Recent Lattice QCD Results for the Muon g-2

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November 9, 2026, KMI2026

Key References

Muon g-2 Theory Initiative

- T. Aoyama et al, Phys. Rept. **887** (2020), 1-166, 2006.04822
- R. Aliberti et al, Phys. Rept. **1143** (2025), 1-158, 2505.21476
- Theory Initiative Workshop 2026 (https://indico.phys.uconn.edu/event/3/).

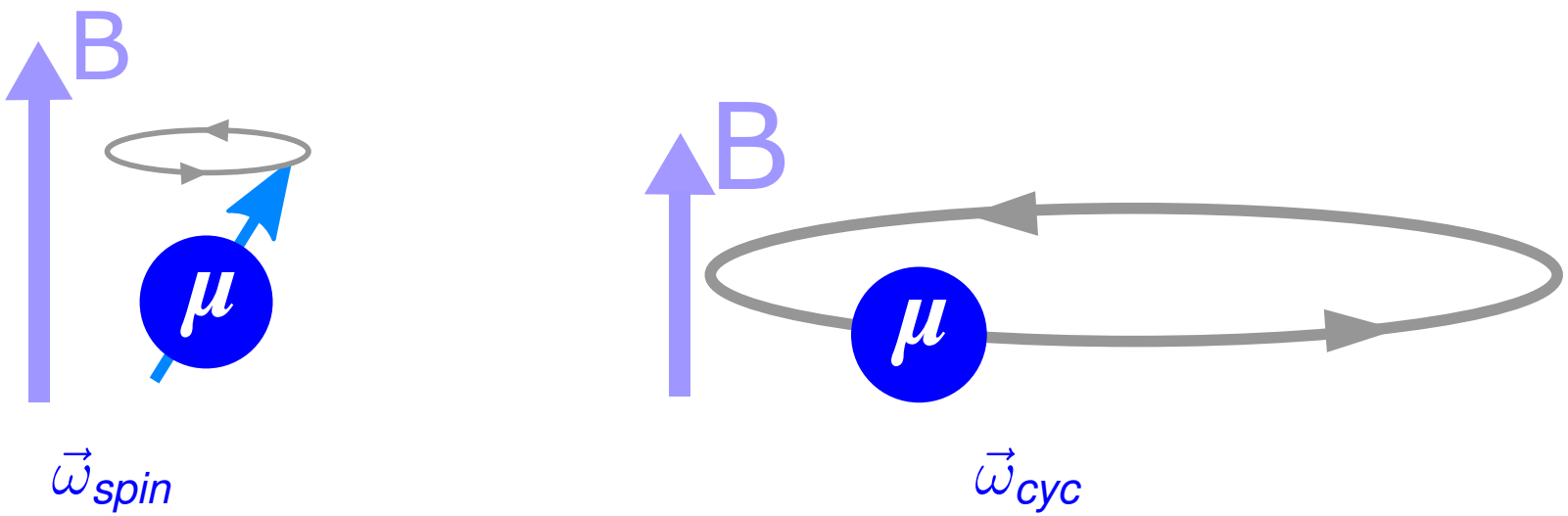
Lattice QCD: Budapest-Marseille-Wuppertal (BMW)

- S. Borsanyi et al, Phys. Rev. D **96** (2017) no.7, 074507, 1612.02364
- S. Borsanyi et al. Phys. Rev. Lett. **121** (2018) no.2, 022002, 1711.04980
- S. Borsanyi et al, Nature **593** (2021) no.7857, 51-55, 2002.12347

Lattice QCD: Mainz/Coordinated Lattice Simulations (CLS)

- M. Cè et al, JHEP **08** (2022), 220, 2203.08676.
- M. Cè et al, Phys. Rev. D **106** (2022) no.11, 114502, 2206.06582
- A. Conigli et al, PRD**114** (2026) no.3, 034508; PRL**137** (2026) no.7, 071901.

Muon Anomalous Magnetic Moment



- Interaction correction to spin precession → anomalous magnetic moment

$$\vec{\omega}_a = \vec{\omega}_{spin} - \vec{\omega}_{cyc} = a_\mu \frac{e\vec{B}}{m_\mu c}, \quad a_\mu = \frac{g_\mu - 2}{2}. \tag{1}$$

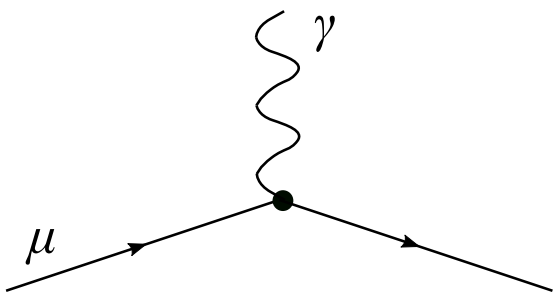
- Pauli equation (nonrelativistic limit of the Dirac equation)

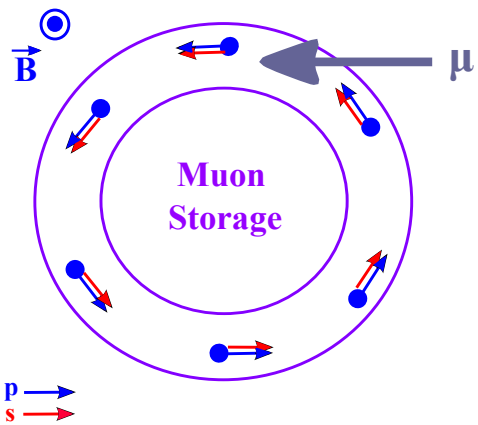
$$i\hbar\frac{\partial\phi}{\partial t} = \left[\frac{(-i\hbar c\vec{\nabla} - e\vec{A})^2}{2m_\mu c} - \vec{M}_\mu \cdot \vec{B} + eA_0\right]\phi, \quad \vec{M}_\mu = g_\mu \frac{e}{2m_\mu c}\frac{\hbar\vec{\sigma}}{2}. \tag{2}$$

Quantum Field Effects on the Muon Anomalous Magnetic Moment

Dirac Theory (Relativistic Quantum Mechanics)

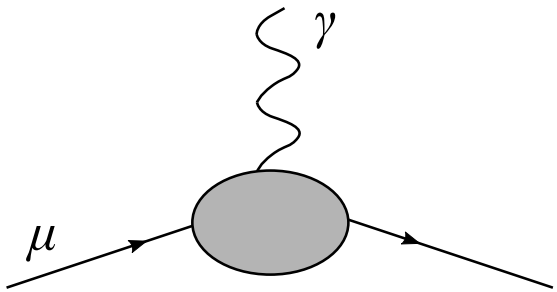
$\vec{\omega}_a = \vec{0}\ ,\ a_\mu = 0\ ,\ g_\mu = 2\ .$

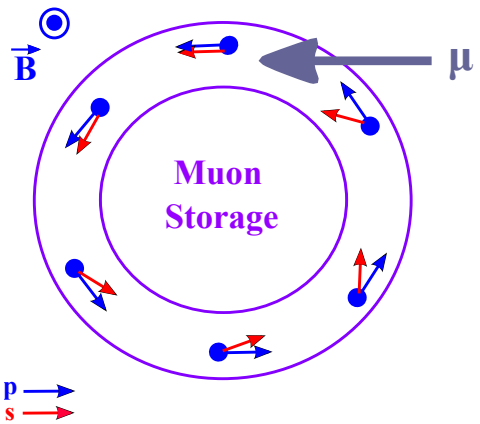




Quantum Field Theory (QFT)

$\vec{\omega}_a \neq \vec{0}\ ,\ a_\mu > 0\ ,\ g_\mu > 2\ .$





FNAL-E989 Arrival-Time Spectrum

p^+(beam) + target nucl. $\rightarrow$ fragmentations $\ni \pi^+ \rightarrow \mu^+ + \nu_\mu$,

μ^+(storage) $\rightarrow e^+$(measured) $+ \nu_e$.

Model function for positron energy spectrum:

$$N_{e^+}(t, E_{th}) = N_0(E_{th}) e^{-t/(\gamma\tau_\mu)} \left(1 + A(E_{th}) \cos[\omega_a t + \phi(E_{th})]\right) . \quad (3)$$

fitted to data about $600\mu s \sim 10\gamma\tau_\mu$.

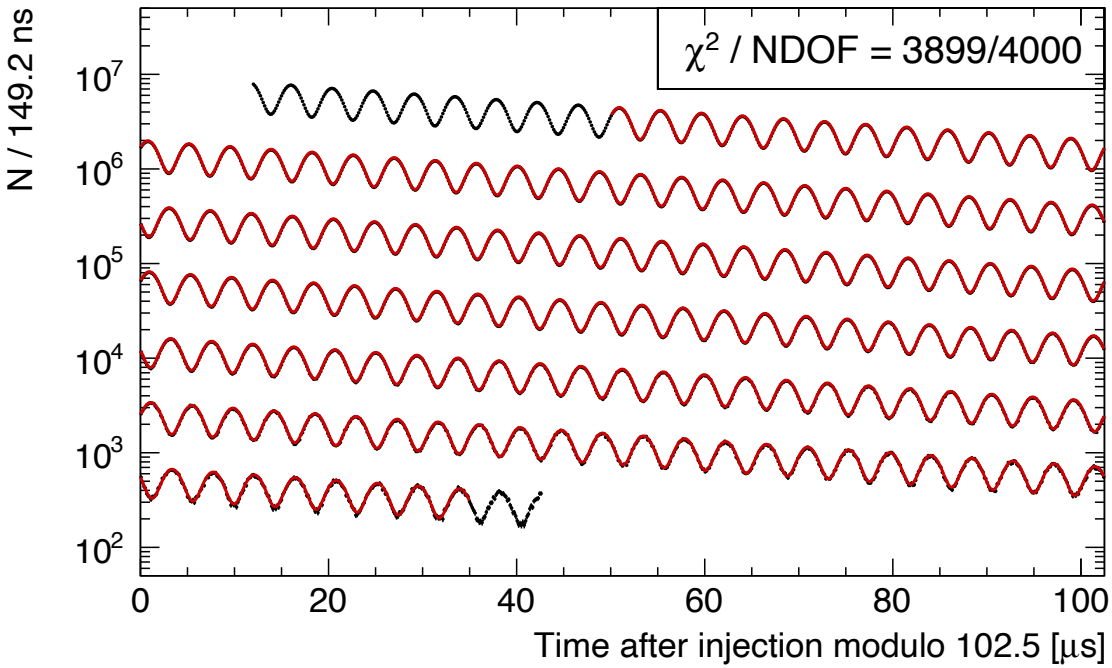
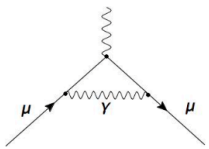
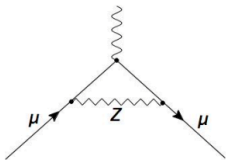
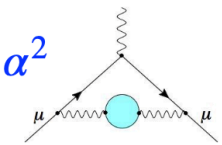
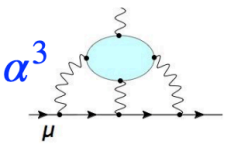


Figure: Quoted From FNAL-E989 Paper: PRD2021.

Why a_μ? Why Hadron Vacuum Polarization (HVP) in Particular?

- Search for Beyond Standard Model (BSM) physics by comparing precision measurements (BNL/FNAL) with Standard Model (SM) calculations.
- Flavor and CP independent: many BSM processes contribute as $(m_{\ell=e,\mu,\tau}/\Lambda_{BSM})^2$.
- White Paper 2020 (WP2020) reported a 5σ tension between a_μ^{ex} and a_μ^{SM}. (It disappeared in WP2025. This talk explains why.)

QED		+… (5 loops)	$116584718.8\,(2) \times 10^{-11}$	2 ppb
EW		+… (2 loops)	$154.4\,(4) \times 10^{-11}$	3 ppb
HVP	α^2 	+… (NNLO)	$7045\,(61) \times 10^{-11}$ [0.9%]	523 ppb
HLbL	α^3 	+… (NLO)	$115.5\,(9.9) \times 10^{-11}$ [8.6%]	84 ppb

Figures borrowed from the lattice 2025 slide by A.El-Khadra

Guide

Hadron Vacuum Polarizaion (HVP):

Nonperturbative quantum chromodynamics (QCD) of quarks and gluons → lattice QCD

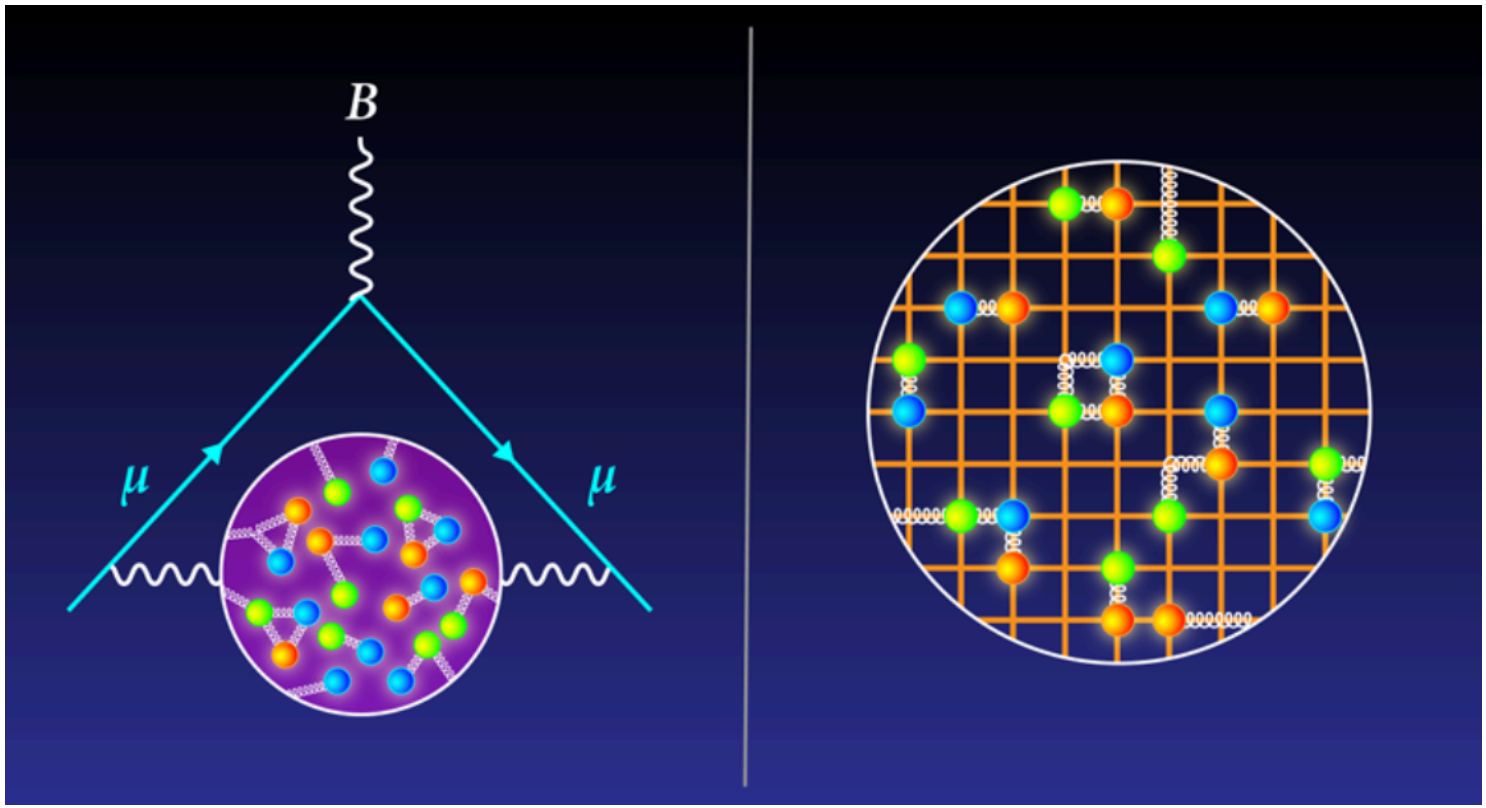


Fig: APS Archives (https://physics.aps.org/articles/v17/6). Credit: APS/Alan Stonebraker

Euclidean path integral defined by the lattice QCD action:

$$Z = \int DU\, e^{-S_{\mathrm{LQCD}}[U]}\,, \quad S_{\mathrm{LQCD}}[U] = S_G[U] - \mathrm{Log\ Det}\, D[U]\,, \tag{4}$$

This resembles a statistical system, so a (Hybrid) Monte Carlo ((H)MC) algorithm can evaluate the integral:

$$P[U] = \frac{e^{-S_{\mathrm{LQCD}}[U]}}{Z}\,, \quad \langle O \rangle = \int DU\, P[U] O[U] = \frac{1}{N}\sum_{n=1}^{N} O[U_n] + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right). \tag{5}$$

For muon $g-2$, O is the vector-current correlator (discussed later).

$$\text{Lattice QCD}\Big|_{a \xrightarrow[M_H/\Lambda=\text{const.}]{} 0} \text{: a nonperturbative def. of continuum QCD.}$$

Lattice QCD Determination of HVP and $a_\mu^{\text{LO-HVP}}$

$\{U^{(i)}\}$: generated by HMC

↓

$D_f[U] \equiv D[U, m_f]$: Dirac operator

↓ Solve the Dirac equation

$D_f^{-1}[U]$: Quark Propagator.

↓

Vector Current Correlator

$$G^f_{\mu\nu}(x) = \langle (\bar{\psi}\gamma_\mu\psi)_x \, (\bar{\psi}\gamma_\nu\psi)_{y=0} \rangle$$

↓ Wick contraction

$$C^f_{\mu\nu}(x) = -\left\langle \mathrm{ReTr}\left[\gamma_\mu \, D^{-1}_{f,x0}[U] \, \gamma_\nu \, D^{-1}_{f,0x}[U]\right] \right\rangle ,$$

$$D^f_{\mu\nu}(x) = \left\langle \mathrm{Re}\left[\mathrm{Tr}\left[\gamma_\mu \, D^{-1}_{f,xx}[U]\right] \mathrm{Tr}\left[\gamma_\nu \, D^{-1}_{f,yy}[U]\right]_{y=0}\right] \right\rangle ,$$

↓

HVP: $\Pi^f_{\mu\nu}(Q) = \mathcal{F}.\mathcal{T}.[G^f_{\mu\nu}(x)]$, g-2: $a^{\text{LO-HVP}}_{\mu,f} = (\alpha/\pi)^2 \sum_t W_{1loop}(t, m_\mu^2) G^f(t)$.

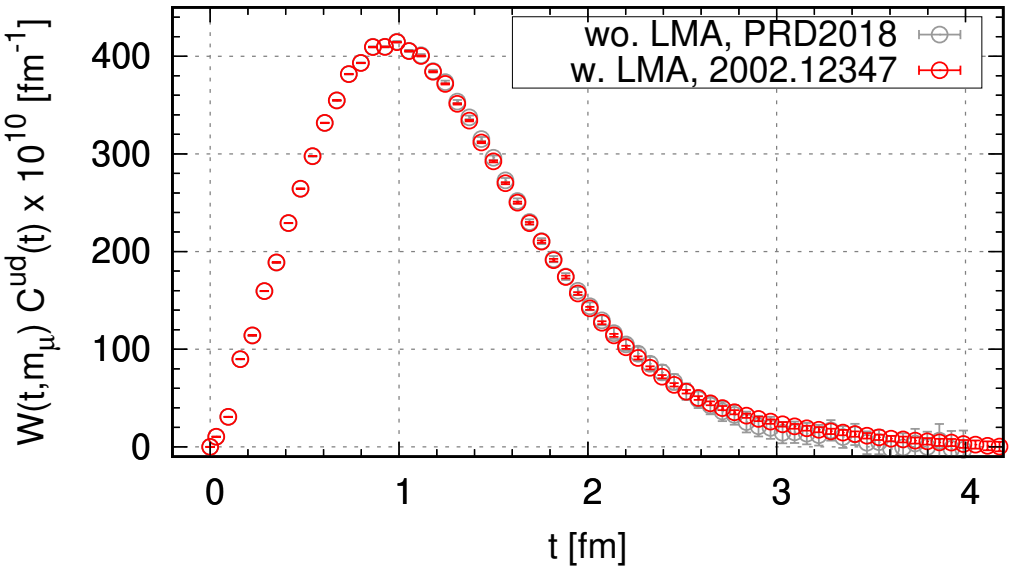


Fig: Integrand of $a^{\text{LO-HVP}}_{\mu,\,ud} = \sum_t W(t, m_\mu^2) C^{ud}(t)$. BMW-2020/2018 data with the finest lattice $a = 0.064$ *fm*.

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   - BMW: Paradigm Shift in $a_\mu^{\text{LO-HVP}}$
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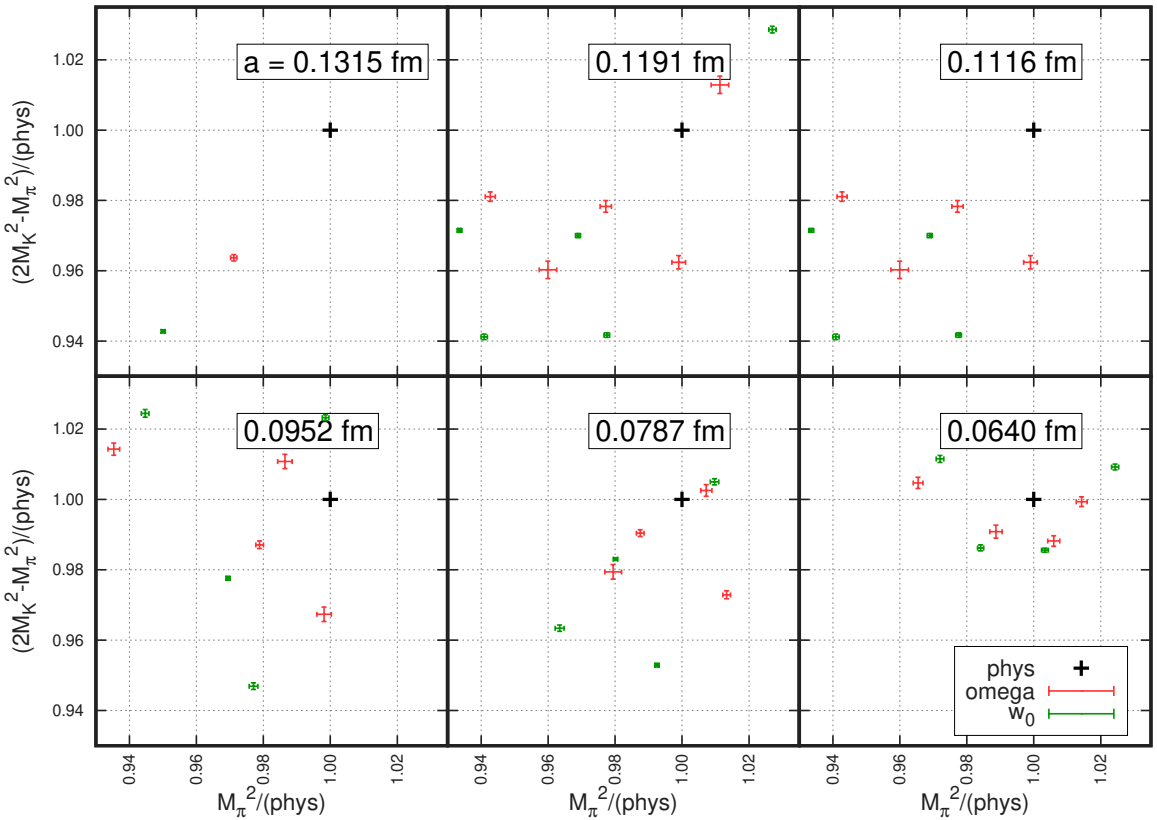
4 Summary

BMW Simulation Setup

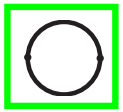


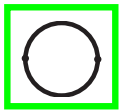
- 6 lattice spacings, 28 simulations around phys. pt.
- $N_f = (2+1+1)$ staggered quarks. ($m_u = m_d$)
- Large Volume: $(L, T) \sim (6, 9-12) fm$. c.f. Helium nucleous diameter is $3.8 fm$.
- Scale Setting (0.4%):
  $M_\Omega^{lat} = \frac{M_{\Omega_}^{phys} a[fm]}{\hbar c}$.

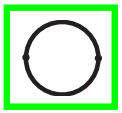
Input Quark Mass (m_{ud}^0, m_s, m_c) Tuning

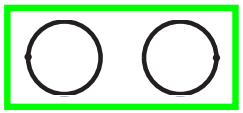
$$\left[\frac{M_\pi^2}{M_\Omega^2}\right]_{lat} \simeq \left[\frac{M_{\pi^0}^2}{M_{\Omega_}^2}\right]_{phys}, \quad \left[\frac{M_K^2 - M_\pi^2/2}{M_\Omega^2}\right]_{lat} \simeq \left[\frac{(M_{K^+}^2 + M_{K^0}^2 - M_{\pi^0}^2)/2}{M_{\Omega_}^2}\right]_{phys}, \quad \frac{m_c}{m_s} = 11.85\ .$$

Isospin symmetric

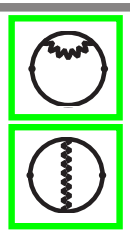

connected light
633.7(2.1)(4.2)

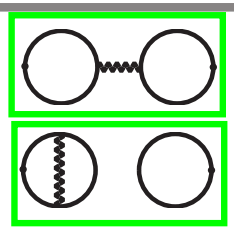

connected strange
53.393(89)(68)


connected charm
14.6(0)(1)

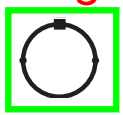

disconnected
-13.36(1.18)(1.36)

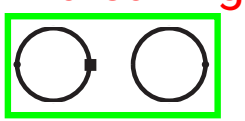
QED isospin-breaking: valence


connected -1.23(40)(31)

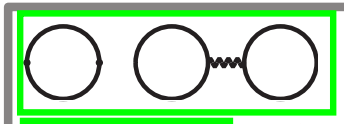

disconnected -0.55(15)(10)

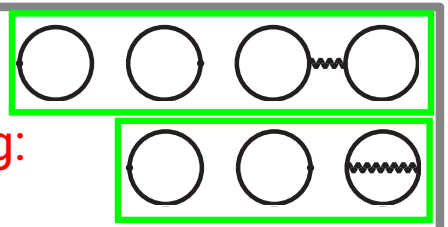
Strong isospin-breaking


connected
6.60(63)(53)


disconnected
-4.67(54)(69)

QED isospin-breaking: sea


connected 0.37(21)(24)

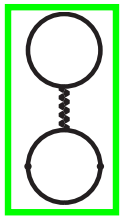

disconnected -0.040(33)(21)

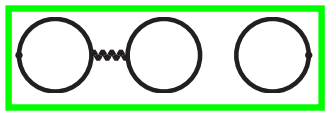
Etc.

bottom; higher order; perturbative

0.11(4)

QED isospin-breaking: mixed


connected -0.0093(86)(95)


disconnected 0.011(24)(14)

Finite-size effects

isospin-symmetric
18.7(2.5)

isospin-breaking
0.0(0.1)

$10^{10} \times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$

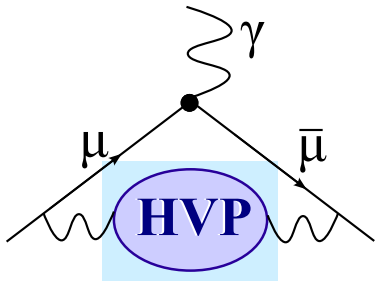


Fig: Quoted from Nature **593** (2021) no.7857.

c.f. 693.1(3.4) (WP2020, Pheno)

711.6(18.4) (WP2020, LQCD)

Whitepaper-2025 Scheme

- The sum on the right is the observable compared with nature, with no ambiguity. Individual terms are scheme dependent.

$$a_\mu^{\text{LO-HVP}} = a_{\mu,\,isoQCD}^{\text{LO-HVP}} + \delta a_\mu^{QED} + \delta a_\mu^{SIB}. \tag{6}$$

  - Edinburgh Consensus for IsoQCD (MeV Units):

    $M_{\pi^+} = 135.0,\ M_{K^+} = 494.6,\ M_{D_s^+} = 1967,\ f_{\pi^+} = 130.5(\text{scale setting}).$

  - WP2025 Scheme for IsoQCD:

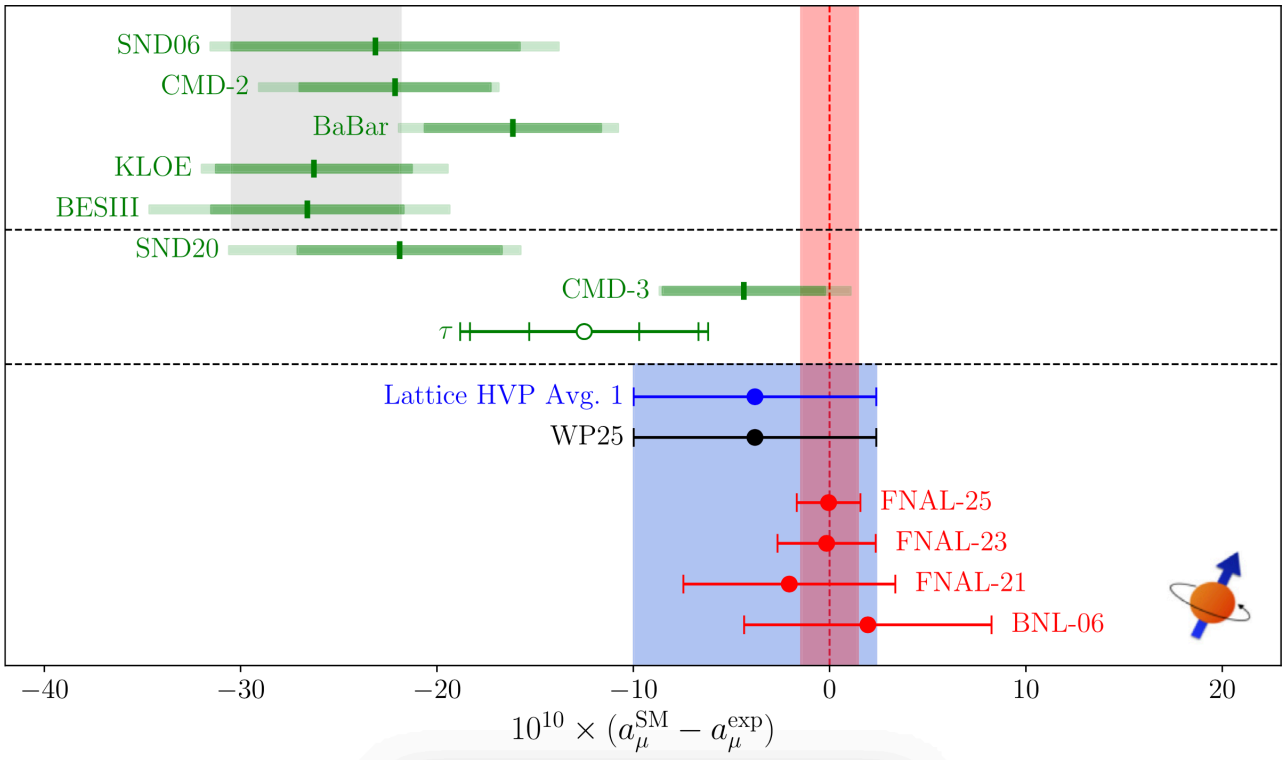
    $M_{\pi^+} = 135.0,\ M_{K^+} = M_{K^0} = 494.6,\ M_{D_s^+} = 1967,\ w_0 = 0.17236(70)fm.$

- Window Method:
  $a_{\mu,f}^{\text{LO-HVP}}|_{SD} = \sum_{t\le 0.4fm} W(t,m_\mu)G_f(t),\quad a_{\mu,f}^{\text{LO-HVP}}|_{ID} = \sum_{t\in[0.4,1.0]fm} W(t,m_\mu)G_f(t),$
  $a_{\mu,f}^{\text{LO-HVP}}|_{LD} = \sum_{t\ge 1.0fm} W(t,m_\mu)G_f(t).$

- Blind Analyses:

  For example, Mainz/CLS hides a random modulation in the kernel $W(t,m_\mu)$ of $a_{\mu,f}^{\text{LO-HVP}} = \sum_t W(t,m_\mu)G_f(t)$. Analysts remove it only at the final stage.

Muon g-2 Whitepaper-2025



- Lattice QCD reached 0.86% precision for HVP in muon $g-2$. WP2025 adopted this consensus.
- The SM prediction agrees with experiment within uncertainties. No BSM signal remains.
- The data-driven approach has NOT been rejected. Its consensus is still developing.

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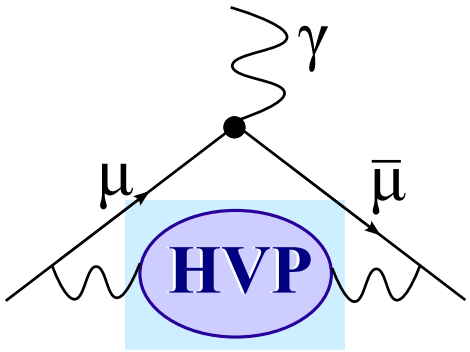
HVP & QED Running Coupling

Hadron Vacuum Polarization (HVP)



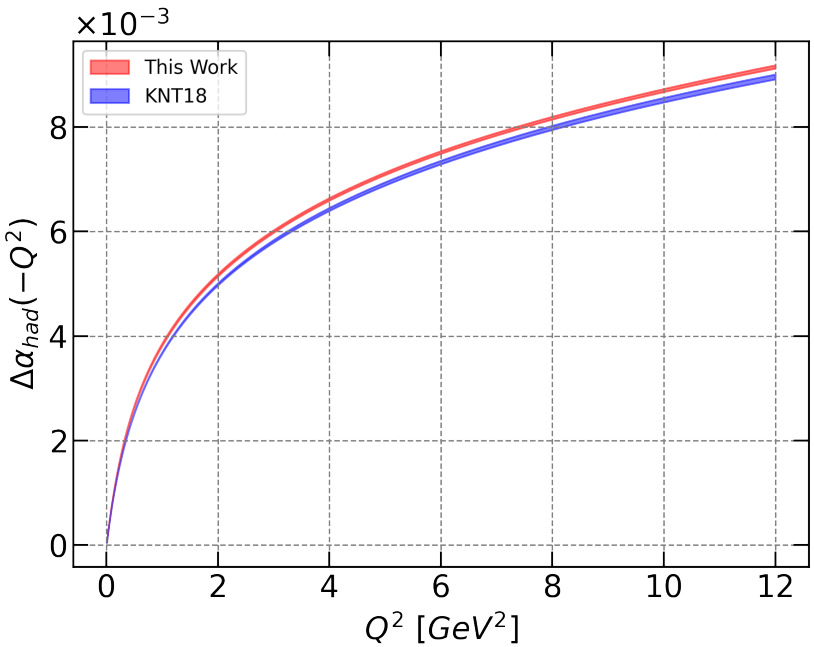


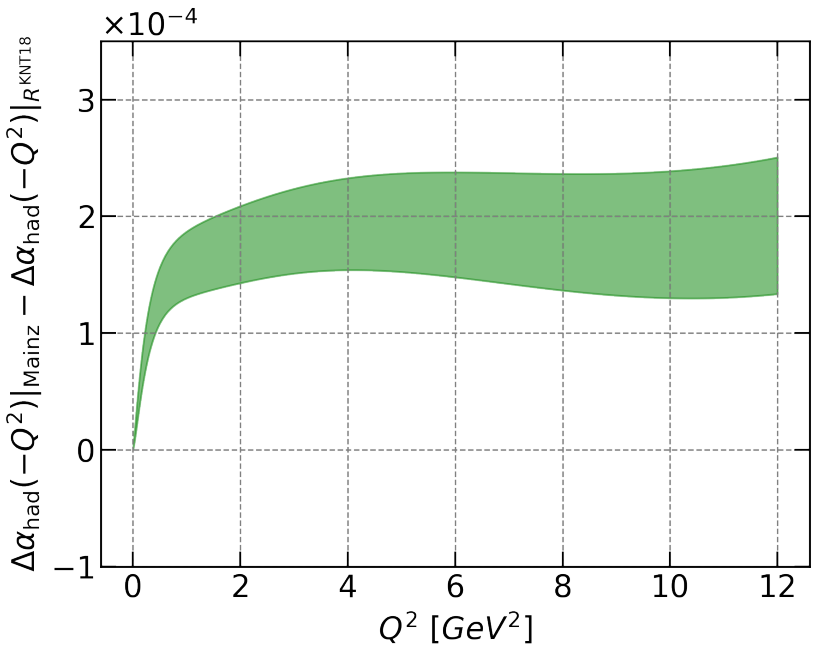
$$= \Pi_{\mu\nu}(q^2) = (q_\mu q_\nu - \eta_{\mu\nu} q^2)\Pi(q^2) \ .$$

$$\Delta\alpha_{\rm had}(q^2) = 4\pi\alpha_0\big(\Pi(q^2) - \Pi(0)\big) \ , \quad \alpha_0 \simeq \frac{1}{137.036} \ . \tag{7}$$

$$\alpha_{EM}(q) = \frac{\alpha_0}{1 - \Delta\alpha_{lep}(q^2) - \Delta\alpha_{\rm had}(q^2)} \ . \tag{8}$$

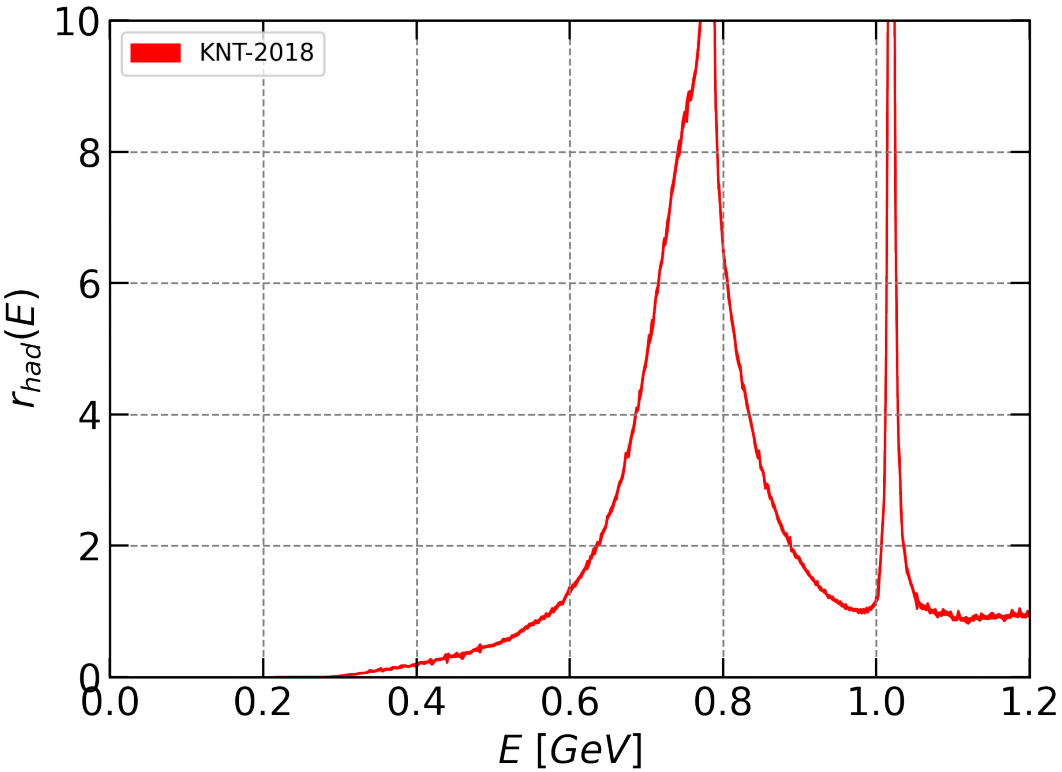
LQCD(Mainz2025) vs. Data Driven(R-ratio)





Lattice QCD (Mainz/CLS-2025) is significantly larger than the data-driven result (KNT-2018). This agrees with the tension seen in $a_\mu^{\text{LO-HVP}}$.

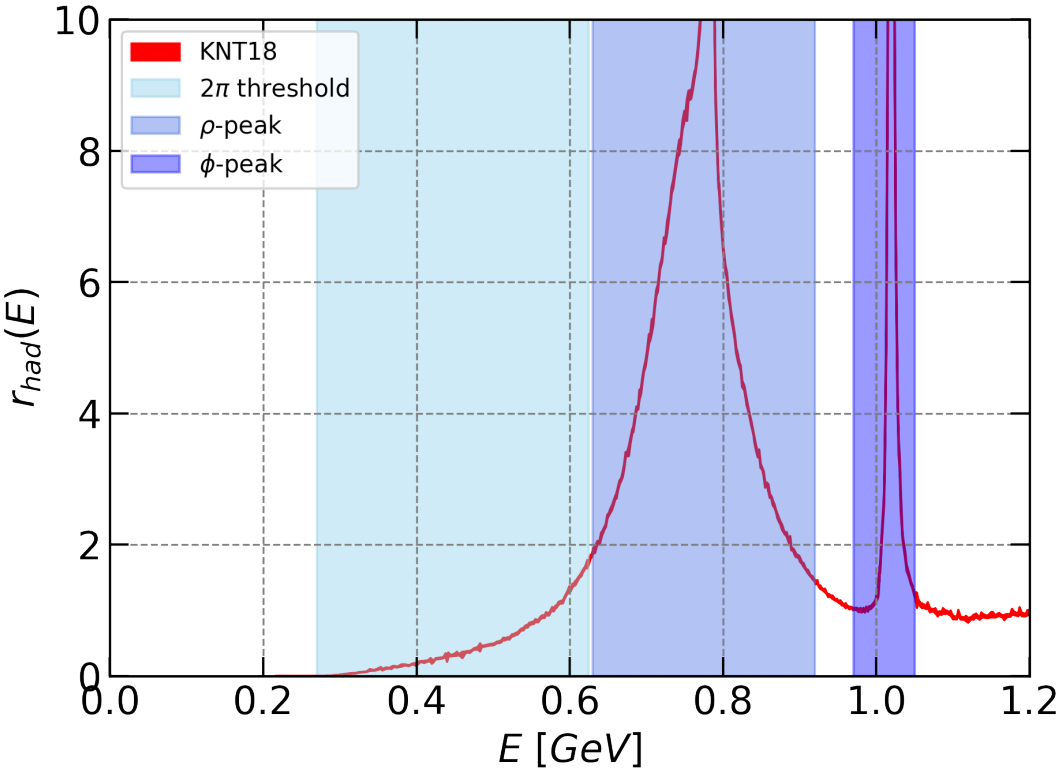
Data Driven Approach



Data-driven approach: R-ratio ($\propto$ cross section[$e^+e^- \to$ hadrons])

$$\Delta\alpha_{\rm had}(-Q^2)|_R = \frac{\alpha Q^2}{3\pi}\int_0^\infty ds \frac{R(s)}{s(s+Q^2)} \ . \tag{9}$$

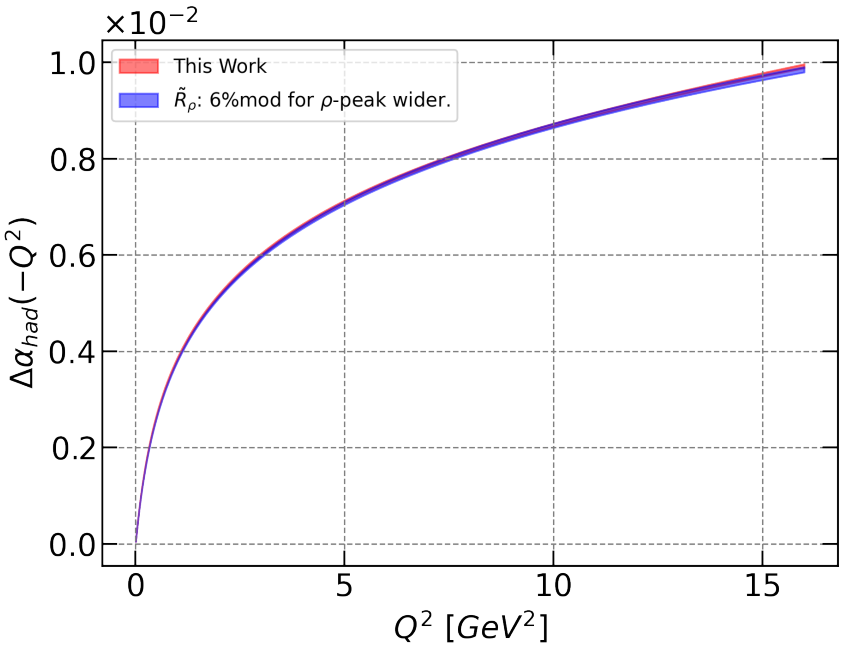
R-Ratio Modification

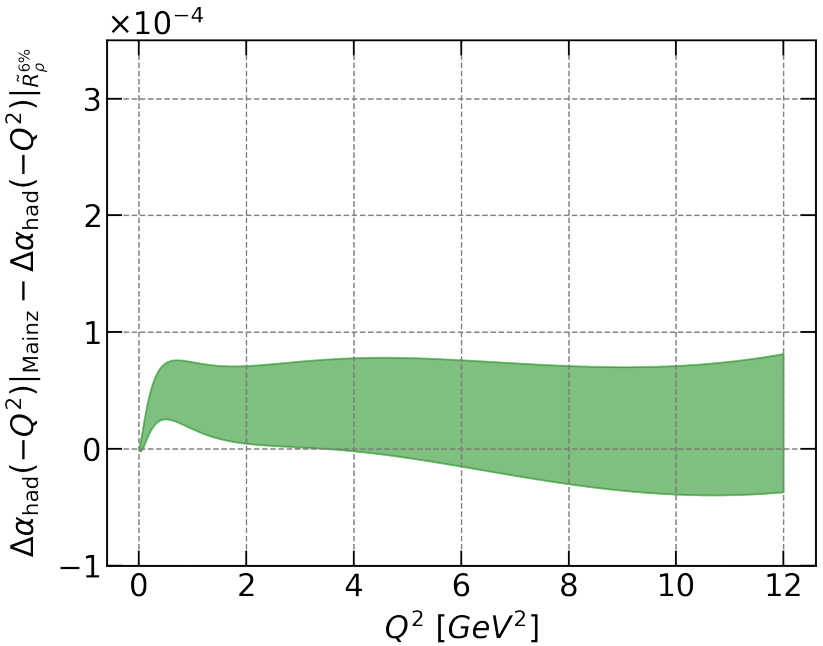


As a heuristic test, increase part of the R-ratio by 6%:

$$\Delta\alpha_{\rm had}(-Q^2)|_{\tilde{R}^{6\%}} = \frac{\alpha Q^2}{3\pi}\int_0^\infty ds \frac{\tilde{R}^{6\%}(s)}{s(s+Q^2)} \ . \tag{10}$$

LQCD(Mainz2025) vs. Data-Driven($\tilde{R}_\rho^{6\%}$)



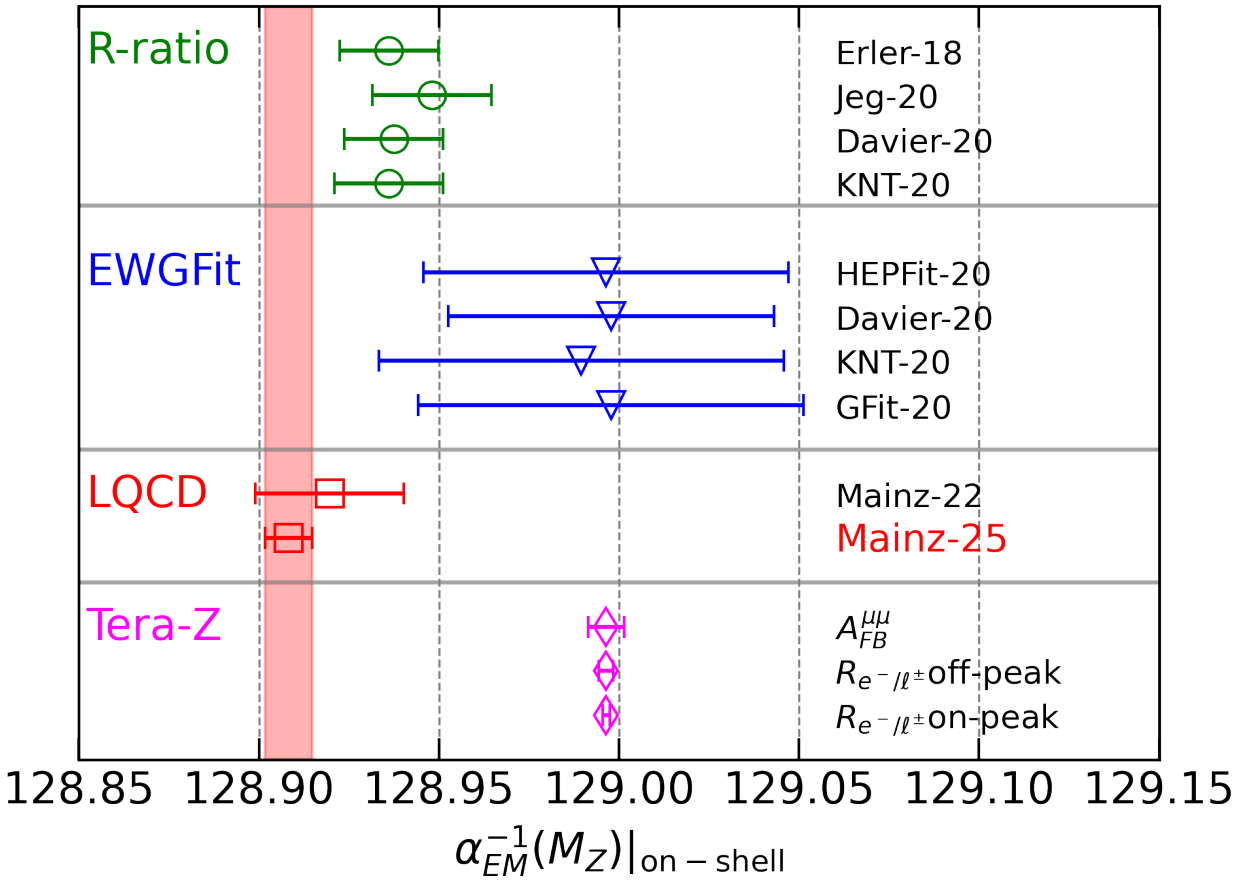


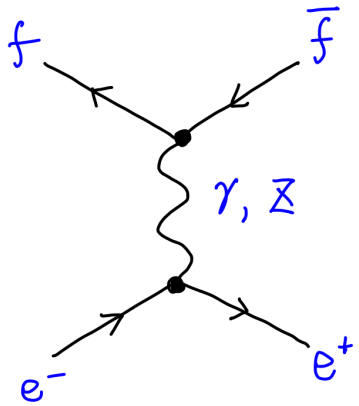
The data-driven result agrees with lattice QCD (Mainz/CLS-2025) only when the 6% modification is applied to the ρ peak. (Similar analyses: BMW-Pheno Collab. PRD'24).

Toward Tera-Z Precision

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = \underbrace{\Delta\alpha^{(5)}_{\rm had}(-9GeV^2)}_{\text{LQCD: } 0.008456(31), 0.4\%} + \underbrace{\left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-9GeV^2)\right]}_{\text{pQCD}'\text{[Adler]: } 0.019304(35)} + \underbrace{\left[\Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2)\right]}_{\text{pQCD: } 4.5(0.2)\cdot 10^{-6}}$$

$$= 0.027813(48), 0.17\% .$$





Tera-Z project in Fcc-ee.

Guide

Parallel Array Computer System (PACS) Collaboration
Lattice QCD Grouparound University of Tsukuba

- Spacelike pion form factor: Almost done (Sato-san).
- $I = 2$ Two-pions scattering length:
  This talk with PACS5 Ensembles, Preliminary.
- Timelike pion form factor: Next one year.
- Muon g-2 up/down quark contributions: Next five years.
- Muon g-2 HVP contributions in 0.2% precision: 10 years.

PACS10 and PACS5

N_f	2+1 (PACS5)	2+1 (PACS10)			2+1+1 (PACS10$_c$)		
$L^3 \cdot T$	64^4	128^4	160^4	256^4	128^4	160^4	256^4
L [fm]	5.4	10.9	10.2	10.5	10.7	10.5	$\sim$10.5
a [fm]	0.085	0.085	0.06	0.04	0.08	0.06	$\sim$0.04
m_π [GeV]	0.135	0.135	0.138	0.141	0.137	0.137	$\sim$0.137
m_K [GeV]	0.497	0.497	0.505	0.514	0.498	0.497	$\sim$0.497
N_{conf}	80	20	20	20	40	40	$\sim$20

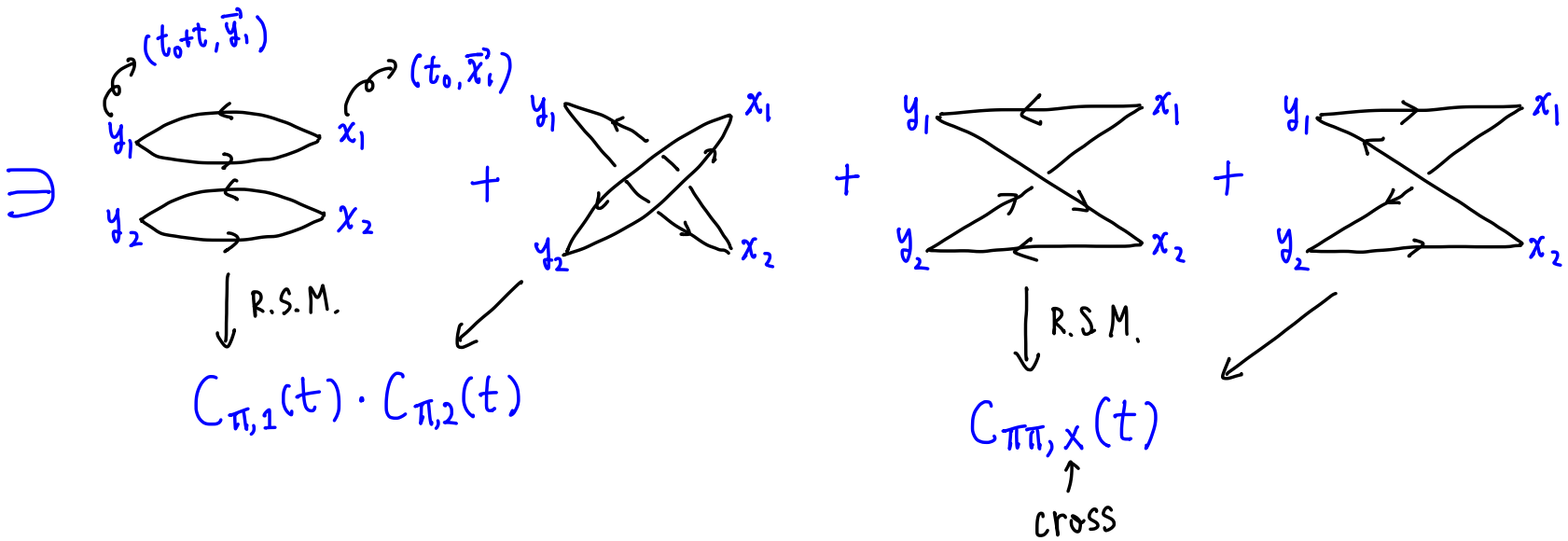
PACS Ensembles

- $N_f = 2 + 1(+1)$ six-stout-smeared non-perturbarively $\mathcal{O}(a)$-inproved Wilson quark + Iwasaki gauge action.
- PACS10/L128,L160,L256 are generated in the Fugaku Supercomputer. PACS10$_c$/L256 is still under way (Publication: PACS10/L128 [PRD101,9,094504(2020)] PACS10/L160 [PRD106,9,094501(2022)]).
- Ensambles are mainly generated and managed by Ukita-san.

$I = 2$ Two-Pion Scattering Length

$$C_{\pi\pi}(t) = \left\langle (\pi^+\pi^+)(\vec{P}, t+t_0)\ (\pi^+\pi^+)^\dagger(\vec{P}, t_0) \right\rangle$$

- $(\pi^+\pi^+)(\vec{P}, t) = \pi^+(\vec{P}_1, t)\,\pi^+(\vec{P}_2, t+1)$
- $\pi^+(\vec{P}, t) = \frac{1}{L^{3/2}} \sum_{\vec{x}} e^{-i\vec{P}\cdot\vec{x}} (\bar{d}\,\gamma_5\,u)^\dagger(\vec{x}, t)$



$I = 2$ Two-Pion Scattering Length

$$C_{\pi\pi}(t) = 2\left(C_{\pi,1}(t) \cdot C_{\pi,2}(t) + C_{\pi\pi,x}(t) \right)$$

large t

$$\sim e^{-E_{\pi\pi} t} \qquad \sim e^{-m_\pi t}$$

Specified as $1, 2, 4, 6, 8, \cdots$

$$R(t) = \left\langle \frac{C_{\pi\pi,x}(t) - C_{\pi\pi,x}(t+\delta t)}{C_{\pi,1}(t) \cdot C_{\pi,2}(t) - C_{\pi,1}(t+\delta t)\, C_{\pi,2}(t+\delta t)} \right\rangle \sim e^{-\Delta E_{\pi\pi} t}$$

$$\Delta E_{\pi\pi} = E_{\pi\pi} - 2m_\pi : \text{Binding Energy}$$

$I = 2$, two Pion Scattering Length

$$\Delta E_{\pi\pi} L = \frac{4\pi}{m_\pi L} \frac{a_{\pi\pi}}{L} \left(1 - 2.837297 \frac{a_{\pi\pi}}{L} + 6.375183 \left(\frac{a_{\pi\pi}}{L} \right)^2 \right)$$

Lüscher [Commun. Math. Phys. **104 & 105**, (1986)]

$I = 2$ Two-Pion Scattering Length

$$C_{\pi\pi}(t) = 2\left(C_{\pi,1}(t) \cdot C_{\pi,2}(t) + C_{\pi\pi,x}(t) \right)$$

large t

$$\sim e^{-E_{\pi\pi} t} \qquad \sim e^{-m_\pi t}$$

Specified as 1, 2, 4, 6, 8, ...

$$R(t) = \left\langle \frac{C_{\pi\pi,x}(t) - C_{\pi\pi,x}(t+\delta t)}{C_{\pi,1}(t) \cdot C_{\pi,2}(t) - C_{\pi,1}(t+\delta t)\, C_{\pi,2}(t+\delta t)} \right\rangle \sim e^{-\Delta E_{\pi\pi} t}$$

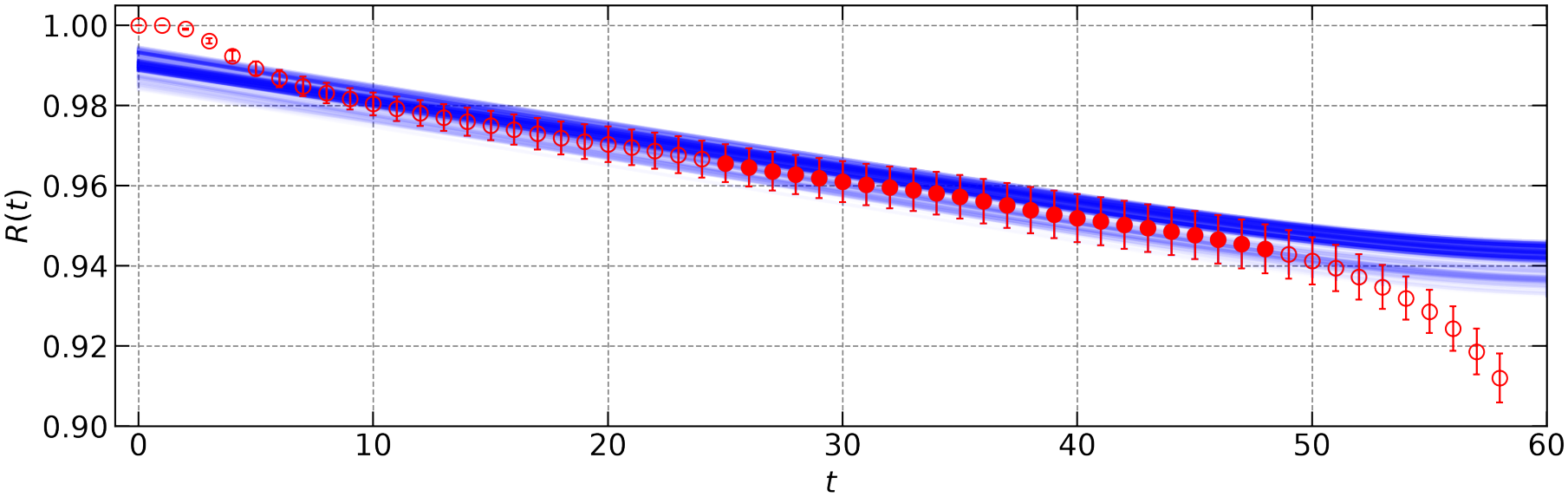
$\Delta E_{\pi\pi} = E_{\pi\pi} - 2m_\pi$: Binding Energy

$I = 2$, two Pion Scattering Length

$$\Delta E_{\pi\pi} L = \frac{4\pi}{m_\pi L} \frac{a_{\pi\pi}}{L} \left(1 - 2.837297 \frac{a_{\pi\pi}}{L} + 6.375183 \left(\frac{a_{\pi\pi}}{L} \right)^2 \right)$$

- NA48/2: Cusp analyses, $\mathcal{M}[K^\pm \to \pi^\pm \pi^0 \pi^0] \ni (a_{\pi\pi}^{I=0} - a_{\pi\pi}^{I=2})/3$.
- DIRAC: Pionium life-time analyses, $\Gamma_{\pi^+\pi^- \to \pi^0\pi^0} \propto |a_{\pi\pi}^{I=0} - a_{\pi\pi}^{I=2}|$.

$I = 2$ Two-Pion Scattering Length

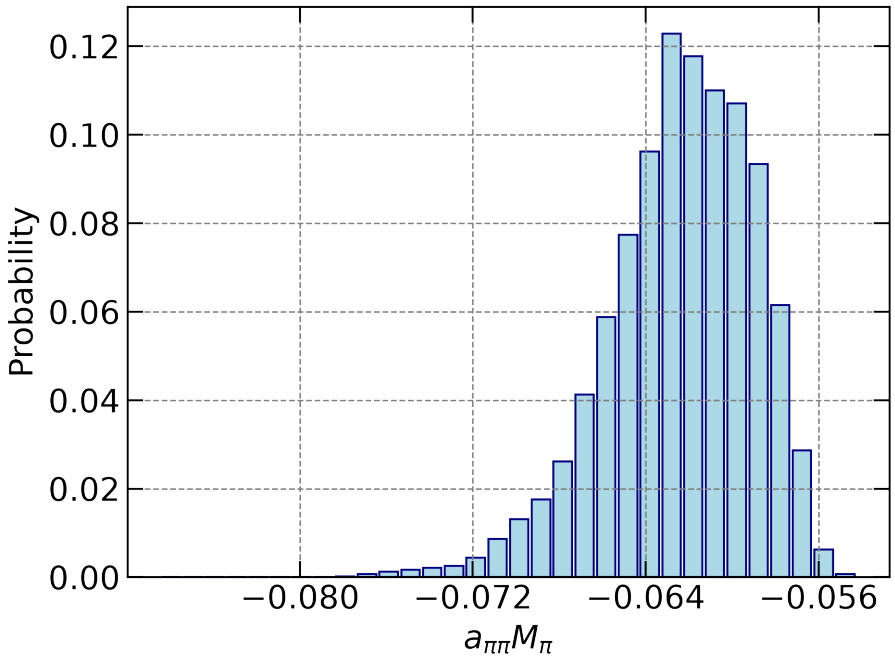


Fit Model (Feng et al. '09):

$$R_{fit}(t) = A\Big(\cosh\big[\Delta E_{\pi\pi}\tilde{t}\big] + \sinh\big[\Delta E_{\pi\pi}\tilde{t}\big]\coth\big[2m_\pi\tilde{t}\big]\Big)\ , \tag{11}$$

with $\tilde{t} = t + \delta t/2 - 1/2 - L_t/2$.

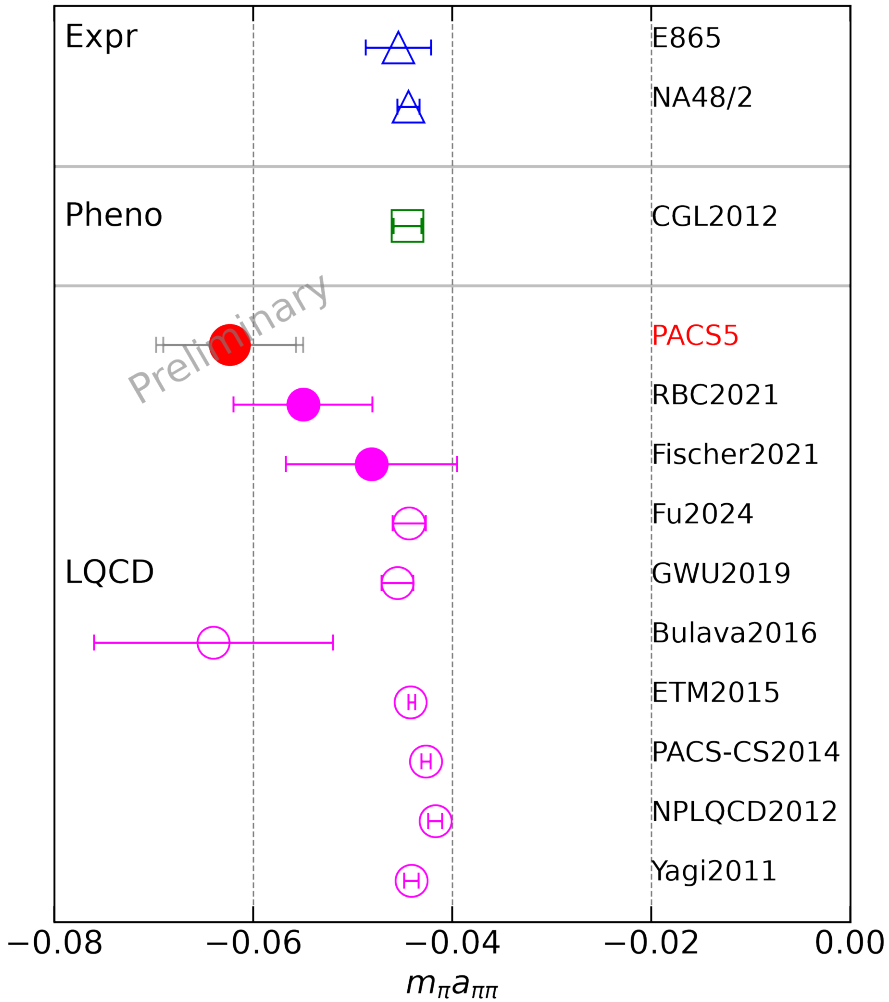
$I = 2$ Two-Pion Scattering Length



Distribution with AIC-Weight:

$$W \propto \exp\left[-\chi^2 + 2(n_{\mathrm{dof}} - n_{\mathrm{cut}})\right] .$$

Variance = Total Error.



Filled symbols: with physical pion masses.

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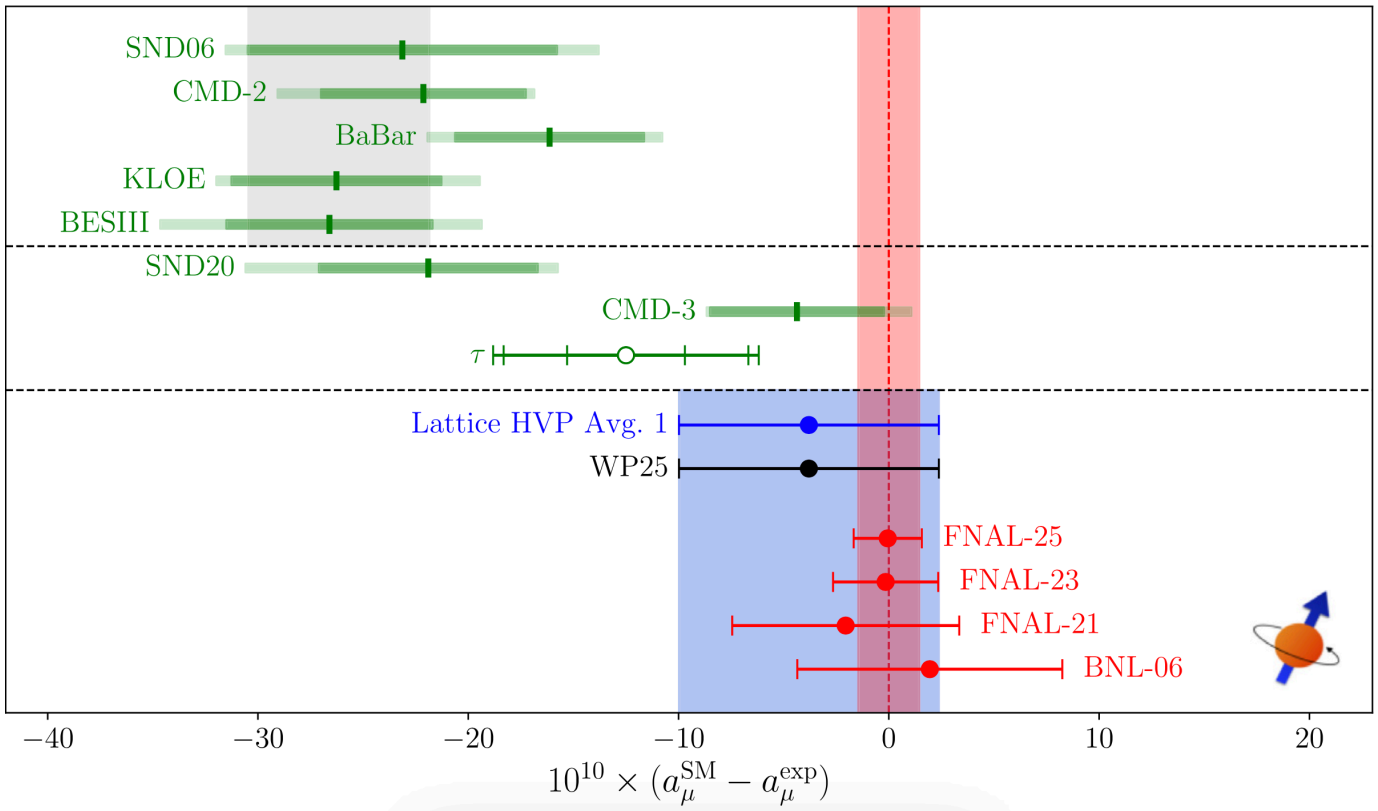
- PACS Collab. for Pion Precision Physics

4 Summary

Summary



- Lattice QCD reached 0.86% precision for HVP in muon $g-2$. WP2025 adopted this consensus.
- The SM prediction agrees with experiment within uncertainties. No BSM signal remains.
- The data-driven approach remains valid. Its consensus is still developing.
- The lattice and data-driven tension comes from the $\rho \to \pi^+\pi^-$ contribution.

Future Prospect

- Consistency I: Building a data-driven consensus (1) Await the Belle II radiative-return results. (2) Form a working group under the Theory Initiative.
- Consistency II: Building consensus between lattice QCD and the data-driven approach The current tension reflects incomplete work during consensus building. Resolving it is essential to precision physics.
- The consensus must reach experimental-level uncertainty: 0.2% for HVP.
- Consistency III: Comparing the Z-pole QED running coupling $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ Next year's Theory Initiative will consider a theory value for the Particle Data Group and comparison with Tera-Z.
- For more precise muon $g-2$ physics, the PACS-J-PARC Joint Collaboration has launched (Mibe, Yoshioka, Yamazaki, Miura).