop — part of "charming penguins"

First piece: it depends on local FFs, contains hard gluon exchange in QCDF.

Second piece: one soft gluon contribution emitted by the $c\bar{c}$ loop.

- first estimated by Khodjamirian with $\tilde{\mathcal{V}} \sim \mathcal{O}(10^{-4})$
- updated inputs + higher twist contribution for B-LCDA by Gubernari et al with $\tilde{\mathcal{V}} \sim \mathcal{O}(10^{-6})$

... the only existing 1st order computation of a "charming penguin" shows that it is irrelevant

$\mathcal{C}_9^{\mathrm{U}}$ Possible Origins?

Non-exclusive (and non-exhaustive) possibilities

- **1) LFU-NP: Direct NP contribution?** a Z' with vectorial lepton couplings

- **2) LFU-NP: Loop-effects with SM particles + NP $bs\tau\tau$ effective coupling**
  - τ-loops with or without connection with $R_{D^{(*)}}$.

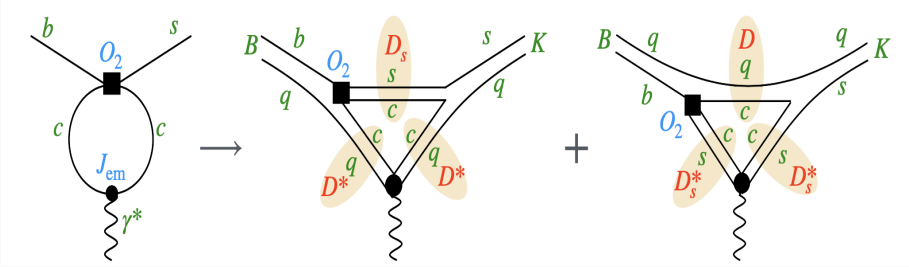
  [Capdevila, Crivellin, Descotes-Genon, Hofer, Matias, PRL'18, arxiv 1712.01919]
  [Crivellin, Greub, Muller, Saturnino, PRL'19, arxiv 1807.02068]

- **3) Contrived** constant unknown hadronic contribution ("charming penguins" or smgt else of hadronic origin).

A recent example of 3) an an example of a failed explanation: Triangle diagrams with D, D_s, D^*

Non-local matrix elements $< B|TJ_\mu^{em}O_2|B >$:

It was hypothesized by [L. Silvestrini et al.] that triangle diagrams could explain data due to possible large anomalous thresholds:

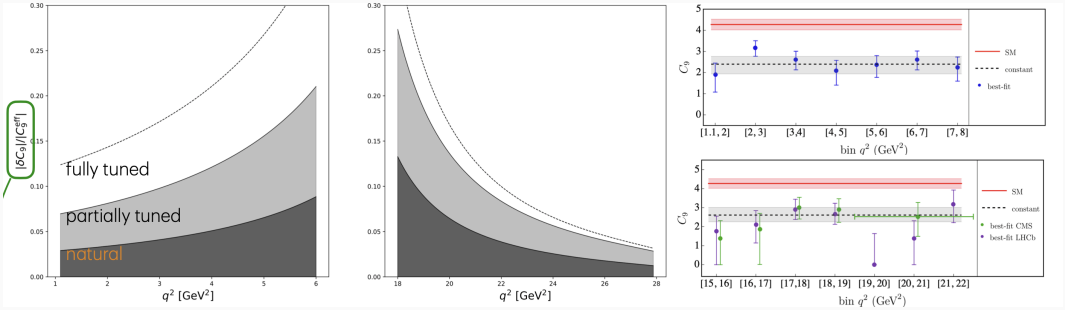


In [Ciuchini et al., 2212.10516] these diagrams were drawn with no computation at all and

In [Isidori et al., 2507.17824] they were computed using HHChPT in an over-conservative way.

Two important points why those triangle diagrams are NOT an explanation of B-anomalies

Point 1: Clash with Global fits that show **same deficit** at large and low q^2 while DD_s^* and D_sD^* rescattering computed by [Isidori, Polonsky, Tinari] show opposite sign (monopole dominant) and same(dipole) and global fits require $|\delta C_9/C_9| \simeq 0.25$:



Point 2: The structure of anomalous thresholds of the hadronic and partonic diagram are **exactly the same** [Mutke et al. 2406.14608, Hoferichter et al. 2604.01284]. Double counting?

Two paths to DISCOVER New Physics and DISCARD hadronic explanations

Path I: $b \to s\tau\tau$ processes

- Third generation fermions are natural portals to New Physics.
- Persistent hints of LFUV in FCCC observable $R(D^{(*)})$ governed by

$$b \to c\tau\nu$$

- Natural to expect LFUV in FCNC involving third generation leptons

$$b \to s\tau^+\tau^-$$

- Tensions in $b \to s\nu\bar{\nu}$ also recently emerged:

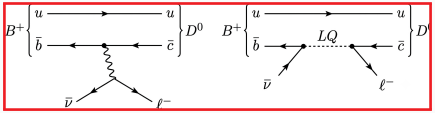
$$R^{\nu}_{K^{(*)}} = \frac{\mathcal{B}(B \to K^{(*)}\nu\bar{\nu})}{\mathcal{B}(B \to K^{(*)}\nu\bar{\nu})^{\mathrm{SM}}}$$

which at the moment are measured to be

$$R^{\nu,\mathrm{exp}}_{K} = 2.9 \pm 0.9\,, \qquad R^{\nu,\mathrm{exp}}_{K^*} < 2.7\ (90\,\%\ \mathrm{CL})\,.$$

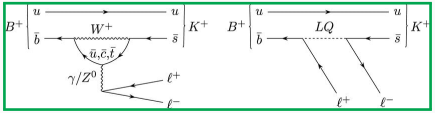
SMEFT link

SMEFT Link: Manifest $SU(2)_L$ gauge invariance correlates the left-handed $bc\tau\nu$ current to $b \to s\tau^+\tau^-$ transitions and $b \to s\nu\bar{\nu}$.



$$\mathcal{O}^{(1)}_{2333} = [\bar{Q}_2\gamma_\mu Q_3][\bar{L}_3\gamma^\mu L_3]$$
$$= [\bar{s}_L\gamma_\mu b_L][\bar{\tau}_L\gamma^\mu\tau_L] + [\bar{s}_L\gamma_\mu b_L][\bar{\nu}_\tau\gamma^\mu\nu_\tau] + \ldots$$

$$\mathcal{O}^{(3)}_{2333} = [\bar{Q}_2\gamma_\mu\sigma^I Q_3][\bar{L}_3\gamma^\mu\sigma^I L_3]$$
$$= 2V_{cs}[\bar{c}_L\gamma_\mu b_L][\bar{\tau}_L\gamma^\mu\nu_\tau] + [\bar{s}_L\gamma_\mu b_L][\bar{\tau}_L\gamma^\mu\tau_L] -$$
$$-[\bar{s}_L\gamma_\mu b_L][\bar{\nu}_\tau\gamma^\mu\nu_\tau] + \ldots$$



NP enhancements of $b \to s\tau^+\tau^-$ up to some **orders of magnitude** are predicted.

Experimental Status

Predicted enhancement consistent with the 90 % *CL* experimental limits.

Current limits:

$$\mathcal{B}\big(B_s \to \tau^+\tau^-\big)_{\rm exp} \leq 5.2 \times 10^{-3} \quad \text{LHCb'17}\,,$$
$$\mathcal{B}\big(B \to K\tau^+\tau^-\big)_{\rm exp} \leq 5.6 \times 10^{-4} \quad \text{Belle} + \text{Belle II'26}\,,$$
$$\mathcal{B}\big(B \to K^*\tau^+\tau^-\big)_{\rm exp} \leq 2.5 \times 10^{-4} \quad \text{LHCb'25}\,,$$
$$\mathcal{B}(B_s \to \phi\tau^+\tau^-)_{\rm exp} \leq 4.1 \times 10^{-4} \quad \text{LHCb'25}.$$

SM predictions are at the level

$$10^{-6} - 10^{-7}.$$

Therefore:

Room for large NP enhancements.

And Belle-II is/will be a key player in the measurement of the $b \to s\tau\tau$ processes

Problem: Experimental Challenges in $b \to s\tau^+\tau^-$

- Unlike e and μ modes, τ modes involve missing energy (neutrinos), making q^2 **reconstruction difficult**:
  - While Belle II **can discriminate** q^2 **regions** via tagging,
  - Hadronic environments (LHCb, CMS) **cannot** easily resolve resonances due to the difficult q^2 discrimination.
- **Our Strategy:** Instead of avoiding resonance regions via q^2 cuts, we include resonant contributions (specifically $\psi(2S)$) directly into the predictions for $b \to s\tau^+\tau^-$ with and without NP.

Treatment of resonances: Data-driven approach and inclusion of $\psi(2S)$ in the PS integral

- Include the nonlocal hadronic amplitudes extracted from the fit to $B \to K^{(*)}\mu^+\mu^-$ data from LHCb analysis. [R. Aaij et al. (LHCb), 2603.12477] [R. Aaij et al. (LHCb), JHEP 09, 026]

  For $B^+ \to K^+\tau^+\tau^-$: $\mathcal{C}_{9\tau}^{\rm eff}(q^2) = \mathcal{C}_{9\tau} + Y^0_{c\bar{c}}(q_0^2) + \Delta Y^{\rm 1p}_{c\bar{c}}(q^2) + \Delta Y^{\rm 2p}_{c\bar{c}}(q^2)$

- Parametrization of the resonances: Dispersion relation method

  $\Delta Y^{\rm 1p}_{c\bar{c}}(q^2) = \sum_j \eta_j e^{i\delta_j} \frac{q^2-q_0^2}{m_j^2-q_0^2} A_j^{\rm res}(q^2)\,, \qquad A_j^{\rm res}(q^2) = \frac{m_j \Gamma_{0j}}{m_j^2 - q^2 - i\, m_j \Gamma_j(q^2)}$

  η_j and δ_j are the magnitude and phase of each resonance w.r.t. the local contribution.

$$\mathcal{B}(B^+ \to K^+\tau^+\tau^-)_{\rm SM} = 1.86^{+0.17}_{-0.16} \times 10^{-6} \quad [4m_\tau^2, q^2_{\rm max,K}]$$

$$\mathcal{B}(B \to K^*\tau^+\tau^-)_{\rm SM} = 1.50^{+0.23}_{-0.22} \times 10^{-6} \quad [4m_\tau^2, q^2_{\rm max,K^*}]$$

$q^2_{\rm min}$	$\mathcal{B}(B^+ \to K^+\tau^+\tau^-) \times 10^7$	$\mathcal{B}(B^0 \to K^{*0}\tau^+\tau^-) \times 10^7$
14.18	$1.54\,^{+0.16}_{-0.13}$	$1.36\,^{+0.61}_{-0.40}$
15	$1.37\,^{+0.14}_{-0.12}$	$1.16\,^{+0.54}_{-0.36}$

Including resonances ($\psi(2S)$) enhances SM prediction by an order of magnitude

SMEFT Framework: Matching relations

Dimension-six SMEFT:

$$\mathcal{L}^{(6)}_{\text{SMEFT}} \supset \sum_i \frac{\tilde{C}_i}{\Lambda^2}\mathcal{O}_i = \sum_i C_i \mathcal{O}_i$$

Relevant operators are the four-fermion operators:

$$\begin{aligned}
\left[\mathcal{O}^{(1)}_{\ell q}\right]_{2333} &= \left[\bar{Q}_2\gamma_\mu Q_3\right]\left[\bar{L}_3\gamma^\mu L_3\right], \\
\left[\mathcal{O}^{(3)}_{\ell q}\right]_{2333} &= \left[\bar{Q}_2\gamma_\mu \sigma^I Q_3\right]\left[\bar{L}_3\gamma^\mu \sigma^I L_3\right], \\
\left[\mathcal{O}^{(3)}_{\ell q}\right]_{3333} &= \left[\bar{Q}_3\gamma_\mu \sigma^I Q_3\right]\left[\bar{L}_3\gamma^\mu \sigma^I L_3\right], \\
\left[\mathcal{O}_{\ell d}\right]_{2333} &= \left[\bar{d}_{2R}\gamma_\mu d_{3R}\right]\left[\bar{L}_3\gamma^\mu L_3\right],
\end{aligned}$$

Q and L denote the quark and lepton $SU(2)_L$ doublet and d the down quark $SU(2)_L$ singlet.

Only taus so we simplify notation:

$$\left[C^{(1)}_{\ell q}\right]_{2333} \equiv C^{(1)}_{23}\,, \left[C^{(3)}_{\ell q}\right]_{2333} \equiv C^{(3)}_{23}\,, \left[C^{(3)}_{\ell q}\right]_{3333} \equiv C^{(3)}_{33}\,, \left[C_{\ell d}\right]_{2333} \equiv C^{\ell d}_{23}\,.$$

Matching Conditions

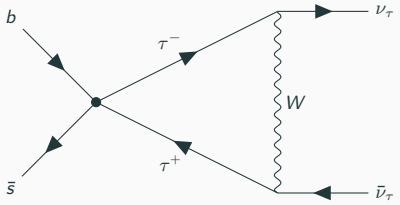
$$C^{\tau}_{9(10)} = C^{\mathrm{SM}}_{9(10)} + (-)\frac{1}{2\mathcal{N}}\left(C^{(1)}_{23} + C^{(3)}_{23}\right), \qquad C^{\tau}_{9'(10')} = +(-)\frac{1}{2\mathcal{N}}C^{\ell d}_{23},$$

$$C^{\nu_\tau}_{L} = C^{\mathrm{SM}}_{L} + C^{\nu_\tau\,\mathrm{W}}_{L} + \frac{1}{2\mathcal{N}}\left(C^{(1)}_{23} - C^{(3)}_{23}\right), \qquad C^{\nu_\tau}_{R} = \frac{1}{2\mathcal{N}}C^{\ell d}_{23},$$

$$\text{with } \epsilon_L = -\tfrac{1}{\sqrt{2}G_F}\left(\tfrac{V_{cs}}{V_{cb}}C^{(3)}_{23} + C^{(3)}_{33}\right) \text{ (for } b \to c\tau\nu\text{) and}$$

$$\mathcal{N} \propto \alpha G_F V_{tb} V^*_{ts}.$$

We include a W vertex correction for $b \to s\nu\bar{\nu}$

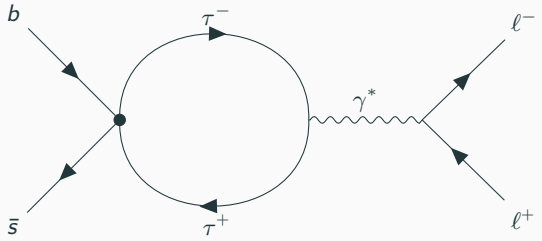


$$C^{\nu_\tau\,\mathrm{W}}_{L} = \frac{3}{4\sqrt{2}G_F \sin^2\theta_W}\frac{1}{V_{tb}V^*_{ts}}\log\left(\frac{m^2_W}{\Lambda^2}\right)\left(C^{(1)}_{23} + C^{(3)}_{23}\right).$$

Large Wilson coefficient $C_9^{\tau\,\mathrm{NP}}$ has important loop effects relevant for $b \to s\ell^+\ell^-$ transitions with light leptons.

$$C_9^{\mathrm{U}} = -\frac{\alpha}{3\pi} \log\left(\frac{\Lambda^2}{\mu_b^2}\right) C_9^{\tau\,\mathrm{NP}},$$

where we identify $C_9^{\mathrm{U}} \equiv C_9^{\mu\,\mathrm{NP}} = C_9^{e\,\mathrm{NP}}$.



The tau-loop off-shell photon penguin with the photon coupling to $\ell^+\ell^-$, generating a mixing of $\mathcal{O}_9^{\tau}$ into $\mathcal{O}_9^{\mathrm{U}}$.

The space for an unknown hadronic contribution gets squeezed by this NP tau loop.

Phenomenology: Four scenarios in presence of NP

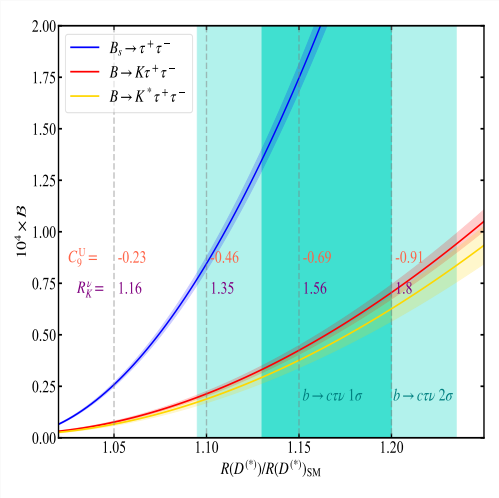
Scenarios

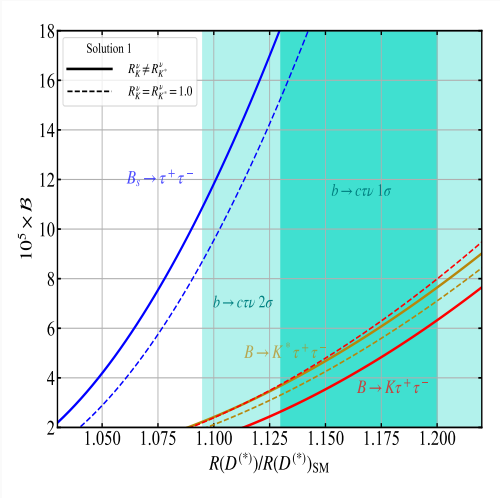
- Scenario I: $C_{23}^{(1)} = C_{23}^{(3)}$ and $|C_{23}^{(3)} V_{cs}| \gg |C_{33}^{(3)} V_{cb}|$
  - $B \to K^{(*)} \nu\bar{\nu}$ **NOT generated at tree level, while** $R_K^\nu = R_{K^*}^\nu \neq 1$ generated at loop level.
  - Enhances all $b \to s\tau\tau$ modes by $\mathcal{O}(10^2 - 10^3)$ with respect to SM.
- Scenario II: $C_{23}^{(1)} \neq C_{23}^{(3)}$ and $|C_{23}^{(3)} V_{cs}| \gg |C_{33}^{(3)} V_{cb}|$
  - $B \to K^{(*)} \nu\bar{\nu}$ **is generated at tree level.**
  - Still **LH currents assumption** $R_K^\nu = R_{K^*}^\nu$. Two solutions.
- Scenario III: $C_{23}^{(1)} \neq C_{23}^{(3)}$ with $|C_{23}^{(3)} V_{cs}| \gg |C_{33}^{(3)} V_{cb}|$ and $C_{23}^{\ell d} \neq 0$
  - $B \to K^{(*)} \nu\bar{\nu}$ **is generated at tree level.**
  - **RH currents** enter and $R_K^\nu \neq R_{K^*}^\nu$ and $C_{9'(10')}^\tau$ is generated.
  - $C_{23}^{ld} \neq 0$ leads to a non-zero $C_R^{\nu\tau}$ also. Four solutions.
- Scenario IV: $|C_{33}^{(3)}| \gg |C_{23}^{(3)}|$. Also generates a tree-level $B \to K^{(*)} \nu\bar{\nu}$. Third-generation dominance if there is a **hierarchical structure**, $C_{33}^{(3)} \gg C_{23}^{(3)} \to$ additional contribution via CKM rotations

$$\mathcal{L}_{\text{eff}}^{\text{NP}} \supset 2 C_{33}^{(3)} V_{cb} (\bar{c}_L \gamma_\mu b_L)(\bar{\tau}_L \gamma^\mu \nu_{L\tau}) \,.$$

  $\Rightarrow$ the one-to-one correlation between $b \to c\tau\nu$, $b \to s\tau^+\tau^-$ and $b \to s\nu\bar{\nu}$ is broken.

Scenario I: $\mathcal{B} - R(D^{(*)})/R(D^{(*)})_{\rm SM}$ and Scenario III: $\mathcal{B} - R(D^{(*)})/R(D^{(*)})_{\rm SM}$: Solution 1





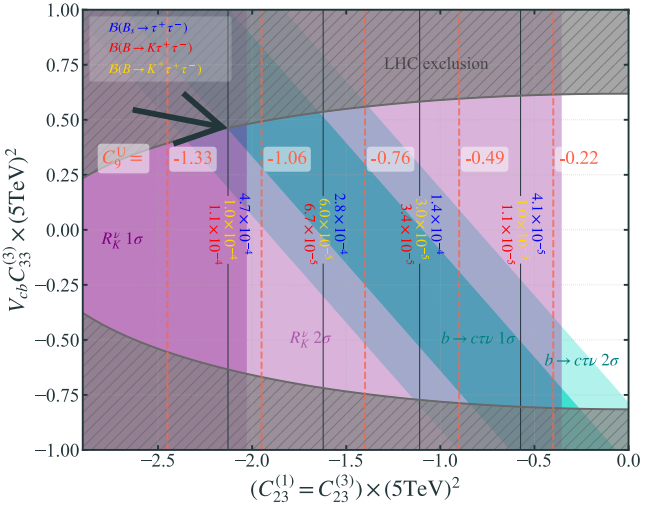
Scenario I: $C_{23}^{(1)} = C_{23}^{(3)} \gg C_{33}^{(1,3)} V_{cb}$

No tree-level $B \to K^{(*)}\nu\bar{\nu}$ only loop level.

Scenario III: $C_{23}^{(1)} \neq C_{23}^{(3)}$ and $C_{23}^{\ell d} \neq 0$

$B \to K^{(*)}\nu\bar{\nu}$ is generated at tree level & $R_K^\nu \neq R_{K^*}^\nu$

Scenario IV: Preferred/excluded region in $\left(C_{23}^{(1)} = C_{23}^{(3)}\right) - V_{cb} C_{33}^{(3)}$



Large positive allowed values of $C_{33}^{(3)}$ could lead to an alternative to the $C_{23}^{(1)} \neq C_{23}^{(3)}$ condition for enhancing $B \to K^{(*)}\nu\bar{\nu}$

Path II: Inclusive versus exclusive, to discover NP and a second killer of charming penguins

Exclusive ($B \to K^{(*)}\ell\ell$)

- **Low q^2:** SCET-II framework; power corrections involve non-local form factors ("charming penguins"). Inputs: Form Factors, LCDA.
- **High q^2:** $1/q^2$ expansion (OPE in q^2): only integrated spectrum; local power corrections only. Inputs: Form Factors, HQET matrix elements.

-Form Factors:

- LCSR (B-meson DA or light-meson DA)
- Lattice QCD.

Inclusive ($B \to X_s\ell\ell$)

- **Low-q^2**: OPE, Leading uncertainties: **Resolved Photons:** Subleading power corrections at low q^2 estimated at $\sim 5\,\%$.
- **High-q^2**: OPE breaks down but the ratio $b \to s\ell\ell / b \to u\ell\nu$ is well behaved. Leading uncertainties: $\mathcal{O}(1/m_b^3)$ HQET ME (WA).

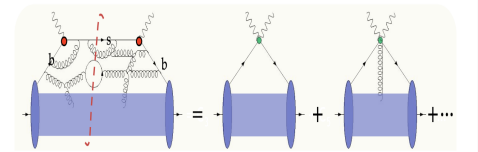
- **Global QPE:** Described by HQE.
- **Kruger-Sehgal (KS) Mechanism:** Factorizable -charm loops determined exactly from $e^+e^- \to$ hadrons data used to incorporate resonances.

Hipothesis: Let's introduce for latter use an extra hadronic contribution with unknown origin or unexpected huge charming penguins $C_{9,\mathrm{extra}}$ that would affect only the exclusive.

Inclusive

OPE free of uncertainties up to power corrections

$$\Gamma[B \to X_s \ell\ell] = \Gamma[b \to s\ell\ell] + \mathcal{O}(\frac{\Lambda^2_{QCD}}{m_b^2}, \frac{\Lambda^2_{QCD}}{m_c^2})$$



The series is known up to and including $\alpha_s^3 \kappa^3$ with $\kappa = \alpha_{em}/\alpha_s$

However due to the expansion parameter (from p^2_{Xs})

$$\frac{\Lambda_{QCD}}{m_b - \sqrt{q^2}}$$

OPE breaks down at high-q^2 generating large power corrections at the endpoint of the spectrum

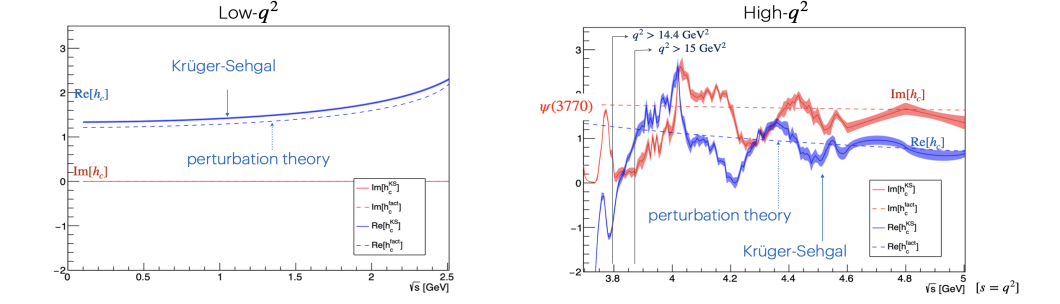
...improved by normalising to $\bar{B}^0 \to X_u \ell\nu$ (same endpoint pathology) & where WA ME cancel in

$$\mathscr{R}(q_0^2) = \frac{\int_{q_0^2}^{m_b^2} dq^2 \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{dq^2}}{\int_{q_0^2}^{m_b^2} dq^2 \frac{d\Gamma(\bar{B}^0 \to X_u \ell \nu)}{dq^2}}$$

Inclusive resonances

They are incorporated using $e^+e^- \to$ hadrons data (BESII, Babar, ALEPH...)

...using Krüger-Sehgal mechanism that works nicely for inclusive.



Finally, non-resonant color-octet effects:

- at high-q^2 can be calculated in PT and scale as Λ^2_{QCD}/q^2 [Buchalla, Isidori, Rey] and
- at low-q^2 are non-local and can be treated using SCET [Hurth, Turczyk et al], power corrections are the so called resolved contributions that are estimated as an irreducible uncertainty of 5 %.

Proposed Observables: The $\mathcal{F}$ Ratios to test unknown rescattering effects

Can we distinguish a NP contribution $\mathcal{C}_9^{\rm NP}$ from an unknown hadronic effect $\mathbf{C_{9,extra}}$?

Ratios of exclusive to inclusive branching fractions designed to cancel normalization uncertainties and be sensitive to $\mathcal{C}_{9,\rm extra}$ dependence:

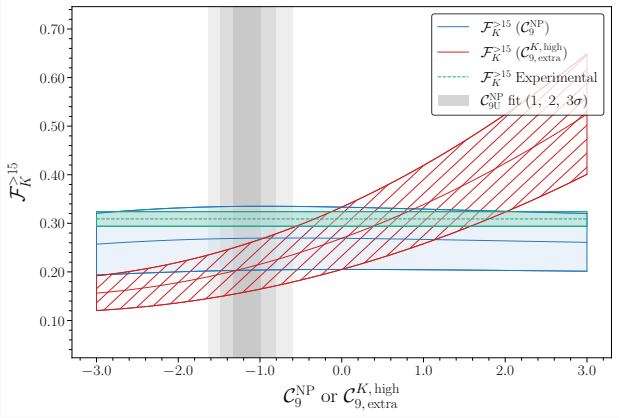
Definitions

$$\mathcal{F}_{K^{(*)}}^{[1.1,6]} = \frac{\mathcal{B}(B \to K^{(*)}\mu\mu)_{[1.1,6]}}{\mathcal{B}(B \to X_s\ell\ell)_{[1,6]}} \quad \text{and} \quad \mathcal{F}_{K^{(*)}}^{>15} = \frac{\mathcal{B}(B \to K^{(*)}\mu\mu)_{>15}}{\mathcal{B}(B \to X_s\ell\ell)_{>15}}$$

The Logic: Exclusive modes are sensitive to hypothetical hadronic shifts $C_{9,\rm extra}$; inclusive modes are not. These ratios have **reduced sensitivity to universal NP** due to partial numerator-denominator cancellation and **enhanced sensitivity to $C_{9,extra}$ via exclusive)**.

Belle-II can play a leading role measuring the inclusive and these $\mathcal{F}$ ratios

Experimental Results for $\mathcal{F}$ Ratios



Insight: High q^2 ratios show a slight preference for NP (1.9σ for $F_K^{>15}$).

Scenario ($C_9^{\rm NP}$, $C_9^{\rm extra}$)	$\mathcal{F}_K^{>15}$	$\mathcal{F}_{K*}^{>15}$	$\mathcal{F}_K^{[1.1,6]}$	$\mathcal{F}_{K*}^{[1.1,6]}$
SM (0, 0)	0.270	0.64	0.099	0.111
NP (-1.16, 0)	0.271	0.65	0.0928	0.113
Hadronic (0, -1.16)	0.210	0.50	0.0764	0.093
Experiment	**0.309**	**0.629**	**0.084**	**0.110**

Consistency Test at high-q^2: Sum of Exclusive vs. OPE (Inclusive)

At high q^2, the inclusive rate is essentially saturated by summing $K, K^*, (K\pi)_S, K\pi\pi$ ($< 3\,\%$):

- **Theoretical Consistency:** How the OPE inclusive prediction with Σ of exclusive modes compare?. This requires to add also the dependence of this residual branching ratio on $C_9^{\rm NP}$ and $\mathcal{C}_{9,\rm extra}$

$$10^7 \times \mathcal{B}_{\rm rest>15} = 0.547 + 0.124\,(\mathcal{C}_9^{\rm NP} + \mathcal{C}_{9,\rm extra}) + 0.015\,(\mathcal{C}_9^{\rm NP} + \mathcal{C}_{9,\rm extra})^2\,.$$

  This amounts to 13 % of the total sum of exclusive channels and is more uncertain.

	Region (q^2)	α	β	γ
$10^7 \times \mathcal{B}(B \to K\mu\mu)$	> 15	1.105	0.252	0.033
$10^7 \times \mathcal{B}(B \to K^*\mu\mu)$	> 15	2.640	0.600	0.082
$10^7 \times \mathcal{B}(B \to K\pi\mu\mu)$	> 15	0.547	0.124	0.015
$10^7 \times \mathcal{B}(B \to X_s\mu\mu)$ (sum excl)	> 15	4.292	0.976	0.130
$10^7 \times \mathcal{B}(B \to X_s\mu\mu)$ (incl)	> 15	4.10	0.96	0.14

$$10^7 \mathcal{B}_{\sum \rm exc} \simeq 4.29 + 0.98 C_9^{\rm NP} + 0.13 (C_9^{\rm NP})^2 \quad \text{vs.} \quad 10^7 \mathcal{B}_{\rm incl} \simeq 4.10 + 0.96 C_9^{\rm NP} + 0.14 (C_9^{\rm NP})^2$$

- **Result:** Remarkable agreement between OPE and the Σ of exclusives in NP.

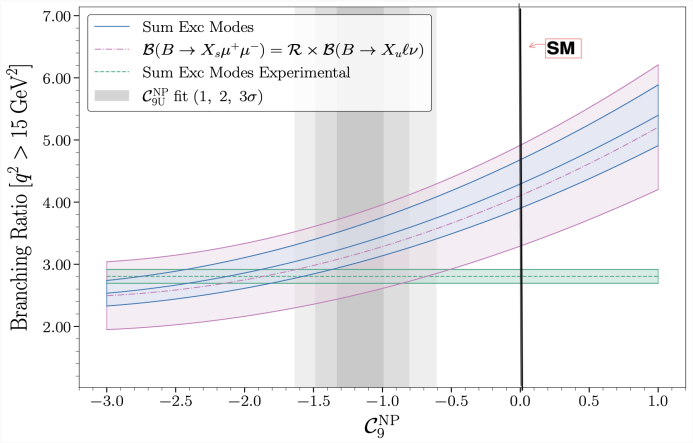


Figura 1: $B \to X_s\mu\mu$ branching ratio for $q^2 > 15\ \mathrm{GeV}^2$ calculated using the OPE ($\mathcal{R} \times \mathcal{B}_{u\ell\nu}$)

The inclusive isospin-averaged **high-q^2** branching ratio as sum of exclusive can be written as:

$$10^7 \mathcal{B}(B \to X_s\mu\mu)_{\rm exp} = \tilde{\mathcal{B}}(B \to K\mu\mu) + \tilde{\mathcal{B}}(B \to K^*\mu\mu) + \tilde{\mathcal{B}}(B \to (K\pi)_S\mu\mu) + \tilde{\mathcal{B}}(B \to K\pi\pi\mu\mu)$$
$$= (2.80 \pm 0.12) \quad \text{vs} \quad \tilde{\mathcal{B}}[>15]^{\rm inc}_{\rm SM} = (4.10 \pm \mathbf{0.81})(1.6\sigma) \,\text{and}\, \tilde{\mathcal{B}}[>15]^{\rm exc}_{\rm SM} = (4.29 \pm \mathbf{0.39})(3.7\sigma)$$

Disfavoring the Hadronic Scenario: A different type of comparison

Using the branching ratios of exclusive and inclusive we can impose two conditions on the constant universal $C_{9,\text{extra}}$:

- **First condition**:

$$(\mathcal{B}_{\text{SM}}^{\sum \text{exc}} - \mathcal{B}^{\sum \text{exc,exp}}) = 0,$$

  $C_{9,\text{extra}}$ has to be in agreement with data. If we assume no NP $\Rightarrow$ $C_{9,\text{extra}} = -1.16$

- **Second condition**: $C_{9,\text{extra}}$ has to be adjusted to fulfill:

$$(\mathcal{B}_{\text{SM}}^{\sum \text{exc}} - \mathcal{B}_{\text{SM}}^{\text{inc}}) = 0.$$

  If we impose that both computations must agree one finds at 68 % CL

$$C_{9,\text{extra}} = -0.20^{+0.90}_{-1.10}.$$

  If we assume a reduction of 50 % in this error thanks to improvement on $\mathcal{B}(B \to X_u \ell \nu)$ measurement and the central value remains untouched one finds at 68 % CL

$$C_{9,\text{extra}} = -0.20^{+0.59}_{-0.62}.$$

compatible with zero and in tension with the value required to explain anomalies.

A NEW test using C_9 dependence with two new observables ($R^{\text{inc},\sum \text{exc}}$) have been proposed [2605.06567]

Future Projections: Belle II

Inclusive Measurement: Dominant uncertainty of $\mathcal{F}$ comes from the inclusive BUT:

- at **low-q^2** Inclusive (stat. limited) at present (400 fb^{-1}) end of year (1 ab^{-1}) uncertainties

Parametric Inputs: Improved determinations of $|V_{ub}|$ and vacuum annihilation matrix elements from Lattice QCD.

Belle II improved measurement of $\mathcal{B}(B \to X_u \ell \nu)$ will strongly impact on inclusive uncertainty @high-q^2.

Outcome: Uncertainties at **high-q^2** $\mathcal{F}$ ratios expected to improve by $> 50\,\%$ in the near future.

A fast taste on some non-leptonic new ideas:
$\bar{B}_{d,s} \to K^{*0}\bar{K}^{*0}$ and $\bar{B}_{d,s} \to \bar{K}^{*0}K^{0}$, $\bar{K}^{0}K^{*0}$

Analogy/parallelism between semi and non-leptonic decays

JHEP 04 (2021) 066,

In $b \to s\ell\ell$ we build observables with reduced hadronic sensitivity optimized using HQS:

- Absence of LO soft FF in optimized observables (P_5') or ratios cancelling hadronic sensitivity in SM.
- LFUV ratios of 2nd gen. lepton (μ^-) vs 1st gen. lepton (e^-)

Lepton Flavour Universality in SM

In **non-leptonic B decays** we build observables optimizing them by reducing sensitivity to IR WA and HSS:

- Absence of LO IR divergences in longitudinal amplitudes
- Ratios comparing 2nd gen. quarks (s) vs 1st gen. quarks (d)

U-spin symmetry

Broken symmetries, but corrections easier in LFUV (QED) than U-spin (QCD)

The L-observable for $B_{d,s} \to VV$ decays

We propose in general for $\bar{B}_Q \to VV$ with $Q = d, s$ and $b \to q$:

$$L_{V_1V_2} = \frac{\mathcal{B}_L(\bar{B}_s \to VV)}{\mathcal{B}_L(\bar{B}_d \to VV)} = \frac{\mathcal{B}_{b\to s}}{\mathcal{B}_{b\to d}} \frac{g_{b\to d} f_L^{b\to s}}{g_{b\to s} f_L^{b\to d}} = \frac{|A_0^s|^2 + |\bar{A}_0^s|^2}{|A_0^d|^2 + |\bar{A}_0^d|^2},$$

- Reduced sensitivity to IR divergences and FFs SU(3) related.
- Constructed upon:
  1. Branching ratios $\mathcal{B}_{b\to s}$
  2. Phase space factor $g_{b\to q}$
  3. Polarization fraction $f_L^{b\to q}$

For the particular case $\bar{B}_Q \to K^{*0}\bar{K}^{*0}$:

$$L_{K^*\bar{K}^*} = \frac{\mathcal{B}_{B_s\to K^{*0}\bar{K}^{*0}}}{\mathcal{B}_{B_d\to K^{*0}\bar{K}^{*0}}} \frac{g_{b\to d} f_L^{B_s}}{g_{b\to s} f_L^{B_d}} = \frac{|A_0^s|^2 + |\bar{A}_0^s|^2}{|A_0^d|^2 + |\bar{A}_0^d|^2},$$

Comparing theory with experiment

$$L^{\rm exp}_{K^*\bar{K}^*} = \mathbf{4.43 \pm 0.92}$$

	B_s Form Factor	B_d Form Factor	ρ_1^2	Observable
This work	$B_s \to K^*$	$B_d \to K^*$		$L_{K^*\bar{K}^*}$
	0.309 ± 0.066 LCSR	0.361 ± 0.064 LCSR	0.78	$18.34^{+7.47}_{-5.83}$ (2.3σ)
	0.356 ± 0.020 LCSR+f.w.e.+Lat.	0.350 ± 0.028 LCSR+f.w.e.+Lat.	1.10	$26.08^{+5.70}_{-4.72}$ (4.4σ)
	0.335 ± 0.039 Lattice	0.351 ± 0.074 Lattice	0.97	$22.88^{+15.35}_{-8.46}$ (2.2σ)

The L-observable: model independent interpretation

$Q^p_{1s,2s}$ are the LH current-current operators, $Q_{3s\ldots6s}$ and $Q_{7s\ldots10s}$ are QCD and EW penguin op., and $Q_{7\gamma s}$ and Q_{8gs} are EM and chromomagnetic dipole op for $b \to s$ and (same for $b \to d$)

$$\Rightarrow \mathbf{Q^c_{1s}} = (\bar{c}b)_{V-A}(\bar{s}c)_{V-A}\,, \qquad Q_{7s} = (\bar{s}b)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}q)_{V+A}\,,$$

$$Q^p_{2s} = (\bar{p}_i b_j)_{V-A}(\bar{s}_j p_i)_{V-A}\,, \qquad Q_{8s} = (\bar{s}_i b_j)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}_j q_i)_{V+A}\,,$$

$$Q_{3s} = (\bar{s}b)_{V-A}\sum_q (\bar{q}q)_{V-A}\,, \qquad Q_{9s} = (\bar{s}b)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}q)_{V-A}\,,$$

$$\Rightarrow \mathbf{Q_{4s}} = (\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V-A}\,, \qquad Q_{10s} = (\bar{s}_i b_j)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}_j q_i)_{V-A}\,,$$

$$Q_{5s} = (\bar{s}b)_{V-A}\sum_q (\bar{q}q)_{V+A}\,, \qquad Q_{7\gamma s} = \frac{-e}{8\pi^2}\, m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b\,,$$

$$Q_{6s} = (\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V+A}\,, \qquad \Rightarrow \mathbf{Q_{8gs}} = \frac{-g_s}{8\pi^2}\, m_b\, \bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b\,,$$

a summation over $q = u, d, s, c, b$ is implied.

Possible explanations: $\mathcal{C}^{\rm NP}_{4s} \sim +0.019$ and $\mathcal{C}^{\rm NP}_{4d} \sim -0.019$ or $\mathcal{C}^{\rm NP}_{8gs} \sim -0.35$ and $\mathcal{C}^{\rm NP}_{8gd} \sim +0.35$ while in SM $\mathcal{C}^{\rm SM}_4(4.2\,\mathrm{GeV}) = -0.036$ and $\mathcal{C}^{\rm SM}_{8g}(4.2\,\mathrm{GeV}) = -0.15$ but still exploring other possibilities...

A promising mechanism in $B \to PV$ with $P = K^0$, $V = K^{*0}$

A strong signal in optimised observables $B \to VP, PV$

Two types depending on the meson that collects the spectator quark:

- $M_1 = \bar{K}^{*0}$ case. We will denote this observable $\tilde{L}_{K^*}$:

$$\tilde{L}_{K^*} = \frac{1}{\rho(m_{K^0}, m_{K^*})} \frac{\mathcal{B}(\bar{B}_d \to \bar{K}^{*0} K^0)}{\mathcal{B}(\bar{B}_s \to K^{*0} \bar{K}^0) + \mathcal{B}(\bar{B}_s \to K^0 \bar{K}^{*0})},$$

The SM prediction (lattice+LCSR) is:

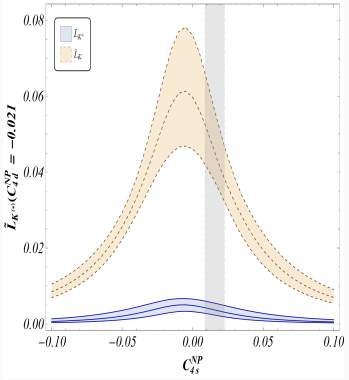
$$\tilde{L}_{K^*}^{\rm SM} = 0.021^{+0.005}_{-0.004} \quad (\downarrow \mathbf{\%10})$$

- $M_1 = K^0$ case: We will denote this observable $\tilde{L}_K$:

$$\tilde{L}_K = \frac{1}{\rho(m_{K^0}, m_{K^*})} \frac{\mathcal{B}(\bar{B}_d \to \bar{K}^0 K^{*0})}{\mathcal{B}(\bar{B}_s \to K^{*0} \bar{K}^0) + \mathcal{B}(\bar{B}_s \to K^0 \bar{K}^{*0})},$$

The SM prediction (lattice+LCSR) is:

$$\tilde{L}_K^{\rm SM} = 0.025^{+0.005}_{-0.003} \quad (\uparrow \mathbf{x2})$$



Strong signal: From a difference of 20 % in SM to 2000 % for the NP value that explains $L_{K^*K^*}$

Here Belle-II can only help in the numerator.

Conclusions

- Two main paths **to discover New Physics** and rule out hadronic explanations of the B-anomalies

Path I: Measurement of $b \to s\tau\tau$ processes, that imply NP per se if sizeable deviate from SM

- but also imply NP in $b \to s\ell\ell$ (with $\ell = e, \mu$) via a tau-loop
- rule out hadronic pollution (charming penguins, etc).
- and exhibit interesting correlations between $b \to s\tau\tau$, $b \to c\tau\nu$ and $b \to s\nu\bar{\nu}$ processes

Path II: Comparison of Inclusive versus Exclusive measurements via:

- new $\mathcal{F}$ observables exploiting their different hadronic sensitivity
- $C_{9,extra}$ affecting only inclusive is constrained by data but also by the sum of exclusives should be equal to the inclusive.
- also the C_9-dependence offer new observables $R^{\rm inc}$ and $R^{\sum \rm exc}$

Belle-II is a major player by measuring $b \to s\tau\tau$, inclusive mode and the normalization $B \to X_u \ell\nu$.

- Non-leptonic decays can also offer a strong signal from U-spin observables like $L_{K^*K^*}$ but also with new promising channels mixing Vector and pseudoscalars: $\tilde{L}_K$ and $\tilde{L}_{K^*}$.

Belle-II can provide precise information on the $b \to d$ U-spin partner of the L observables,

BACK-UP Slides

Setup

- Likelihood taking into account experimental and theoretical uncertainties and correlations in Gaussian approximation

[Algueró, Capdevila, Crivellin, SDG, Masjuan, Matias, Novoa-Brunet, Virto]

$$\text{We fit } C_i = C_i^{\rm SM} + C_i^{\rm NP}$$

Two statistical quantities of interest to asses a NP scenario/hypothesis:

- p-value of a given hypothesis: $\chi^2_{\rm min}$ considering N_{dof} (in %)

goodness of fit: does the hypothesis give an overall good fit ?

and if not, can we exclude it ?

- $\text{Pull}_{\rm SM}$: $\chi^2(\mathcal{C}_i = 0) - \chi^2_{min}$ considering N_{dof} (in σ units)

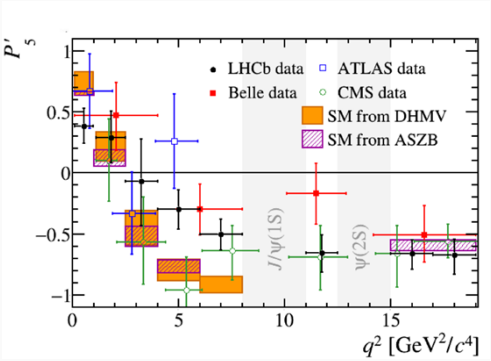
metrology: how well does the hypothesis solve SM deviations ?

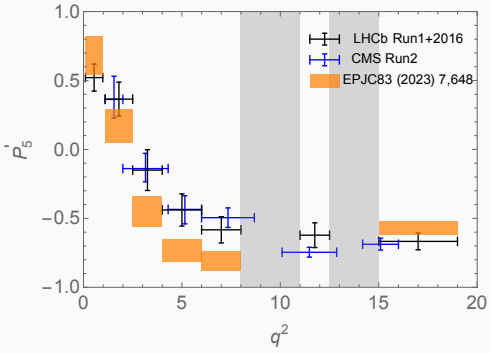
$b \to s\,\mu^+\mu^-$ anomalies: 2024 Picture $\to$ P_5' anomaly reinforced

- Angular observable P_5' in $B \to K^*\mu^+\mu^-$ (two anomaly bins $\sim 2\,\sigma$ each)
  - Most tested anomaly, now including charged channel: $B^+ \to K^{*+}\mu^+\mu^-$.
  - Latest LHCb update increased significantly the coherence w.r.t. other observables.

LHCb, Belle, ATLAS measurements deviate from Standard model (SM) predictions coherently, except for CMS.... till now:

- New CMS result in perfect agreement with LHCb and now deviates from SM.





Treatment of resonances: Data-driven approach

Accurate prediction:

1. Compute the local contributions in the standard way.
2. Include the nonlocal hadronic amplitudes extracted from the fit to $B \to K^{(*)}\mu^+\mu^-$ data from LHCb analysis. [R. Aaij et al. (LHCb), 2603.12477] [R. Aaij et al. (LHCb), JHEP 09, 026]

   For $B^+ \to K^+\tau^+\tau^-$: $\mathcal{C}_{9\tau}^{\rm eff}(q^2) = \mathcal{C}_{9\tau} + Y^0_{c\bar{c}}(q_0^2) + \Delta Y^{1\rm p}_{c\bar{c}}(q^2) + \Delta Y^{2\rm p}_{c\bar{c}}(q^2)$
3. Parametrization of the resonances: Dispersion relation method

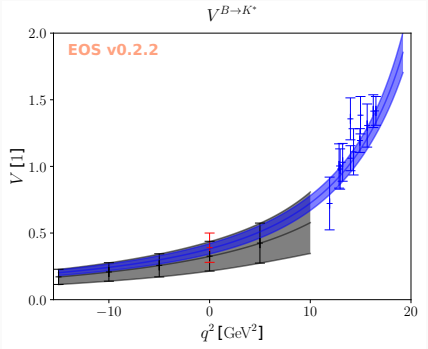
   $\Delta Y^{1\rm p}_{c\bar{c}}(q^2) = \sum_j \eta_j e^{i\delta_j} \frac{q^2-q_0^2}{m_j^2-q_0^2} A_j^{\rm res}(q^2)\,, \qquad A_j^{\rm res}(q^2) = \frac{m_j \Gamma_{0j}}{m_j^2 - q^2 - i\, m_j \Gamma_j(q^2)}$

   η_j and δ_j are the magnitude and phase of each resonance w.r.t. the local contribution.
4. Integrate the spectra over the full kinematically allowed range.

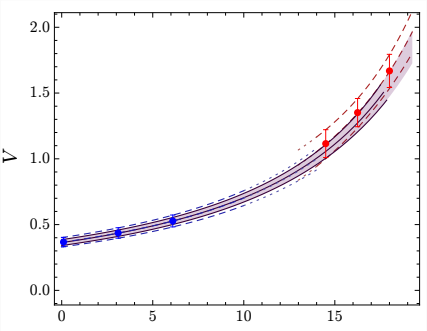
Hadronic uncertainties: form factors

3 form factors for K, 7 form factors for K^* and ϕ

- low recoil (high-q^2): lattice QCD [Horgan, Liu, Meinel, Wingate; HPQCD collab]
- large recoil (low-q^2): Light-Cone Sum Rules (B-meson or light-meson DAs) or lattice ($B \to K$) [Khodjamirian, Mannel, Pivovarov, Wang; Bharucha, Straub, Zwicky; Gubernari, Kokulu, van Dyk]



B-meson LCSR + lattice



Light-meson LCSR + lattice

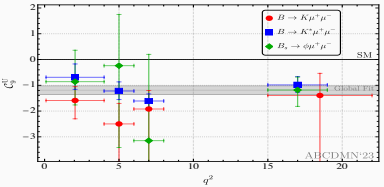
- optimised observables P_i to reduce the impact of form factor uncertainties

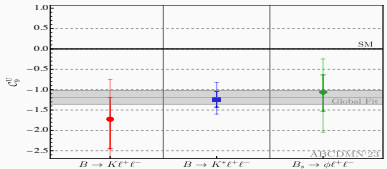
WHY A contrived hadronic contribution to $\mathcal{C}_9^\mu$ must be constant?

An hadronic contribution is naturally expected to be:

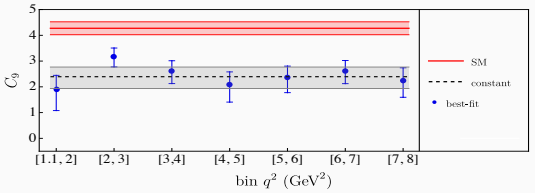
- process-dependent
- bin-dependent
- helicity-dependent (if it is a vector)

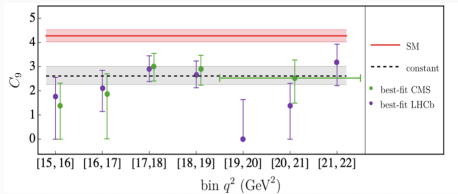
i.e non-universal and not constant, but data requires smtg UNIVERSAL and CONSTANT.





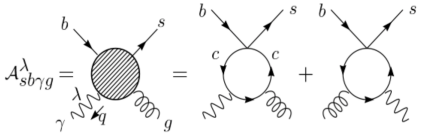
- No signs of non-universality observed in Algueró et al, 2304.07330





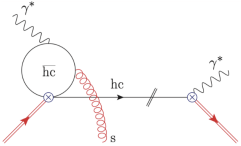
Inclusive: resonances

- Non-resonant color octet effects at **high-q²** can be calculated in perturbation theory for the integrated rate, they are **local** and scale as $\Lambda^2_{\rm QCD}/q^2$ [Buchalla, Isidori, Rey]:



$$\Longrightarrow \frac{\langle B|\bar{b}\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{m_c^2}\left(-6\frac{m_c^2}{q^2}\right) \sim \frac{\lambda_2}{q^2}$$

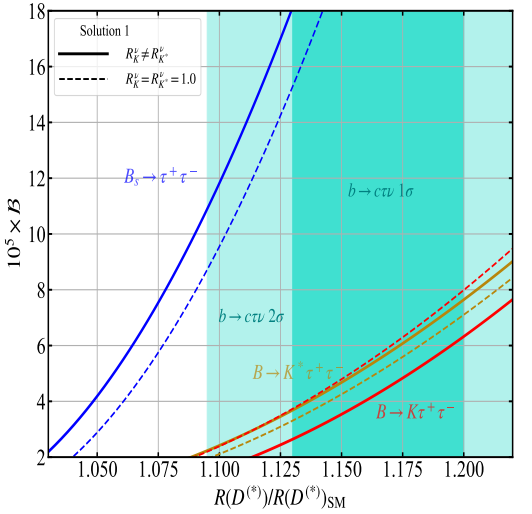
- Non-resonant color octet effects at **low-q²** and with a cut on m_X, are **nonlocal** and can treated using SCET [Hurth, Benzke, Fickinger, Turczyk]:



  - Power corrections remain non-local after m_X cut is released
    ⇒ so-called resolved contributions
  - Depend on mostly unknown subleading B shape functions
  - A rough estimate yields an irreducible uncertainty of about 5%

- Effect of resonant non-factorizable charmonium production at low and high q^2 is being investigated

Scenario III: $\mathcal{B} - R(D^{(*)})/R(D^{(*)})_{\rm SM}$: Solution 1

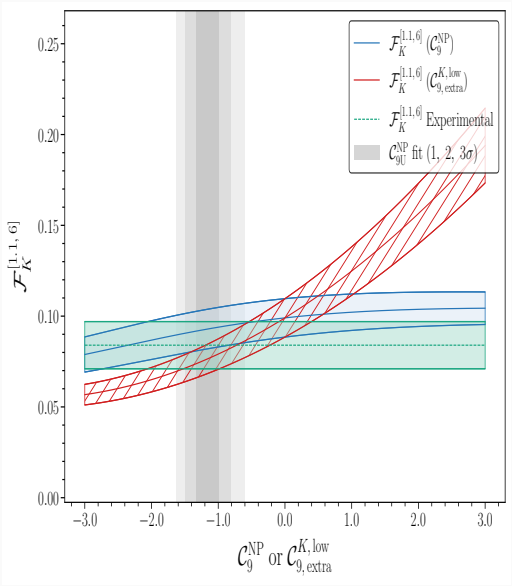


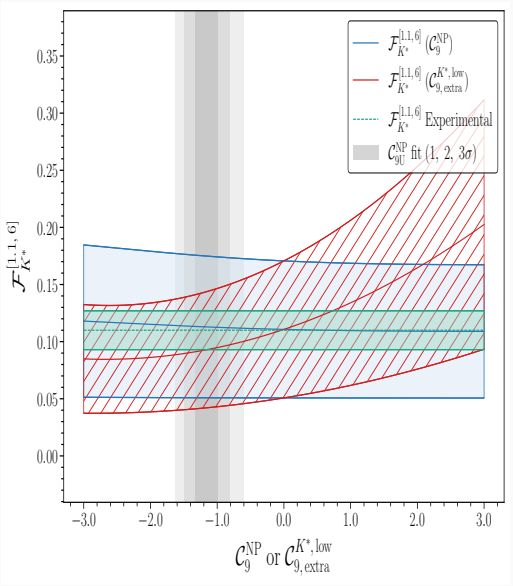
$\mathcal{B}(B_s \to \tau^+\tau^-)$ (blue), $\mathcal{B}(B \to K\tau^+\tau^-)$ (red) and $\mathcal{B}(B \to K^*\tau^+\tau^-)$ (yellow), as a function of $R(D^{(*)})/R(D^{(*)})_{\rm SM}$ for solution 1.

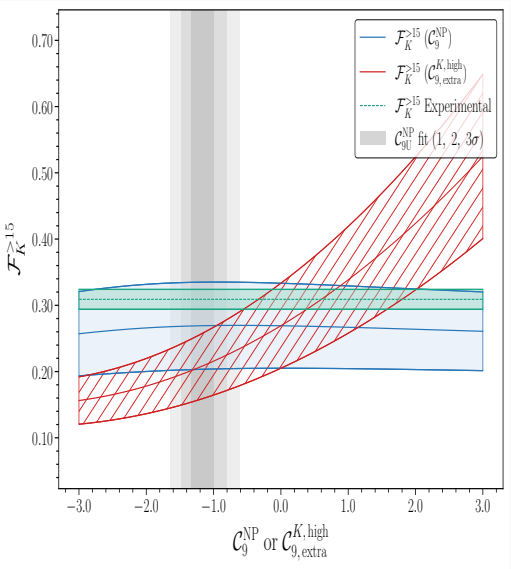
The light-blue region is preferred by the $b \to c\tau\nu$ measurements.

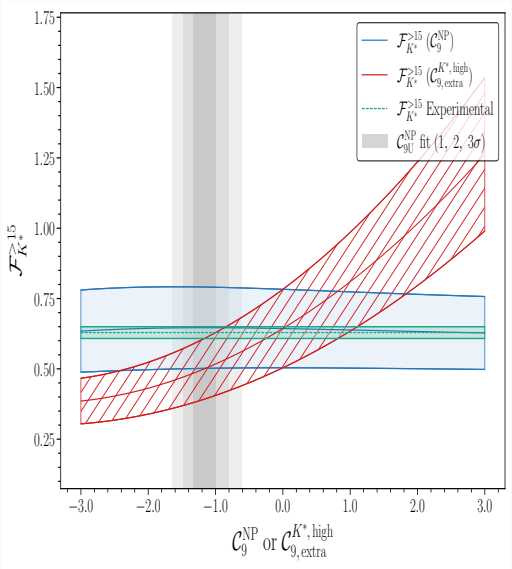
Solid lines represent the case $R_K^\nu = 2.9$ (current measured central value) and $R_{K^*}^\nu = 1.1$ (minimum theoretically allowed value).

Dashed lines depict the case where $R_{K^{(*)}}^\nu = 1$.


$\mathcal{F}_K^{[1.1,6]}$ $(\mathcal{C}_9^{\rm NP})$
$\mathcal{F}_K^{[1.1,6]}$ $(\mathcal{C}_{9,\rm extra}^{K,\rm low})$
$\mathcal{F}_K^{[1.1,6]}$ Experimental
$\mathcal{C}_{9U}^{\rm NP}$ fit $(1,\ 2,\ 3\sigma)$
$\mathcal{F}_K^{[1.1,\ 6]}$
0.25
0.20
0.15
0.10
0.05
0.00
−3.0
−2.0
−1.0
0.0
1.0
2.0
3.0
$\mathcal{C}_9^{\rm NP}$ or $\mathcal{C}_{9,\rm extra}^{K,\rm low}$


$\mathcal{F}_{K^*}^{[1.1,6]}$ $(\mathcal{C}_9^{\rm NP})$
$\mathcal{F}_{K^*}^{[1.1,6]}$ $(\mathcal{C}_{9,\rm extra}^{K^*,\rm low})$
$\mathcal{F}_{K^*}^{[1.1,6]}$ Experimental
$\mathcal{C}_{9U}^{\rm NP}$ fit $(1,\ 2,\ 3\sigma)$
$\mathcal{F}_{K^*}^{[1.1,\ 6]}$
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−3.0
−2.0
−1.0
0.0
1.0
2.0
3.0
$\mathcal{C}_9^{\rm NP}$ or $\mathcal{C}_{9,\rm extra}^{K^*,\rm low}$


$\mathcal{F}_K^{>15}(\mathcal{C}_9^{\rm NP})$
$\mathcal{F}_K^{>15}(\mathcal{C}_{9,\rm extra}^{K,\rm high})$
$\mathcal{F}_K^{>15}$ Experimental
$\mathcal{C}_{9U}^{\rm NP}$ fit (1, 2, 3σ)
$\mathcal{F}_K^{>15}$
$\mathcal{C}_9^{\rm NP}$ or $\mathcal{C}_{9,\rm extra}^{K,\rm high}$


$\mathcal{F}_{K^*}^{>15}(\mathcal{C}_9^{\rm NP})$
$\mathcal{F}_{K^*}^{>15}(\mathcal{C}_{9,\rm extra}^{K^*,\rm high})$
$\mathcal{F}_{K^*}^{>15}$ Experimental
$\mathcal{C}_{9U}^{\rm NP}$ fit (1, 2, 3σ)
$\mathcal{F}_{K^*}^{>15}$
$\mathcal{C}_9^{\rm NP}$ or $\mathcal{C}_{9,\rm extra}^{K^*,\rm high}$

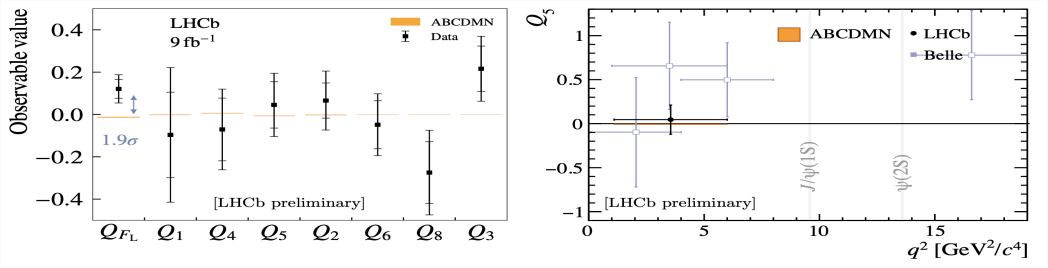
Hints for LFU violation in $b \to s\,\ell^+\ell^-$ decays

Measurement of LFU observable $Q_i = P_i^{(\prime)\mu} - P_i^{(\prime)e}$ **by Belle**. [S. Wehle et al (Belle), PRL 118 (2017)]

- Cancellation of all uncertainties in SM (up to lepton masses) like other LFUV R_{K,K^*}, **and** optimized in presence of NP, contrary to the case of R_{K^*}.
- Isospin averaged but lepton-flavour dependent channels (**Belle**):

$$P_i'^{\ell} = \sigma_+ P_i'^{\ell}(B^+) + (1-\sigma_+) P_i'^{\ell}(\bar{B}^0) \qquad \sigma_+ = 0.5 \pm 0.5$$

still error bars were very large. Now we have **LHCb** results on the Q_i: [PRL 132 (2024) 131801].



LHCb measurements are all consistent (except Q_{F_L}) with the LFU hypothesis.

Theoretical inputs

- **Form factors:** B-meson DA LCSR (state-of-the-art) + lattice + EFT for correlations
- **Charm-loop corrections**: Perturbative contribution including NLO (Asatrian et al.) + magnitude of long-distance contrib inspired by [Khodjamirian, Mannel, Pivovarov, Wang] (These are LCSR computation of "charming penguins")
- **Quark-duality violation at high q^2**: conservative 10 % effect at the level of the amplitude (explicit estimates [Feldman, Buchalla] at the level of 2 %)
- $Br(B_s \to \mu\mu)$ modified to include latest corrections from [Misiak ; Beneke, Bobeth, Szafron]
- $Br(B^+ \to K^{*+}\ell\ell)$ and P_i^+ include mass and lifetime differences, annihilation graphs, hard spectator interactions with $\mathcal{O}_8$ and $\mathcal{O}_{1-6}$

A comparison at the level of C_9-dependencies

We now turn to **a different comparison:** normalization-independent ratios $R_X = \mathcal{B}_X/\mathcal{B}_{X,\mathrm{SM}}$ to compare $\mathcal{C}_9$ dependencies of the inclusive mode and the sum of exclusive modes.

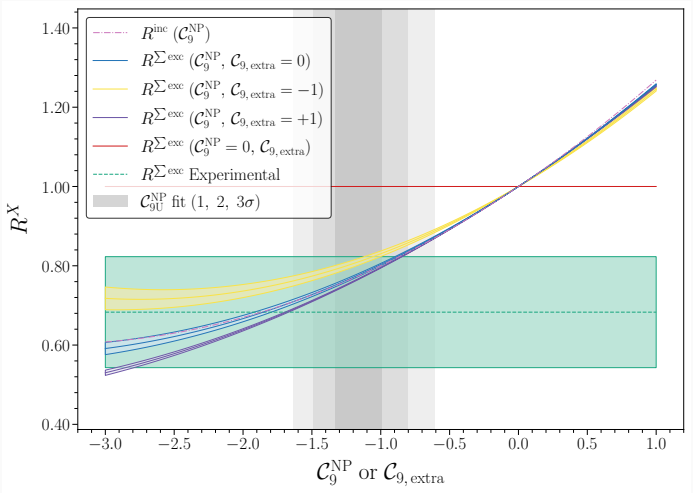
$$
\begin{aligned}
R^{\mathrm{inc}} &= \frac{\mathcal{B}^{\mathrm{inc}}}{\mathcal{B}^{\mathrm{inc}}_{\mathrm{SM}}} = 1 + 0.234\,\mathcal{C}_9^{\mathrm{NP}} + 0.034\,\mathcal{C}_9^{\mathrm{NP}\,2}, \\
R^{\sum \mathrm{exc}} &= \frac{\mathcal{B}^{\sum \mathrm{exc}}}{\mathcal{B}^{\sum \mathrm{exc}}_{\mathrm{SM}}} = \frac{1 + 0.227(\mathcal{C}_9^{\mathrm{NP}} + \mathcal{C}_{9,\mathrm{extra}}) + 0.030(\mathcal{C}_9^{\mathrm{NP}} + \mathcal{C}_{9,\mathrm{extra}})^2}{1 + 0.227\mathcal{C}_{9,\mathrm{extra}} + 0.030\mathcal{C}_{9,\mathrm{extra}}^2}.
\end{aligned}
$$

The two limiting scenarios of interest are: i) $\mathcal{C}_9^{\mathrm{NP}} = 0$, then $\mathcal{C}_{9,\mathrm{extra}}$ mimics NP, implying $R^{\sum \mathrm{exc}} = 1$, and ii) $\mathcal{C}_{9,\mathrm{extra}} = 0$, the purely NP case.

The corresponding experimental value is

$$
R^{\mathrm{exp}} = \frac{\mathcal{B}^{\sum \mathrm{exc,exp}}}{\mathcal{B}^{\mathrm{inc}}_{\mathrm{SM}}} = 0.68 \pm 0.14,
$$

- **The Conflict:** Switching on $C_{9,\text{extra}}$ increases the tension between the inclusive OPE calculation and the sum of measured exclusive channels.
- **Data** The purely hadronic contribution, i.e, $R^{\sum \text{exc}} = 1$ is in tension with the experimental band at 2.3σ



Summary of dominant scenarios before and after 2022

$$\mathcal{C}^{\rm eff}_{9\mu} \to \mathcal{C}^{\rm eff}_{9\mu\, j} = \mathcal{C}^{\rm SM}_{9\mu\,\rm pert} + \mathcal{C}^{\rm NP}_{9\mu} + \mathcal{C}^{c\bar{c}\, B\to K^*}_{9\mu\, j} \qquad \mathcal{C}^{\rm NP}_{9\mu} = \mathcal{C}^{\mathbf{V}}_{9\mu} + \mathcal{C}^{\mathbf{U}}_{9}$$

Picture before December 2022:

Hypotheses	Param.	$\text{Pull}_{\rm SM}$
$\mathcal{C}^{\rm NP}_{9\mu} = -1.01$	1	7.0
$[\mathcal{C}^{\rm V}_{9\mu} = -\mathcal{C}^{\rm V}_{10\mu}, \mathcal{C}^{\rm U}_{9}]^* = [-0.34, -0.82]$	2	7.2
$[\mathcal{C}^{\rm NP}_{9\mu}, \mathcal{C}_{9'\mu} = -\mathcal{C}_{10'\mu}] = [-1.15, +0.17]$	2	7.1

Def. $\text{Pull}_{\rm SM} : \chi^2(\mathcal{C}^{\rm NP}_i = 0) - \chi^2_{min}$ considering N_{dof} (in σ units)

metrology: how well does the hypothesis solve SM deviations ?

With only 1 or 2 parameters one gets excellent fit to data. Scenario (*) moreover link with $b \to c\tau\nu$ anomaly and naturally generates LFU in C_9+imply large $b \to s\tau\tau$. Emerging RHC.

Summary of dominant scenarios before and after 2022

$$\mathcal{C}_{9\mu}^{\rm eff} \to \mathcal{C}_{9\mu\, j}^{\rm eff} = \mathcal{C}_{9\mu\,{\rm pert}}^{\rm SM} + \mathcal{C}_{\mathbf{9}_\mu}^{\rm NP} + \mathcal{C}_{9\mu\, j}^{c\bar{c}\, B\to K^*} \qquad \mathcal{C}_{\mathbf{9}_\mu}^{\rm NP} = \mathcal{C}_{\mathbf{9}_\mu}^{\mathbf{V}} + \mathcal{C}_{\mathbf{9}}^{\mathbf{U}}$$

After December 2022:

Hypotheses	Param.	$\text{Pull}_{\rm SM}$
$\mathcal{C}_{9\mu}^{\rm NP} = \mathcal{C}_{9e}^{\rm NP} = \mathcal{C}_{\mathbf{9}}^{\rm U} = -1.2$	1	5.8
$[\mathcal{C}_{9_\mu}^{\rm V} = -\mathcal{C}_{10\mu}^{\rm V}\,, \mathcal{C}_9^{\rm U}]^* = [-0.08, -1.10]$	2	5.6
$[\mathcal{C}_{9\mu}^{\rm NP}\,, \mathcal{C}_{9'\mu} = -\mathcal{C}_{10'\mu}] = [-0.78, +0.08]$	2	4.3

- New Physics is now prefereably Lepton Flavour Universal.
- Reduction of 2σ of significance, but still $> 5\sigma$.
- $C_{9\mu}^V = -C_{10\mu}^V$ close to zero because of $R_{K^{(*)}}$ and $B_s \to \mu\mu$ and RHC not favoured.

The decomposition in $SU(2)_L$ components is given by

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{NP}} = 2\left(V_{cs}C_{23}^{(3)} + V_{cb}C_{33}^{(3)}\right)(\bar{c}_L\gamma_\mu b_L)(\bar{\tau}_L\gamma^\mu\nu_{L\tau}) \\ + \left(C_{23}^{(1)} + C_{23}^{(3)}\right)(\bar{s}_L\gamma_\mu b_L)(\bar{\tau}_L\gamma^\mu\tau_L) \\ + \left(C_{23}^{(1)} - C_{23}^{(3)}\right)(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_{L\tau}\gamma^\mu\nu_{L\tau}) \\ + C_{23}^{\ell d}\left[(\bar{s}_R\gamma_\mu b_R)(\bar{\nu}_{L\tau}\gamma^\mu\nu_{L\tau}) + (\bar{s}_R\gamma_\mu b_R)(\bar{\tau}_L\gamma^\mu\tau_L)\right] + \text{h.c.}\,.\end{aligned}$$

We work in the down-aligned basis, i.e.

$\hookrightarrow$ CKM elements appear in the charged-current operators.

Scenario I: $C_{23}^{(1)} = C_{23}^{(3)}$ and $|C_{23}^{(3)} V_{cs}| \gg |C_{33}^{(3)} V_{cb}|$

Assume

$$C_{23}^{(1)} = C_{23}^{(3)} = C_{23}.$$

and if $|C_{23}^{(3)} V_{cs}| \gg |C_{33}^{(3)} V_{cb}|$ then C_{33} can be neglected.

Consequences:

- Accommodates $R(D^{(*)})$. Re-express $C_{23} \Rightarrow R(D^{(*)})$.
- Generates large $C_9^\tau = -C_{10}^\tau$ where

$$C_{9(10)}^\tau = C_{9(10)}^{\mathrm{SM}} - (+)\Delta_{R(D^{(*)})}$$

- Enhances all $b \to s\tau\tau$ modes by $\mathcal{O}(10^2 - 10^3)$ with respect to SM.
- $B \to K^{(*)}\nu\bar{\nu}$ NOT generated at tree level, while $R_K^\nu = R_{K^*}^\nu \neq 1$ generated at loop level.
- C_9^{U} and $R_K^\nu = R_{K^*}^\nu$ re-expressed as function of $R(D^{(*)})$.

Scenario II: $C_{23}^{(1)} \neq C_{23}^{(3)}$ and $|C_{23}^{(3)} V_{cs}| \gg |C_{33}^{(3)} V_{cb}|$

Allow

$$C_{23}^{(1)} \neq C_{23}^{(3)}.$$

Features:

- $B \to K^{(*)} \nu\bar{\nu}$ is generated at tree level.
- Still LH currents assumption $R_K^\nu = R_{K^*}^\nu$
- Two parameters $\Rightarrow$ two observables: $C_{23}^{(3)} \Rightarrow R(D^{(*)})$ and $C_{23}^{(1)} - C_{23}^{(3)} \Rightarrow B \to K^{(*)} \nu\bar{\nu}$

Two solutions (two values of $C_{23}^{(1)}$ and $C_{23}^{(3)}$ yield same $R_{K^{(*)}}^\nu$):

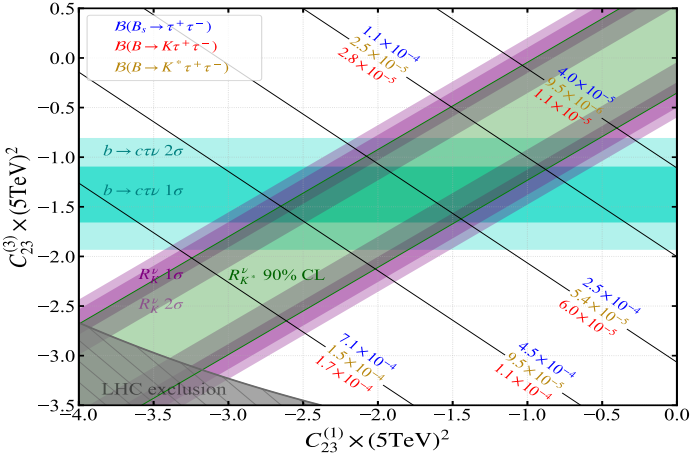
- **Solution 1**: NP interferes constructively with the SM in $C_L^{\nu\tau}$ in $B \to K^{(*)} \nu\bar{\nu}$, such that

$$C_{9(10)}^\tau = C_{9(10)}^{\mathrm{SM}} - (+) \left(\tilde{\Delta}_{R(D^{(*)})} - \tilde{\Delta}_{R_{K^{(*)}}^\nu}^+ \right),$$

- **Solution 2**: NP interferes destructively with the SM in $C_L^{\nu\tau}$ and overcompensates its effect in $B \to K^{(*)} \nu\bar{\nu}$, such that

$$C_{9(10)}^\tau = C_{9(10)}^{\mathrm{SM}} - (+) \left(\tilde{\Delta}_{R(D^{(*)})} - \tilde{\Delta}_{R_{K^{(*)}}^\nu}^- \right),$$

Preferred and excluded region in the $C_{23}^{(1)}$–$C_{23}^{(3)}$ plane



NP scale $\Lambda = 5\,\text{TeV}$, horizontal band preferred by $b \to c\tau\nu$ measurements, the purple strips regions preferred by R_K^ν, while the green region is allowed by $R_{K^*}^\nu$ at 90 % CL. The hatched region on the bottom-left is excluded by non-resonant mono-tau searches at the LHC.

Scenario III: $C_{23}^{(1)} \neq C_{23}^{(3)}$ with $|C_{23}^{(3)} V_{cs}| \gg |C_{33}^{(3)} V_{cb}|$ and $C_{23}^{\ell d} \neq 0$

Allow

$$C_{23}^{(1)} \neq C_{23}^{(3)} \quad \text{and} \quad C_{23}^{ld} \neq 0$$

Features:

- $B \to K^{(*)} \nu \bar{\nu}$ is generated at tree level.
- RH currents enter and $R_K^\nu \neq R_{K^*}^\nu$ and $C_{9'(10')}^\tau$ is generated.
- $C_{23}^{ld} \neq 0$ leads to a non-zero $C_R^{\nu_\tau}$ also.
- Three parameters $\Rightarrow$ three observables: $C_{23}^{(3)}$, $C_{23}^{(1)} - C_{23}^{(3)}$, $C_{23}^{\ell d} \Rightarrow R(D^{(*)}), R_K^\nu, R_{K^*}^\nu$

Four solutions:

Note that there are four solutions due to the possibility of NP interfering destructively with the SM and overcompensating it in R_K^ν and/or $R_{K^*}^\nu$:

	Destructive interference with the SM in $B \to K\nu\bar{\nu}$	Destructive interference with the SM in $B \to K^*\nu\bar{\nu}$
Sol 1	$\times$	$\times$
Sol 2	✓	$\times$
Sol 3	$\times$	✓
Sol 4	✓	✓

Four solutions for $b \to s\tau^+\tau^-$ processes in terms of $R^{\nu}_{K^{(*)}}$ and $R(D^{(*)})$ for $R^{\nu}_{K^{(*)}} > 1$. They are classified depending on whether NP interfere constructively or destructively with the SM in $B \to K\nu\bar{\nu}$ and $B \to K^*\nu\bar{\nu}$ modes.

- **Solution 1 (example)**:

$$\left\{ C^{\tau}_{9(10)}, C^{\tau}_{9'(10')} \right\} = \left\{ C^{\mathrm{SM}}_{9(10)} - (+) \left(\tilde{\Delta}_{R(D^{(*)})} - \tilde{\Delta}^{+}_{R^{\nu}_{K^{(*)}}} \right), +(-)\Delta^{\mathrm{RHC}}_{R^{\nu}_{K^{(*)}}} \right\}.$$

Scenario IV: $|C_{33}^{(3)}| \gg |C_{23}^{(3)}|$

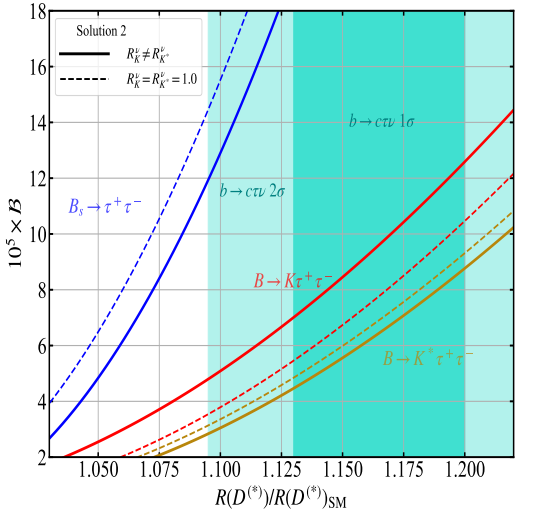
Third-generation dominance

if there is a hierarchical structure, $C_{33}^{(3)} \gg C_{23}^{(3)}$, there is the additional contribution via CKM rotations in the charged current (in the down basis)

$$\mathcal{L}_{\text{eff}}^{\text{NP}} \supset 2C_{33}^{(3)} V_{cb} (\bar{c}_L \gamma_\mu b_L)(\bar{\tau}_L \gamma^\mu \nu_{L\tau}) \,.$$

$\Rightarrow$ the one-to-one correlation between the charged $b \to c\tau\nu$ current and the neutral $b \to s\tau^+\tau^-$ and $b \to s\nu\bar{\nu}$ currents is broken.

- Naturally realized in flavour models.
- Depending on the relative signs of $C_{33}^{(3)}$ and $C_{23}^{(3)}$ the effect on $R(D^{(*)})$ change.

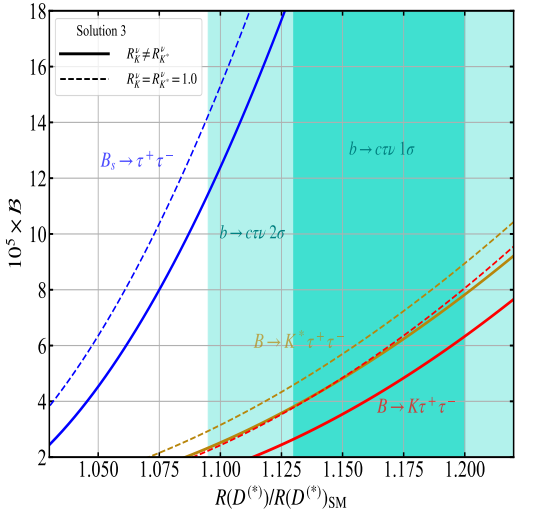


$\mathcal{B}(B_s \to \tau^+\tau^-)$ (blue), $\mathcal{B}(B \to K\tau^+\tau^-)$ (red) and $\mathcal{B}(B \to K^*\tau^+\tau^-)$ (yellow), as a function of $R(D^{(*)})/R(D^{(*)})_{\rm SM}$ for solution 2.

The light-blue region is preferred by the $b \to c\tau\nu$ measurements.

Solid lines represent the case $R_K^\nu = 2.9$ (current measured central value) and $R_{K^*}^\nu = 1.1$ (minimum theoretically allowed value).

Dashed lines depict the case where $R_{K^{(*)}}^\nu = 1$.

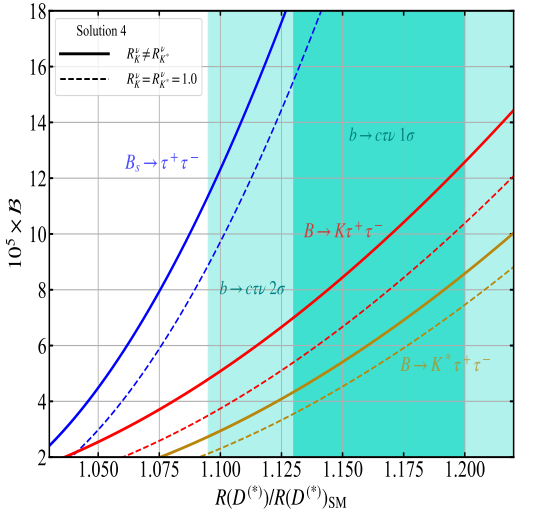


$\mathcal{B}(B_s \to \tau^+\tau^-)$ (blue), $\mathcal{B}(B \to K\tau^+\tau^-)$ (red) and $\mathcal{B}(B \to K^*\tau^+\tau^-)$ (yellow), as a function of $R(D^{(*)})/R(D^{(*)})_{\rm SM}$ for solution 3.

The light-blue region is preferred by the $b \to c\tau\nu$ measurements.

Solid lines represent the case $R_K^\nu = 2.9$ (current measured central value) and $R_{K^*}^\nu = 1.1$ (minimum theoretically allowed value).

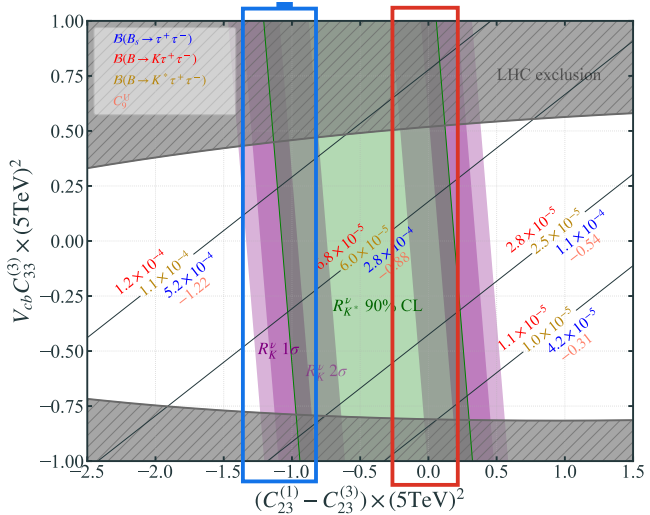
Dashed lines depict the case where $R_{K^{(*)}}^\nu = 1$.



$\mathcal{B}(B_s \to \tau^+\tau^-)$ (blue), $\mathcal{B}(B \to K\tau^+\tau^-)$ (red) and $\mathcal{B}(B \to K^*\tau^+\tau^-)$ (yellow), as a function of $R(D^{(*)})/R(D^{(*)})_{\rm SM}$ for solution 4.
The light-blue region is preferred by the $b \to c\tau\nu$ measurements.
Solid lines represent the case $R_K^\nu = 2.9$ (current measured central value) and $R_{K^*}^\nu = 1.1$ (minimum theoretically allowed value).
Dashed lines depict the case where $R_{K^{(*)}}^\nu = 1$.

$b \to s\tau\tau$ searches can help to disentangle the four solutions, probing NP in neutrino mode.

Preferred/excluded region in $\left(C_{23}^{(1)} - C_{23}^{(3)}\right) - V_{cb}C_{33}^{(3)}$ ($R(D^{(*)})/R(D^{(*)})_{\rm SM} = 1.2$)



- The condition $C_{23}^{(1)} = C_{23}^{(3)}$ selects the constructive-interference R_K^ν solution (right purple fringe) and rules out the overcompensation one (left purple fringe).
- If future $b \to s\tau^+\tau^-$ data favour an overcompensation (blue) of the SM in R_K^ν, the condition $C_{23}^{(1)} = C_{23}^{(3)}$ (red) would have to be abandoned, ruling out hierarchical structures as a viable alternative explanation.