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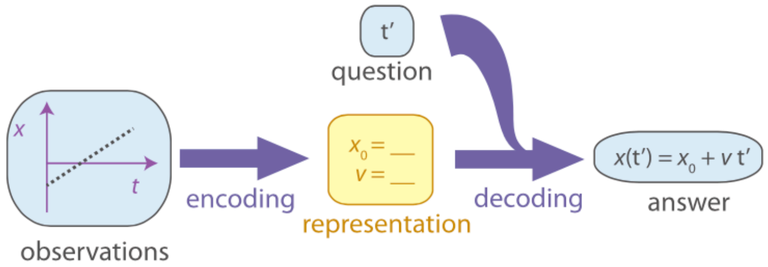
RIKEN Center for Interdisciplinary Theoretical and Mathematical Sciences
Institute for Physics of Intelligence
MLPhYs Foundation of "Machine Learning Physics"
Grant-in-Aid for Transformative Research Areas (A)
文部科学省 MINISTRY OF EDUCATION, CULTURE, SPORTS, SCIENCE AND TECHNOLOGY-JAPAN
BOOST 国家戦略分野の若手研究者及び博士後期課程学生の育成事業 次世代AI人材育成プログラム（若手研究者支援）
国立研究開発法人 科学技術振興機構

From Solving Inverse Problems in QCD to Inverse Modeling

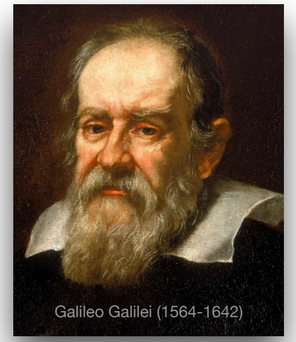
Lingxiao WANG(王 凌霄)
RIKEN/UTokyo

The 7th KMI International Symposium, Nagoya University
Online, 11 November 2026

Machine Learning and Physics



Phys.Rev. Lett. **124**, 010508 (2020)

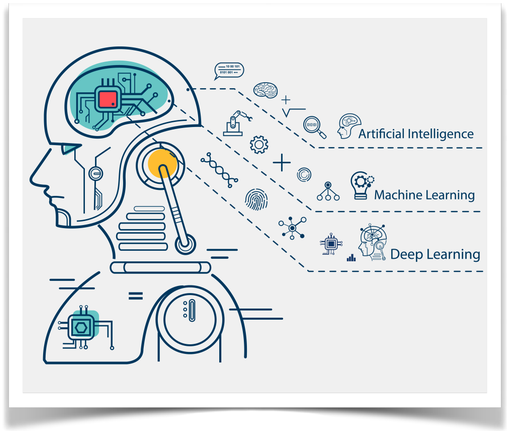


Galileo Galilei (1564-1642)

An inverse problem in science is the process of inferring from a set of observations the causal factors that produced them.



Data, X



Machine, $\{\theta\}$

Prediction $p(X|\theta)$

Estimation

$$\hat{\theta} = \textbf{arg max}_{\theta}\{p(X|\theta)\}$$

Machine Learning and Inference

Maximum Likelihood Estimation(MLE)

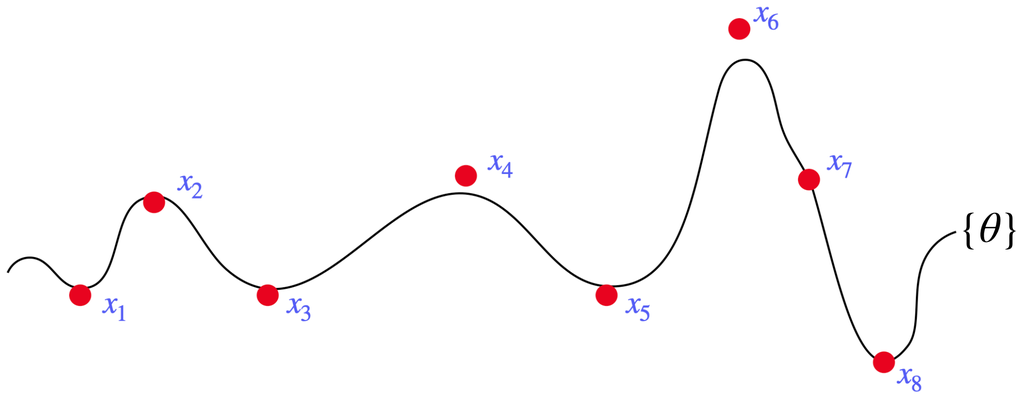
$$\max_{\theta} \prod_{i=1}^{N} p(\mathbf{x}_i \mid \theta)$$

Bayesian Inference

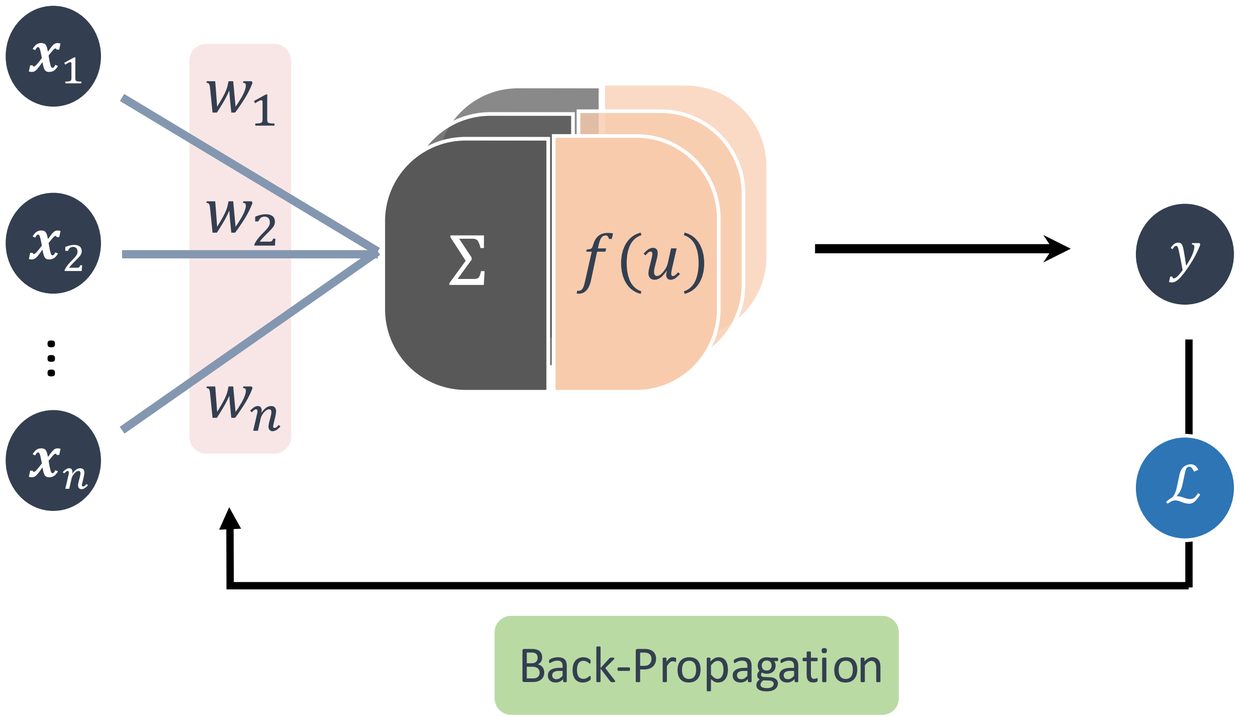
Maximum A Posterior(MAP)

$$p(\theta \mid X) = \frac{p(X \mid \theta)\pi(\theta)}{p(X)}$$

Posterior $p(\theta \mid X)$**, Prior** $\pi(\theta)$ **, Evidence** $p(X)$



Deep Model as Machine



Deep (neural network) Model

- **Inputs,** $\{x\} = x_1, x_2, \ldots, x_n$
- **Weights,** $\{w\} = w_1, w_2, \ldots, w_n$
- **Outputs,** y
- **Summation,** $\Sigma(\cdot)$
- **Non-Linear Activation Functions,** $f(u)$
- **Single Layer** $y = f(\Sigma_{i=1}^{n} x_i w_i)$

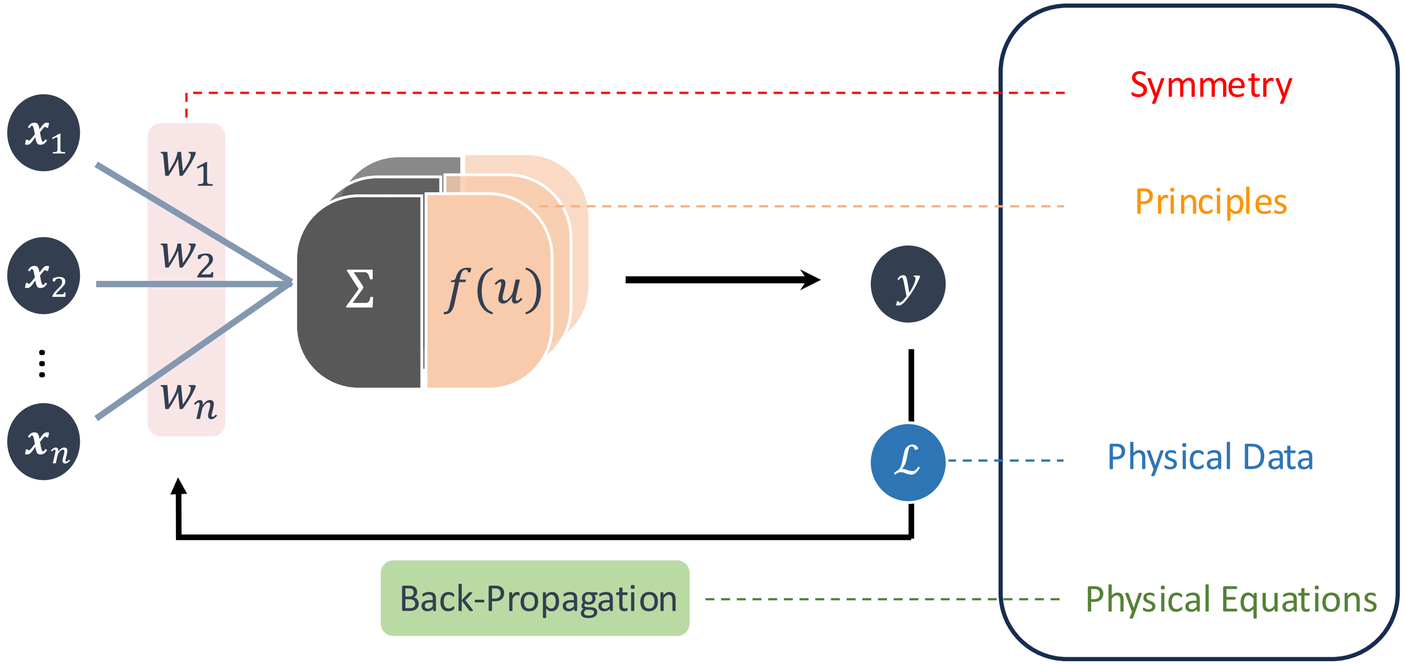
Objective

- **Loss Function,** $\mathscr{L}(y, \hat{y})$
- **Data,** $\hat{y}$

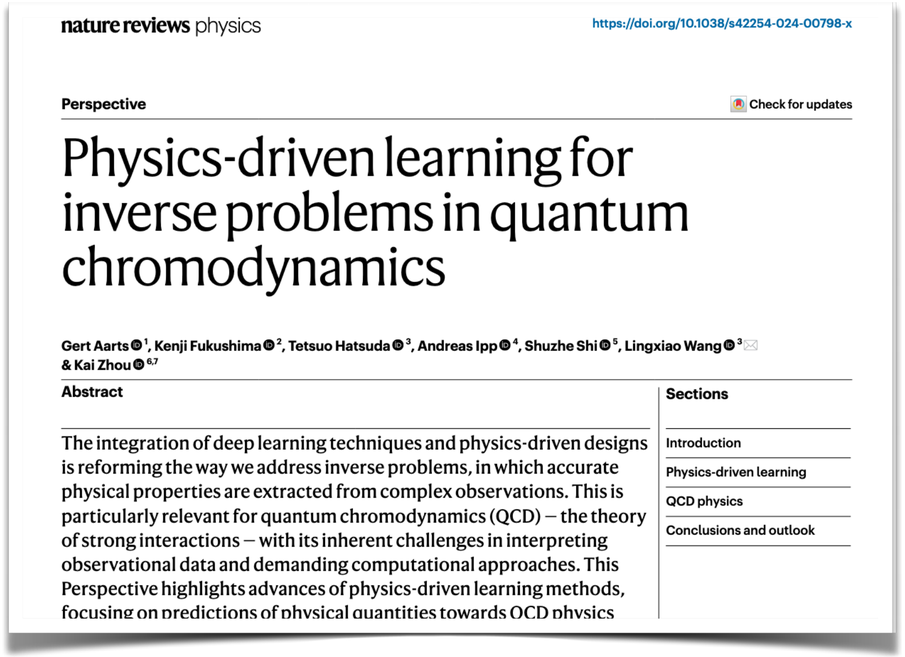
Optimization Algorithm

- **Back-Propagation,** $\dfrac{\partial \mathscr{L}}{\partial \omega}$
- **Stochastical Methods: SGD, Adam, …**

Physics-Driven Desgins



Nature Reviews Physics | Volume 7 | March 2025 | 154–163



nature reviews physics

https://doi.org/10.1038/s42254-024-00798-x

Perspective

Check for updates

Physics-driven learning for inverse problems in quantum chromodynamics

Gert Aarts[1], Kenji Fukushima[2], Tetsuo Hatsuda[3], Andreas Ipp[4], Shuzhe Shi[5], Lingxiao Wang[3] & Kai Zhou[6,7]

Abstract

The integration of deep learning techniques and physics-driven designs is reforming the way we address inverse problems, in which accurate physical properties are extracted from complex observations. This is particularly relevant for quantum chromodynamics (QCD) – the theory of strong interactions – with its inherent challenges in interpreting observational data and demanding computational approaches. This Perspective highlights advances of physics-driven learning methods, focusing on predictions of physical quantities towards QCD physics

Sections

Introduction

Physics-driven learning

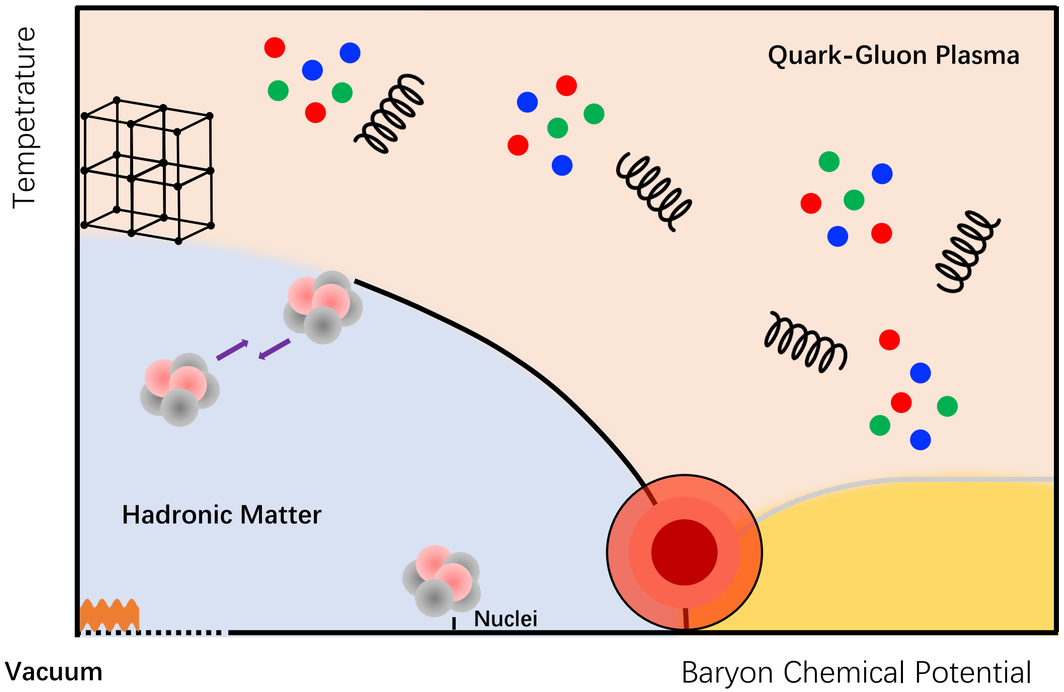
QCD physics

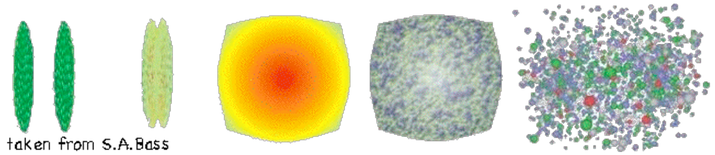
Conclusions and outlook

Physics Knowledge for Designing
More Effcient and Reliable Deep Models

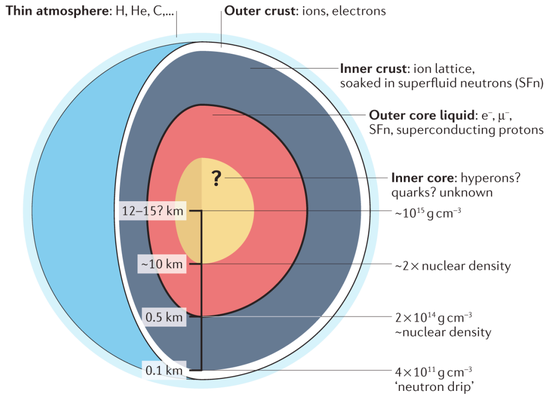
Inverse Problems in QCD

Prog.Part.Nucl.Phys. 104084(2023)

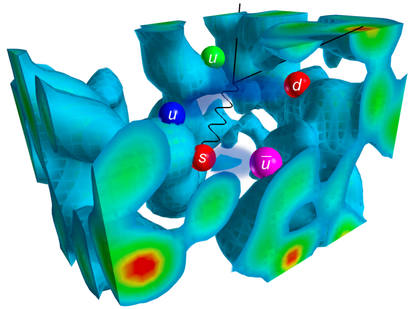




Heavy-Ion Collisions



Nat. Rev. Phys. 4, 237–246 (2022)



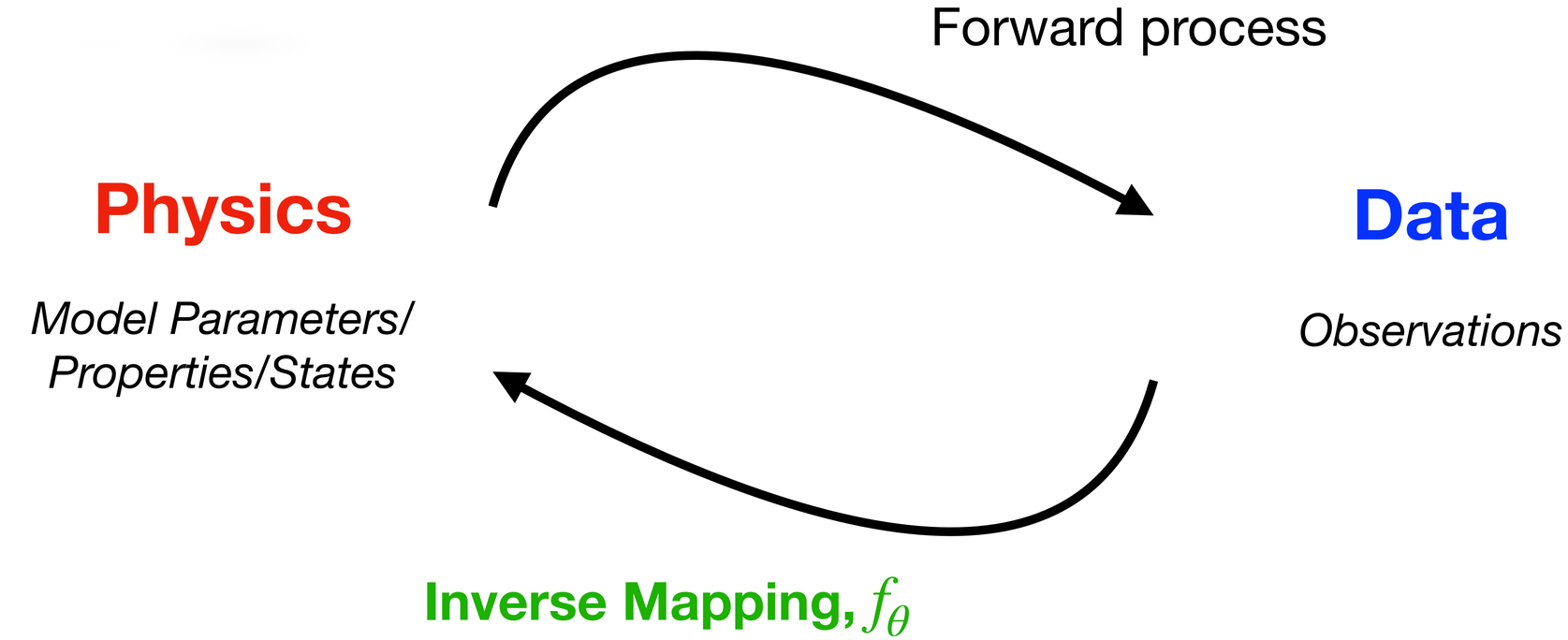
Lattice QCD © Derek Leinweber/CSSM/University of Adelaide

Exploring QCD matter in three "**labs**",

- **Heavy-Ion Collisions** : compress matter to **high-**T and **high-**μ_B
- **Neutron Star** : dense matter, merger events at **low-**T and **high-**μ_B
- **Lattice QCD** : numerically solve QCD Lagrangian at **finite-**T and $\mu_B \sim 0$

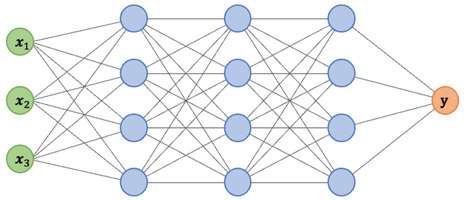
Data-Driven Learning

$$f_\theta : X \rightarrow Y$$

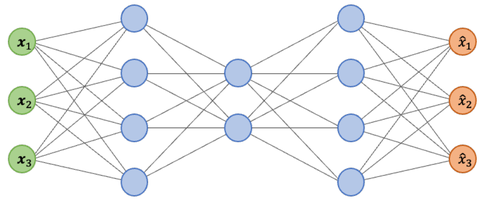


Data-Driven Learning

$$f_\theta : X \rightarrow Y$$



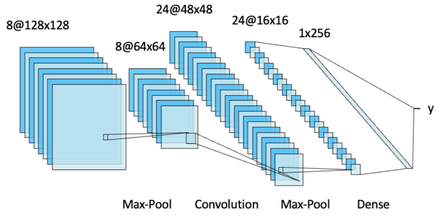
Deep Neural Network



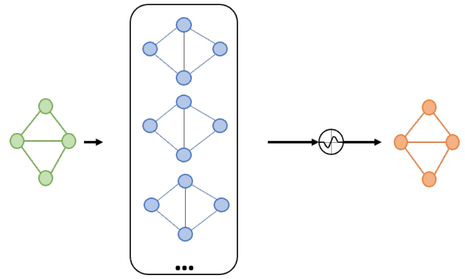
AutoEncoder

Universal Approximation Theorem (1989,1991)

A feed-forward network with *a single hidden layer* containing *a finite number of neurons* can approximate arbitrary continuous functions.



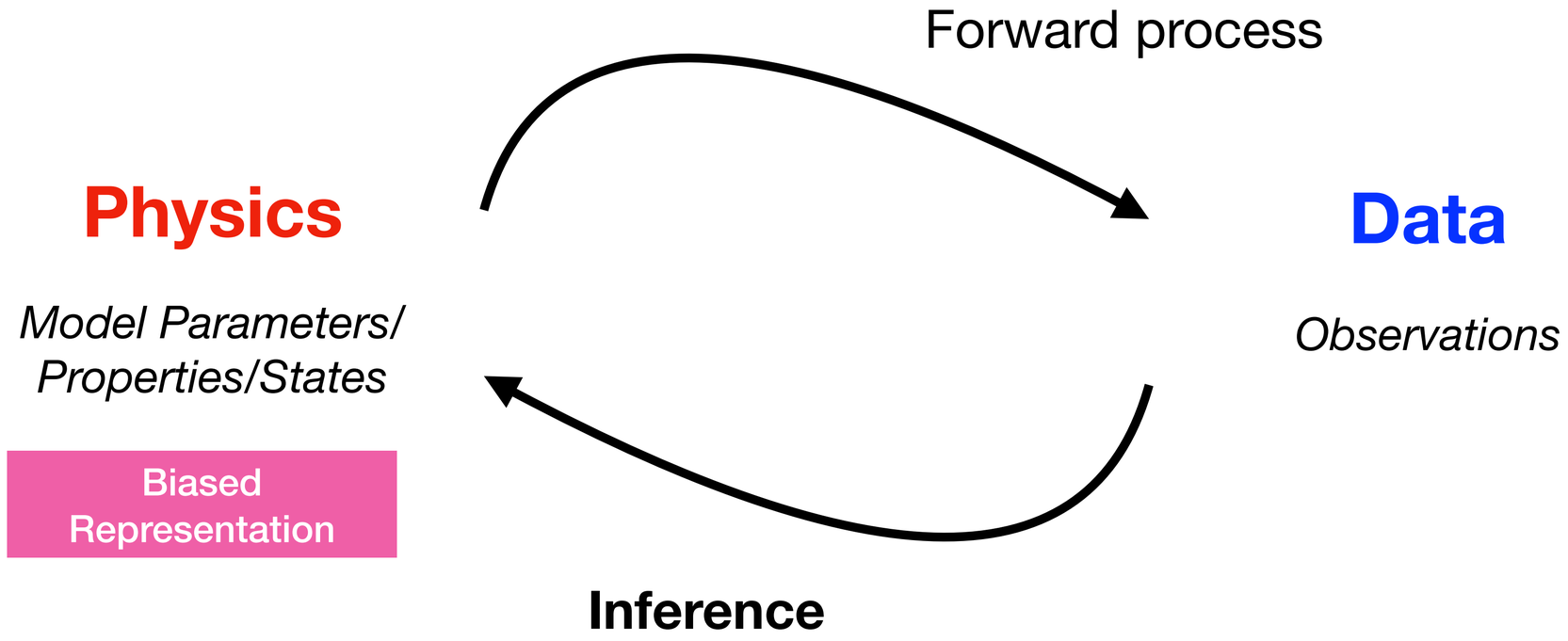
Convolutional Neural Network



Graph Neural Network

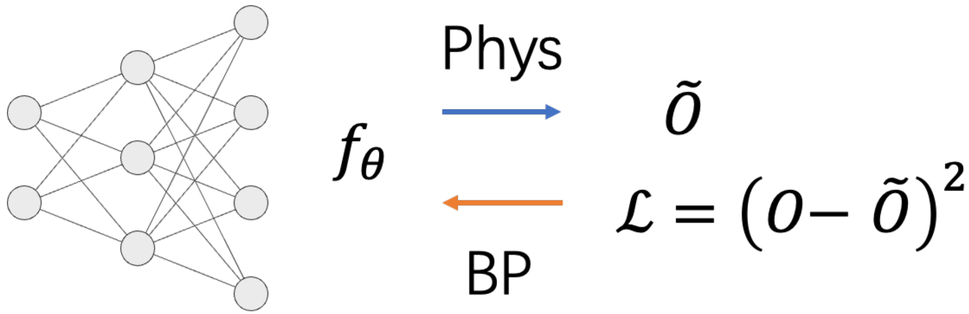
Bayesian Inference

$$\hat{\theta} = \arg\max_{\theta}\{p(X \mid \theta)\}$$



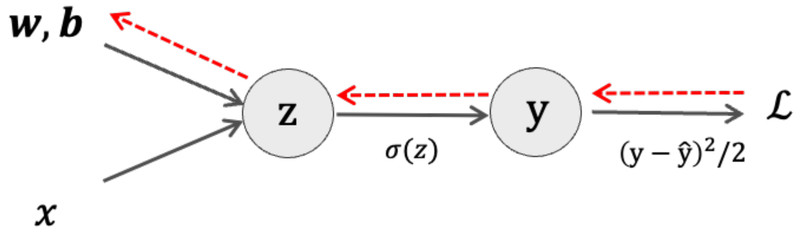
Physics-Driven Deep Learning

$$\hat{\theta} = \arg\max_{\theta}\{p(X \mid \theta)\}$$



Deep Neural Network represented Physics, f_θ

Flexible Representation



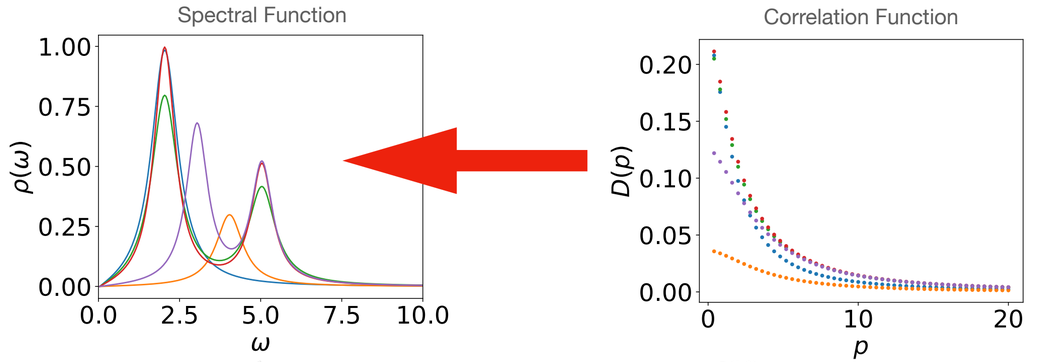
Back-Propagation

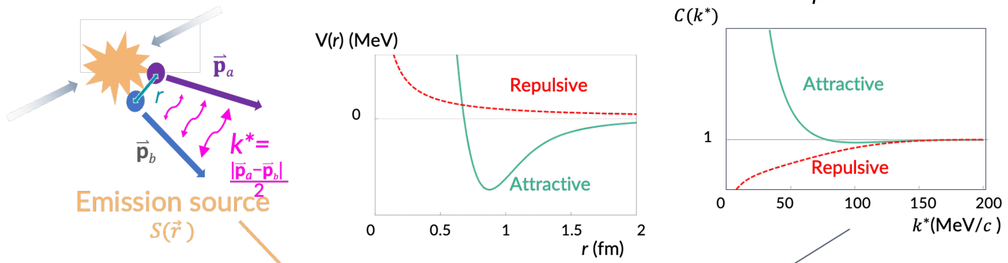
Easy-To-Compute on GPUs

Inverse Problems in QCD

Physics

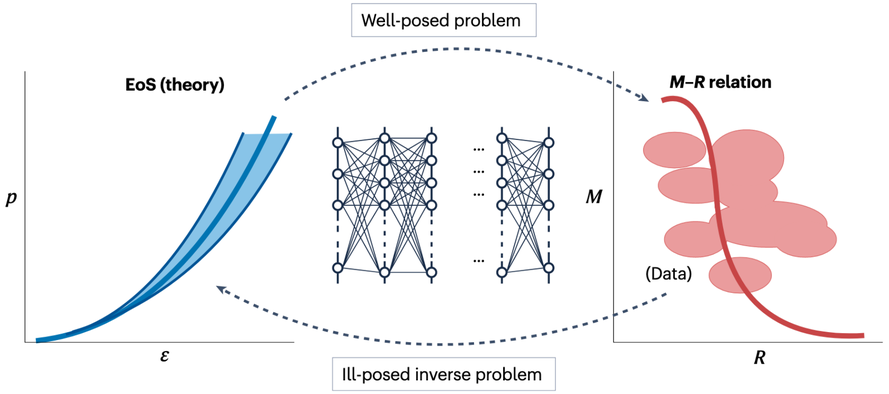
Data





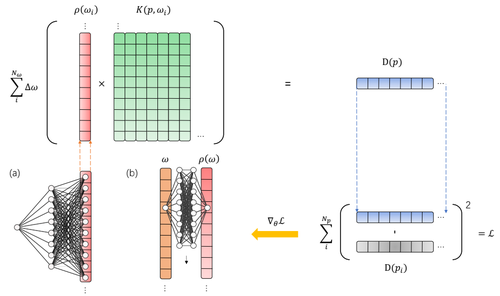
Two-particle wave function

$$C(k^*) = \int S(\vec{r}\,)\left|\psi(\vec{k}^*,\vec{r}\,)\right|^2 d^3\vec{r} = \mathcal{N}(k^*)\frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$



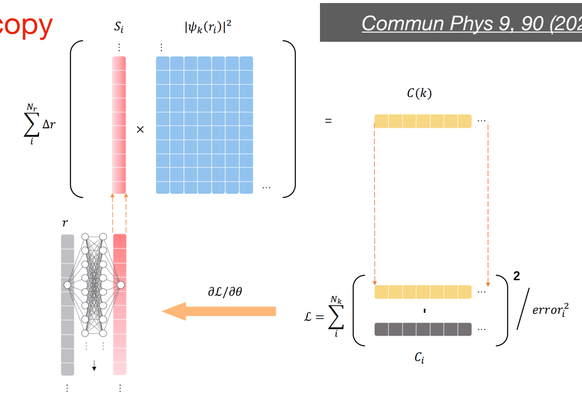
Lattice

Phys. Rev. D 106, L051502; Comput. Phys. Commun. 282, 108547 (2023)

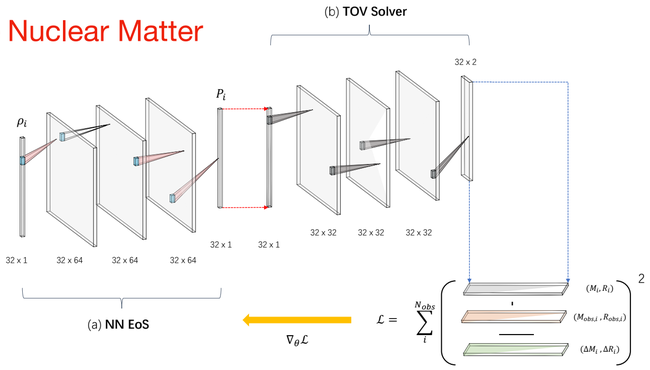


Femtoscopy

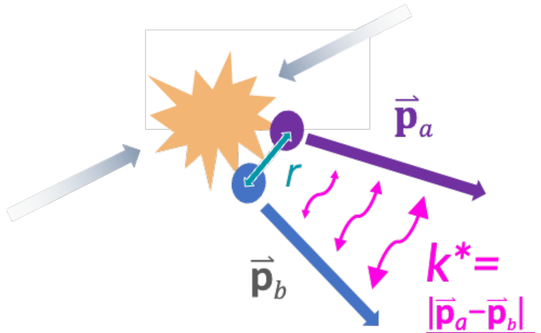
Commun Phys 9, 90 (2026)



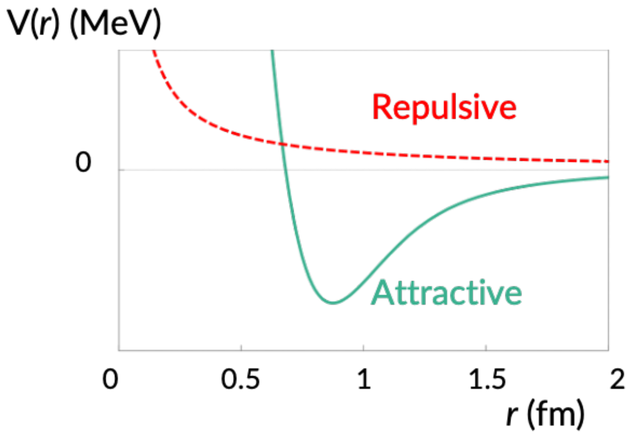
Dense Nuclear Matter

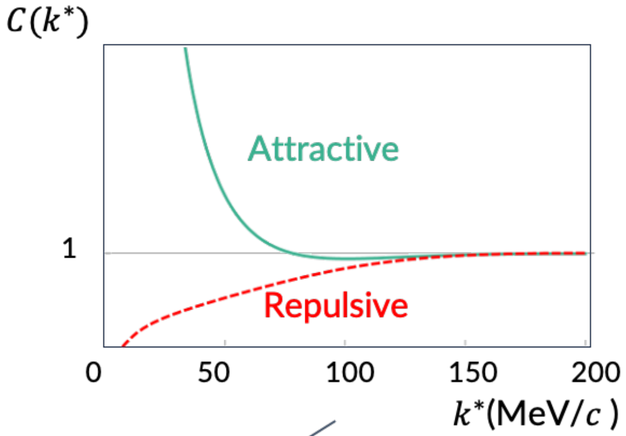


Femtoscopy



Emission source $S(\vec{r})$





Correlations

↓

Scattering Length
Effective Range

Two-particle wave function

$$C(k^*) = \int S(\vec{r}\,)\left|\psi(\vec{k}^*,\vec{r}\,)\right|^2 d^3\vec{r} = \mathcal{N}(k^*)\frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

Lednicky-Lyuboshits(LL) analytic model
(Asymptotic wave-function+
Effective range correlation+
Gaussian source)

Lednicky, Lyuboshits, Sov.J.Nucl.Phys. 35 (1982) 770

Raffaele Del Grande | XQCD 2023

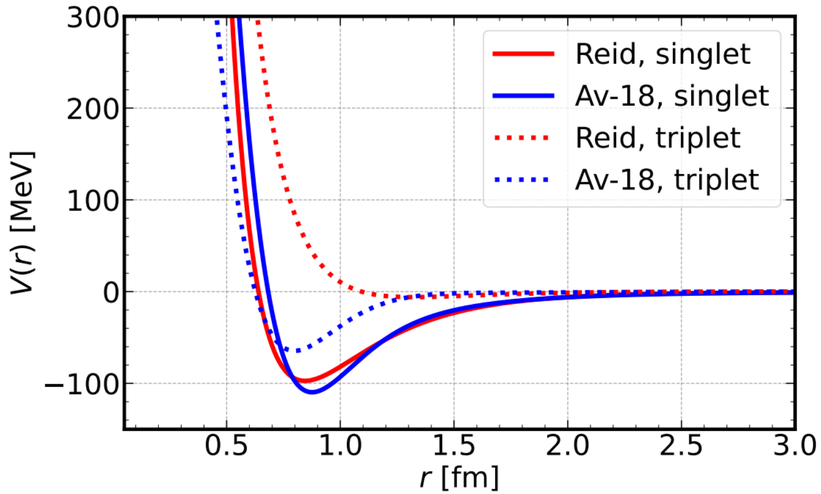
Learning Hadron Emitting Sources

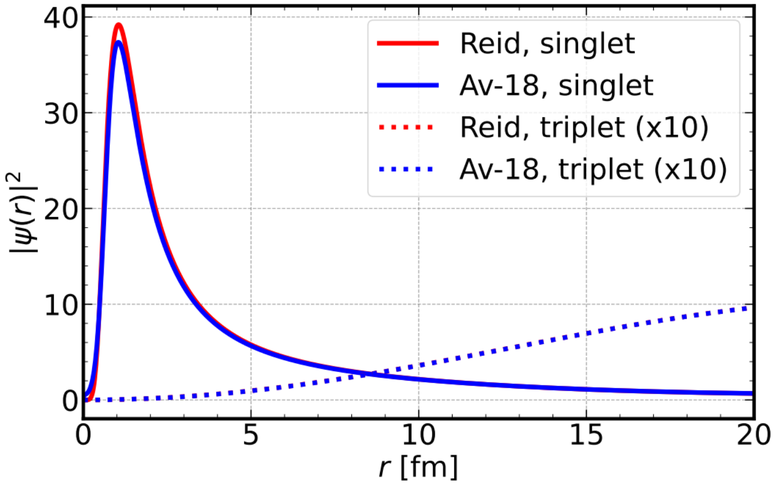
Can we learn hadron-hadron interactions in Femtoscopy

Commun.Phys. 9 (2026) 1, 90

from a source function(SF) perspective

- Identify source function beforehand
  1. Given the known interactions
     - **N-N, e.g., AV-18, Reid, etc.**
  2. Represent SFs with deep neural networks
  3. Optimize SF from CF observations
- Learned source functions for
  - e.g., N-Lambda interactions





$$\frac{d^2u^S_{k,l}(r)}{dr^2} = \left(2\mu V(r) + \frac{l(l+1)}{r^2} - k^2\right) u^S_{k,l}(r)$$

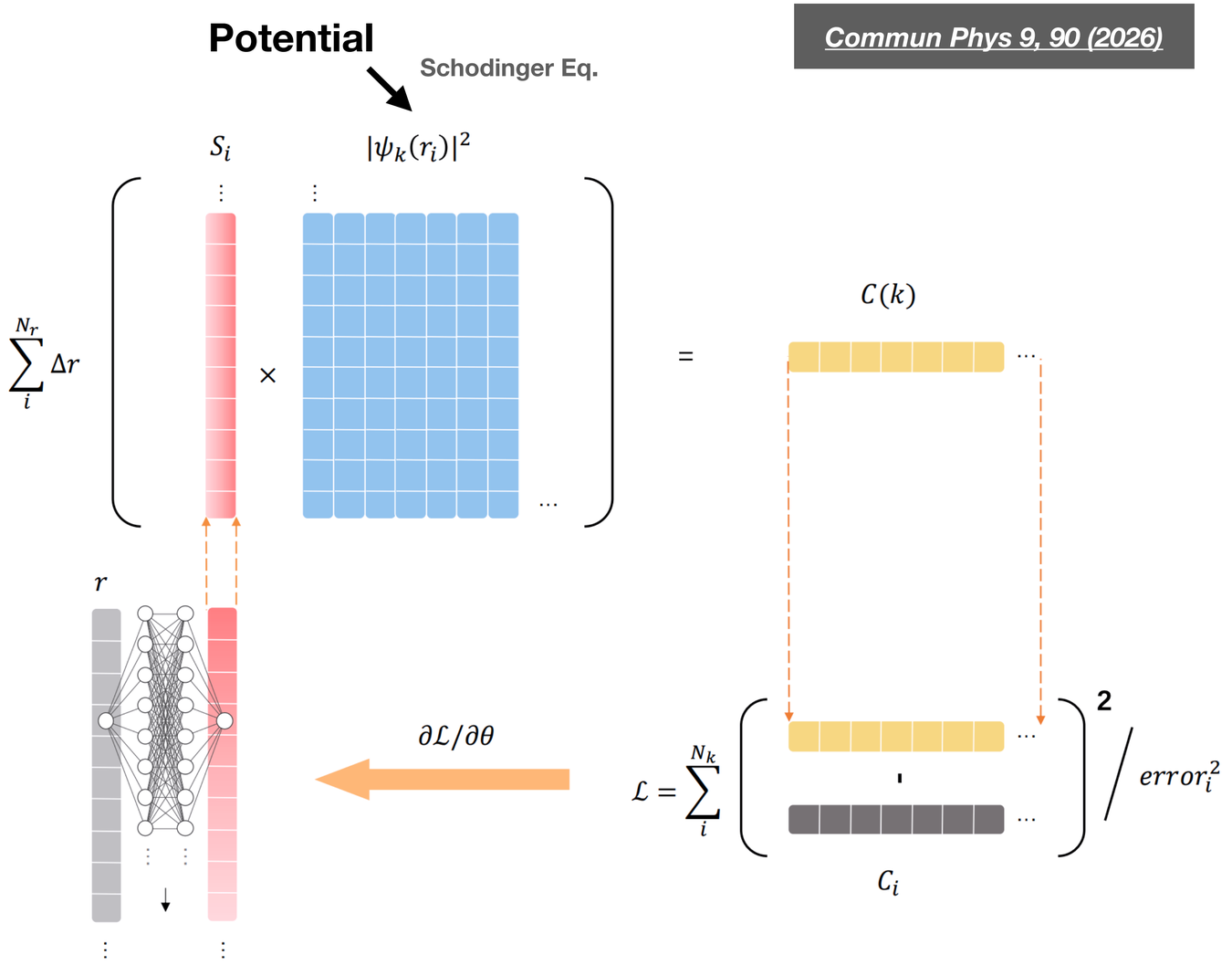
$$\psi^S_k(\mathbf{r}) = \sum_{l=0}^{l_{\max}} (2l+1) i^l \frac{u^S_{k,l}(r)}{r} P_l(\cos\theta)$$

$$C(k) = \int S(r)\left(\frac{1}{4}|\psi^{S=0}_k(r)|^2 + \frac{3}{4}|\psi^{S=1}_k(r)|^2\right) dr$$

Learning Hadron Emitting Sources

Building Source Function(SF)

- Identify source function beforehand
  1. Given the known interactions
     - N-N, e.g., AV-18, Reid, etc.
  2. **Represent SFs with deep neural networks**
  3. Optimize SF from CF observations
- Learned source functions for
  - e.g., N-Lambda interactions



Commun Phys 9, 90 (2026)

Learning Hadron Emitting Sources

Optimization through BP

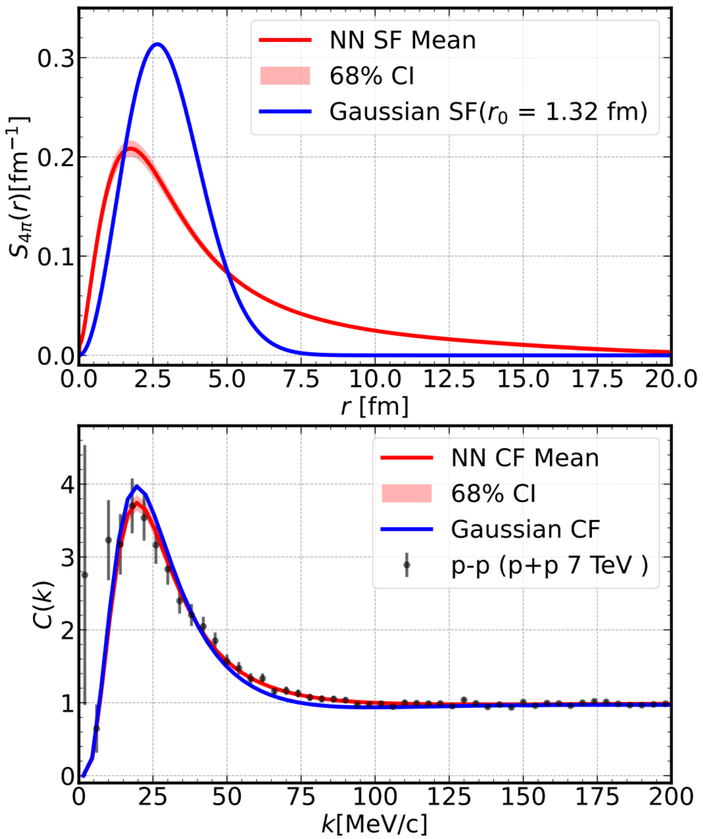
Commun Phys 9, 90 (2026)

- Identify source function beforehand
  1. Given the known interactions
     - N-N, e.g., AV-18, Reid, etc.
  2. Represent SFs with deep neural networks
  3. **Optimize SF from CF observations**

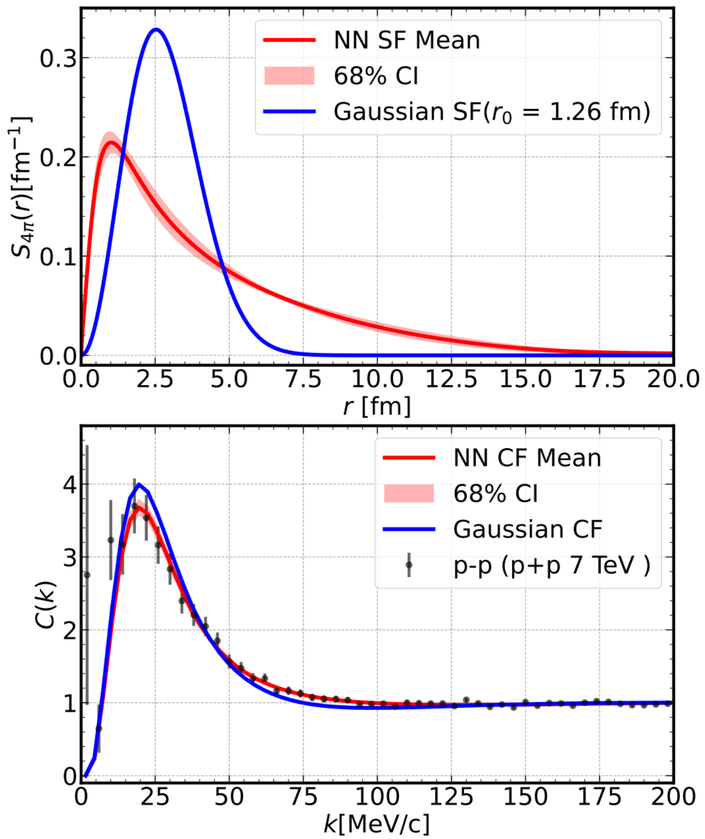
ALICE, PRC 99, 024001(2019)

- Learned source functions for
  - e.g., N-Lambda interactions

Reid



AV-18



Investigating Hadron-Hadron Interactions

N-Lamda Interaction

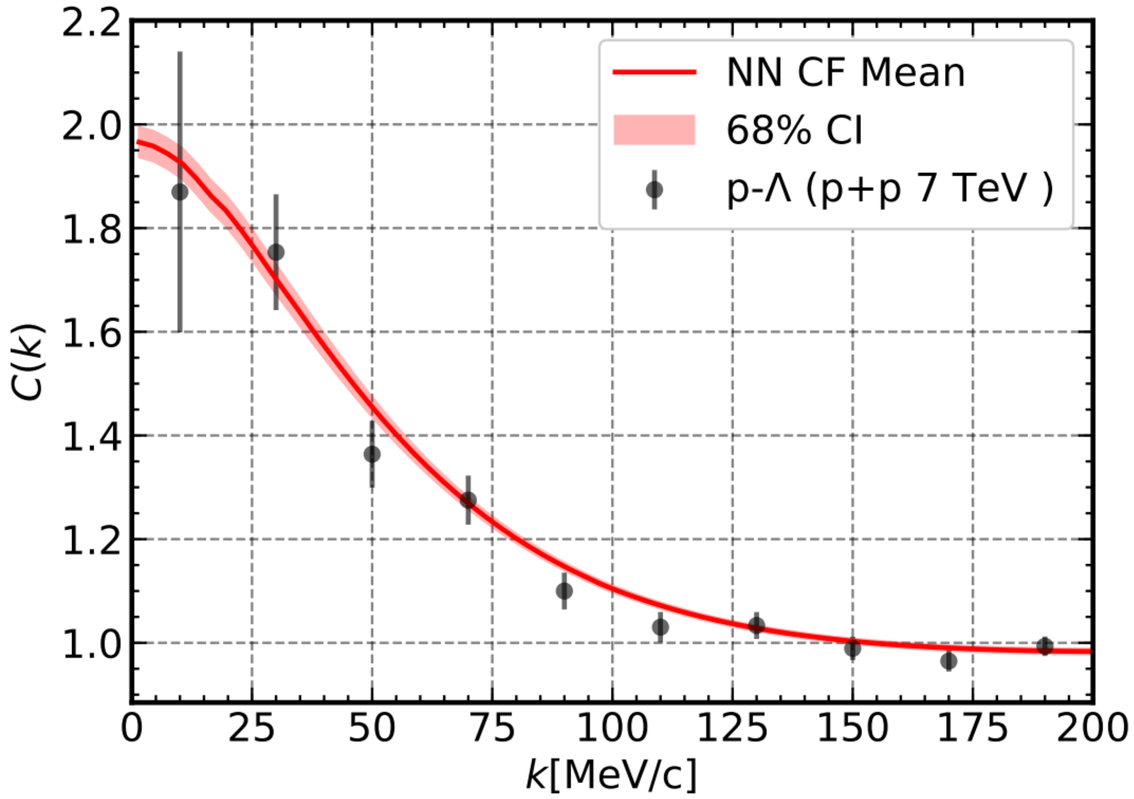
Commun Phys 9, 90 (2026)

- Identify source function beforehand
  1. Given the known interactions
     - N-N, e.g., AV-18, Reid, etc.
  2. Represent SFs with deep neural networks
  3. Optimize SF from CF observations

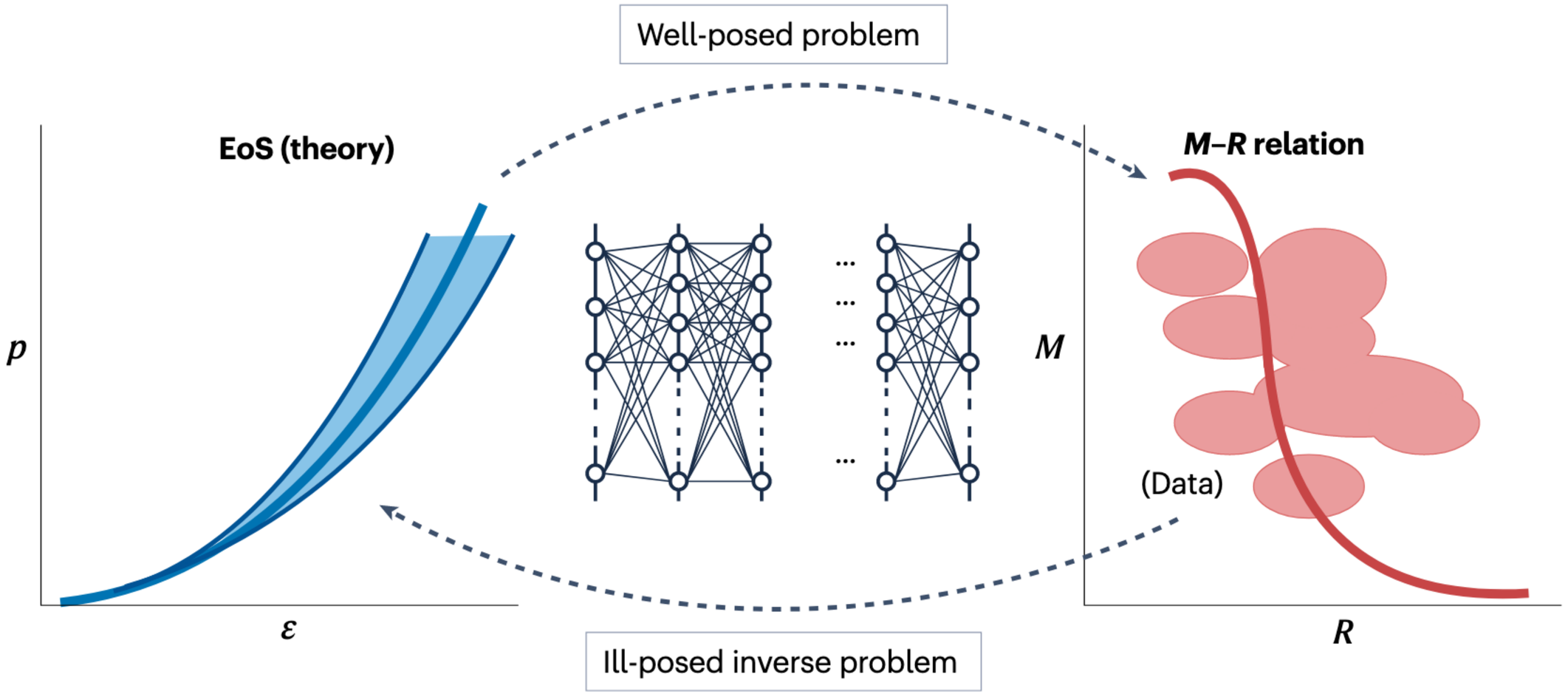
ALICE, PRC 99, 024001(2019)

- Learned source functions for
  - **e.g., p-Lambda interactions**

Usmani form with $W_C = 2250$ MeV



Dense Nuclear Matter EoS



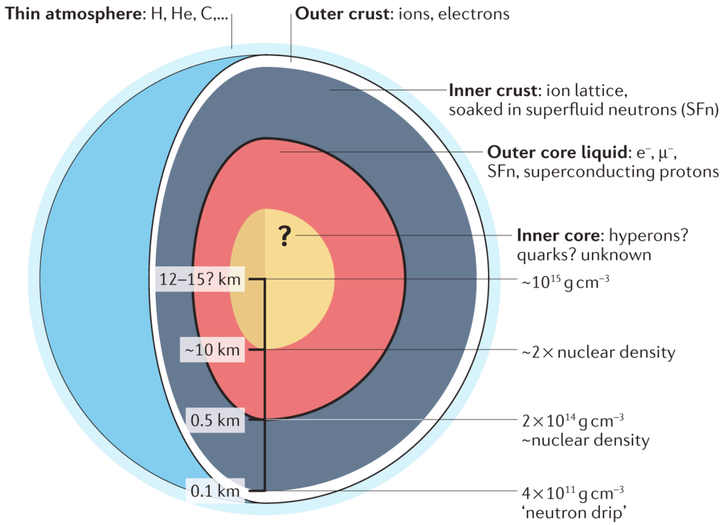
Neutron Star EoS

Tolman–Oppenheimer–Volkoff equations

$$\frac{dP}{dr} = -G\frac{m(r)\varepsilon(r)}{r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{m(r)}\right)\left(1-\frac{2Gm(r)}{r}\right)^{-1}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \epsilon(r)$$

Hydrostatic condition in each shell (dr)



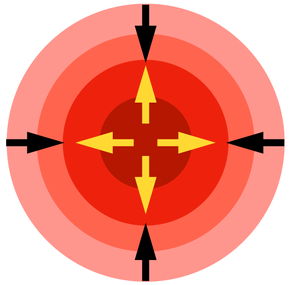
Nat. Rev. Phys. 4, 237–246 (2022)

EoS $P(\varepsilon) = 0$

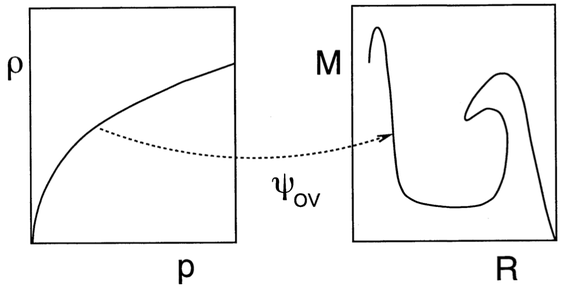
Core $r = 0, \varepsilon(r=0) = \varepsilon_c, P(r=0) = P(\varepsilon_c)$

Surface $r = R, \varepsilon(r=R) \simeq 0, M = \int 4\pi r^2 \varepsilon(r) dr$

$$M, R$$

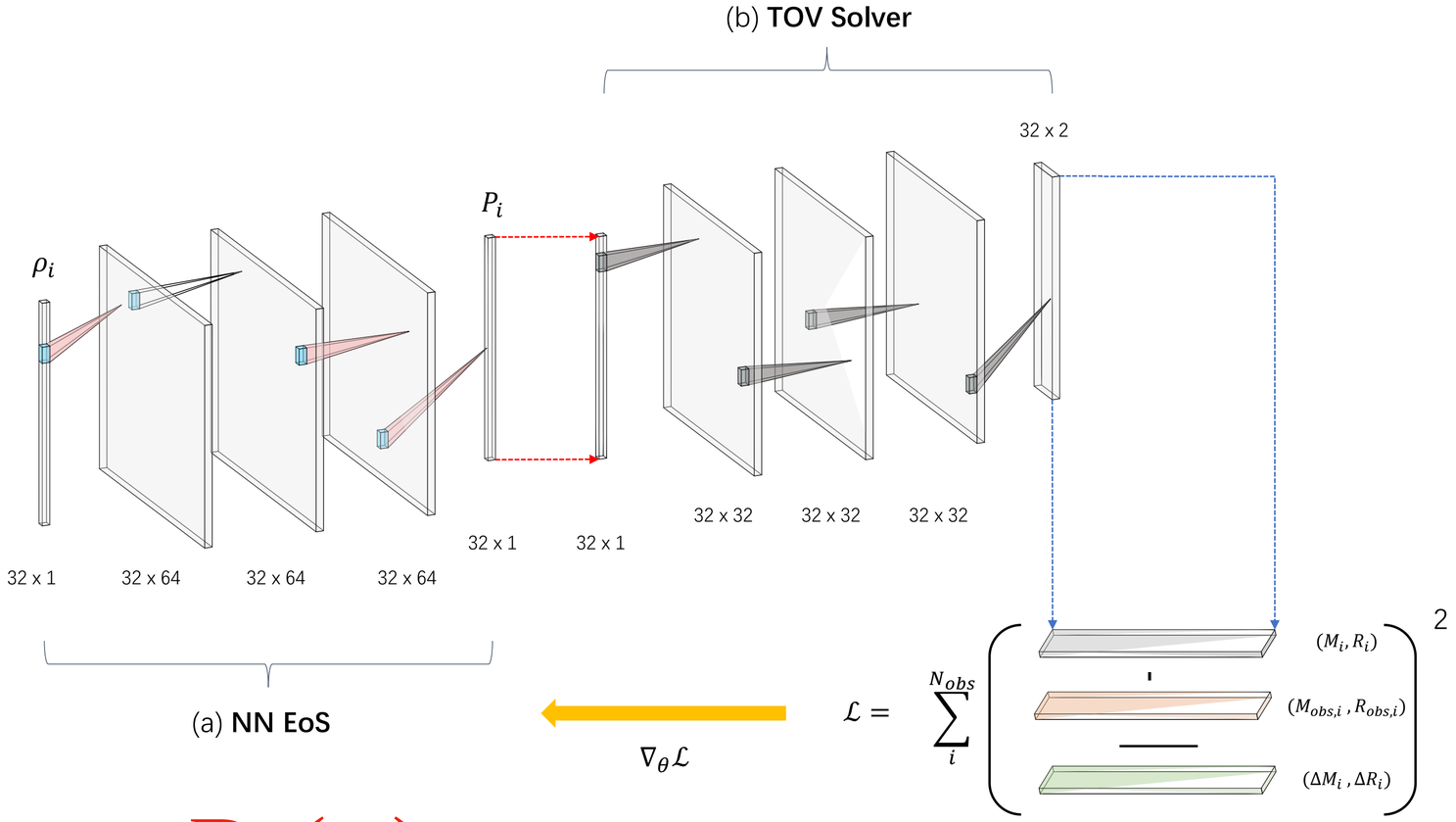


Pressure → ← Gravity



L. Lindblom, *A.J., 398, 569 (1992).*
If the **whole M(R)** is known,it's well-defined problem.

Physics-Driven Deep Learning



$P_\theta(\rho)$

Phys. Rev. D 107, 083028; JCAP08 (2022) 071

$$\mathscr{L} = \chi^2 = \sum_{i=1}^{N_{\rm obs}} \frac{\left(M_i - M_{{\rm obs},i}\right)^2}{\Delta M_i^2} + \frac{\left(R_i - R_{{\rm obs},i}\right)^2}{\Delta R_i^2}$$

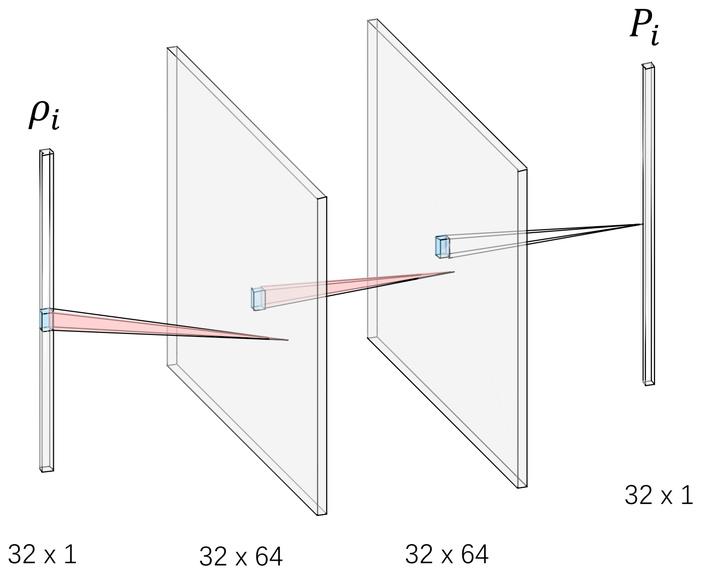
Loss function

- (M_i, R_i) : predicted Mass-Radius
- $(M_{obs,i}, R_{obs,i})$: observed Mass-Radius
- $(\Delta M_i, \Delta R_i)$: uncertainties

Neural Network EoS

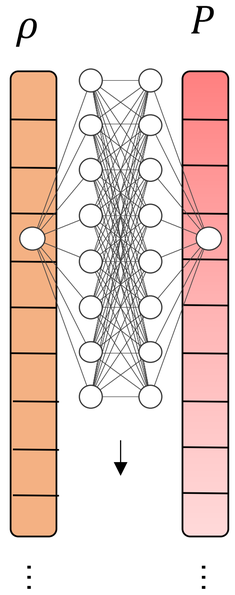
Phys. Rev. D 107, 083028; JCAP08 (2022) 071

Module A. NN EoS



A Trainable Neural Network

$\simeq$



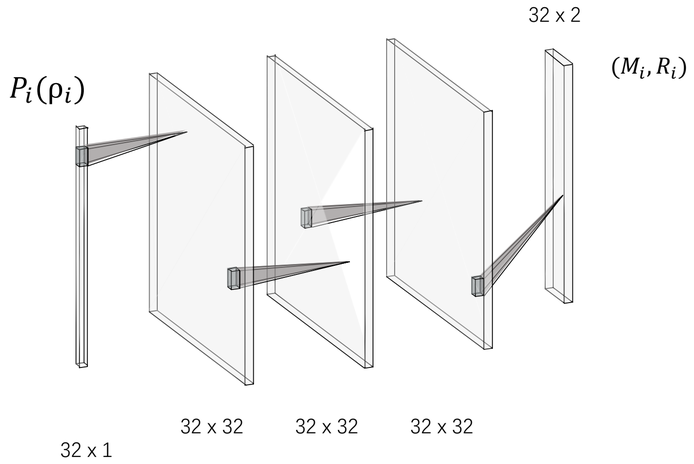
$\{\theta\}$: weights and bias of the neural network
Size of $\{\theta\}$ = 4353

NS crust: **DD2**, inner: $P_\theta(1.1\rho_{\rm sat} \leq \rho)$

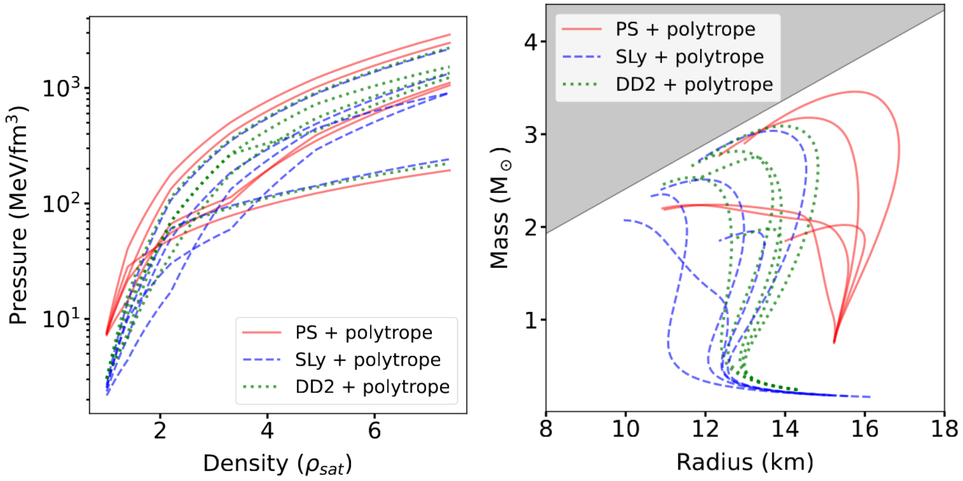
Neural Network TOV Solver

Phys. Rev. D 107, 083028; JCAP08 (2022) 071

Module B. TOV eq. Solver



A **Pre-Trained** Neural Network

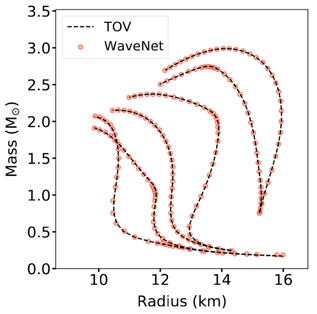


Training data-set

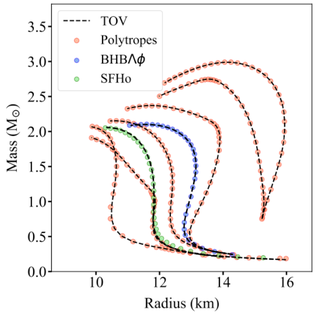
300,000 polytropic EoS functions with 3 low density models

$$P = K_i \rho^{\Gamma_i}, \quad \mathrm{i} = [1,5], \quad 1.1\rho_{\mathrm{sat}} \leq \rho \leq 7.4\rho_{\mathrm{sat}}$$

In-set testing data-set

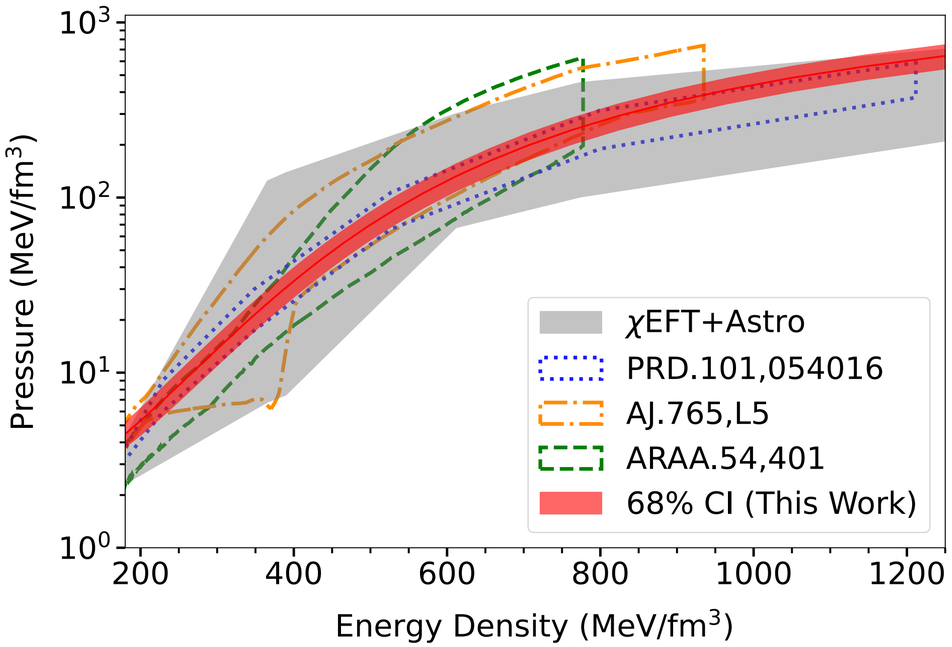


Out-set testing data-set



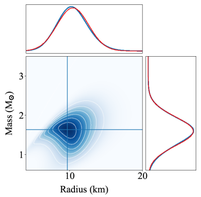
Results: EoS and M-R

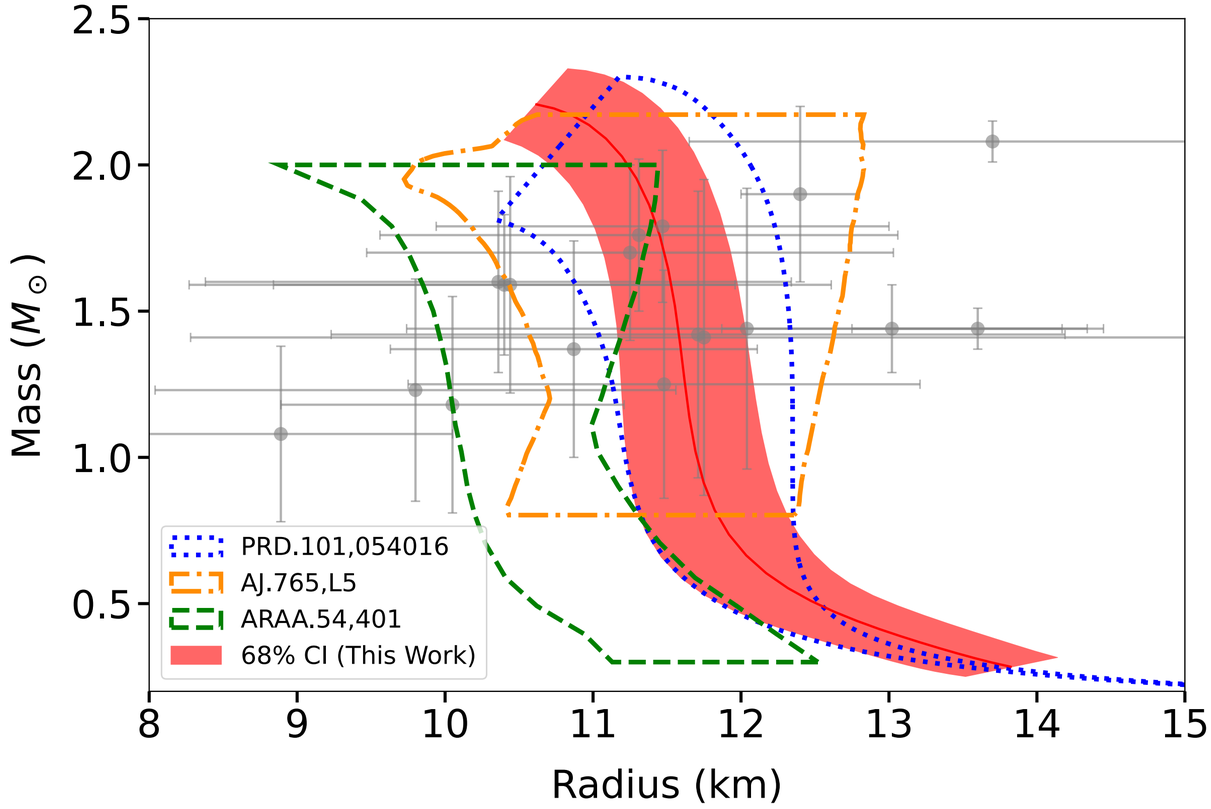
Blue dots: NN results, Fujimoto-Fukushima-Murase
Yellow and Green dashed lines: Bayesian Approaches



Phys. Rev. D 107, 083028

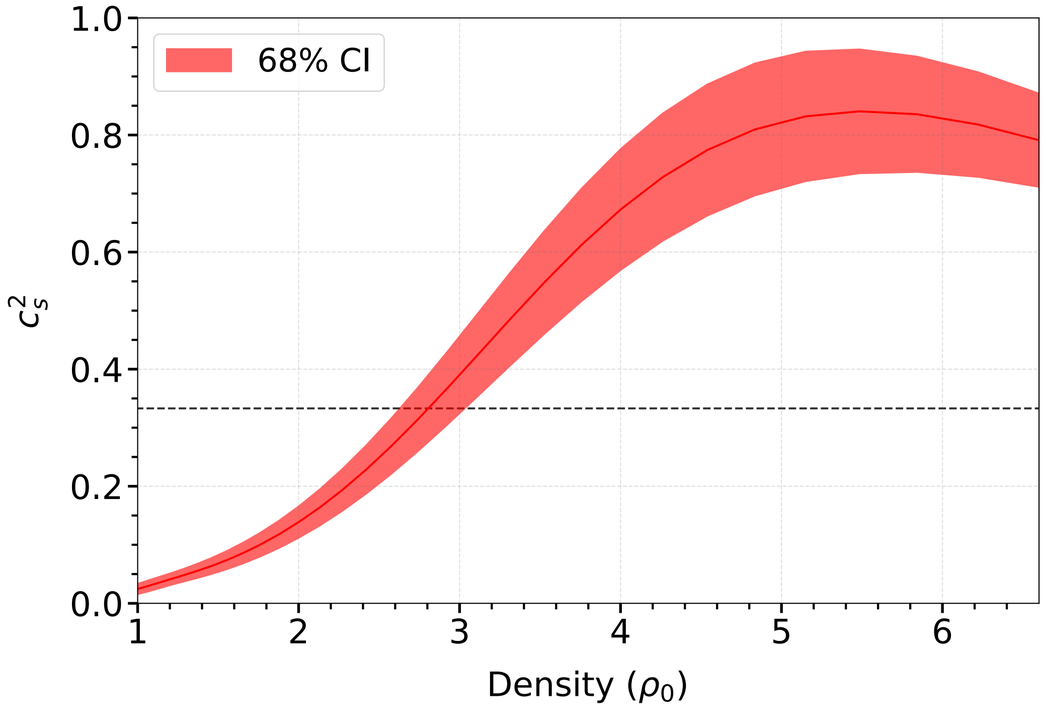
18 (M_i, R_i) , sample size = **10k**
causality ($d\epsilon/dp < 1$)
Maximum mass $\geq 1.9\,M_\odot$

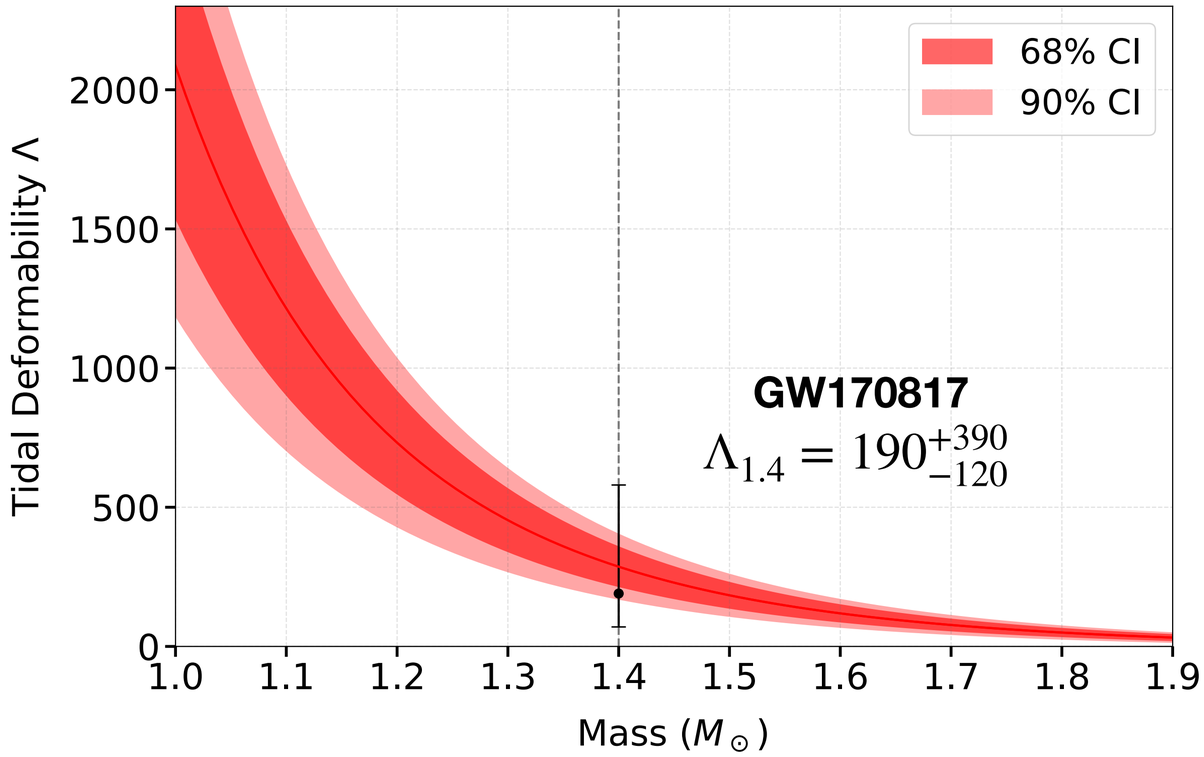




Our results: $R_{1.4} = 11.6 \pm 0.43$ **km (68% CI)**

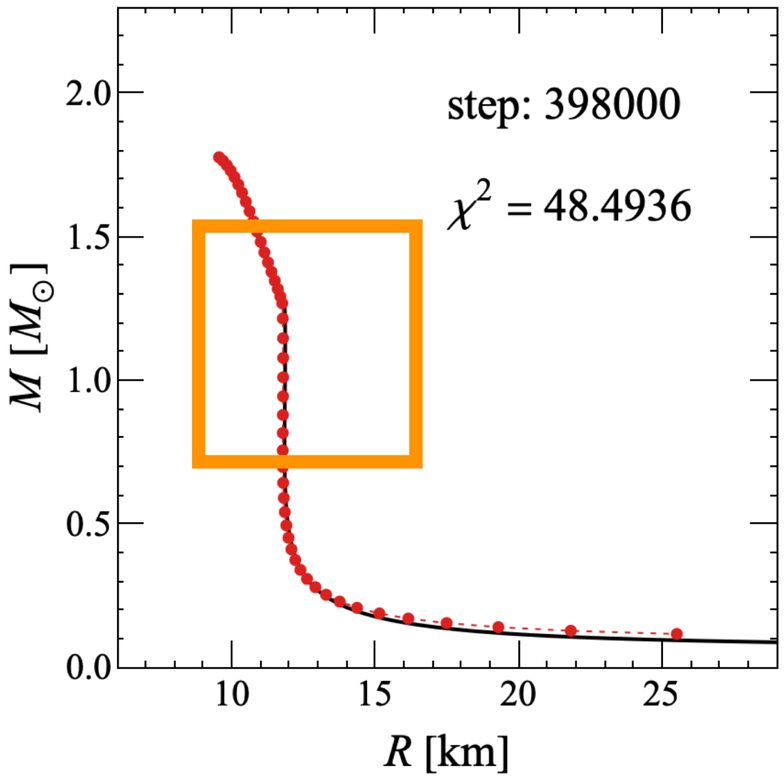
Results: Speed of Sound and Tidal Deformability

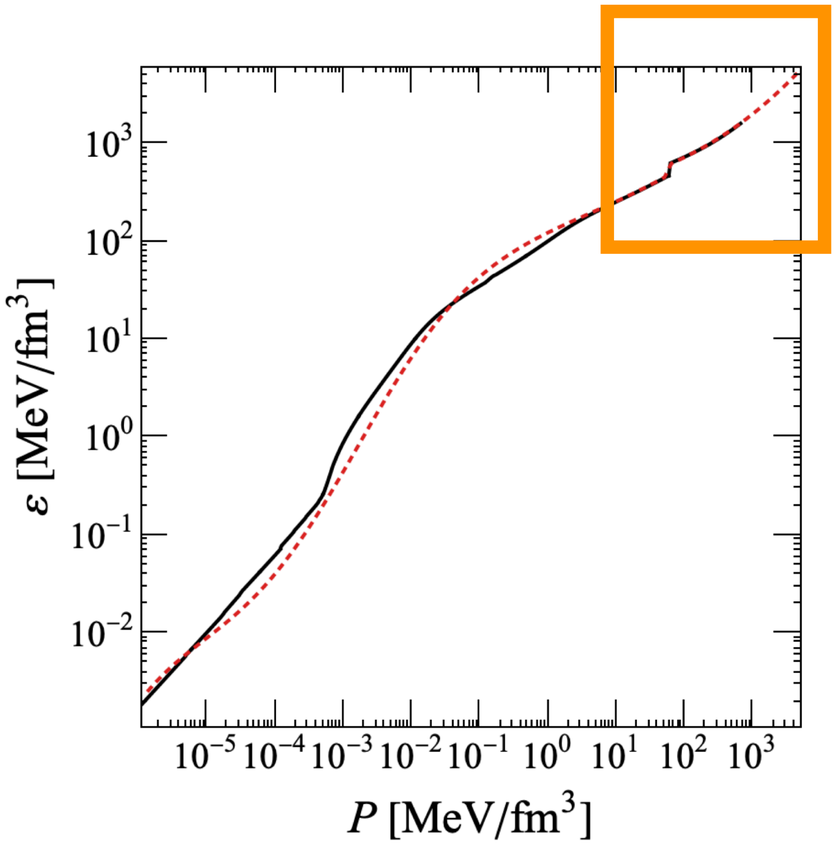




Our results, $\tilde{\Lambda}_{1.4} = 286.47^{+115.9}_{-115.9}$

Ablity to Detect Phase Transition





PhysRevD.111.074026

Able to capture first-order phase transition !

48 Neutron Stars

(24 in $M > M_\odot$)

Represent Speed of Sound

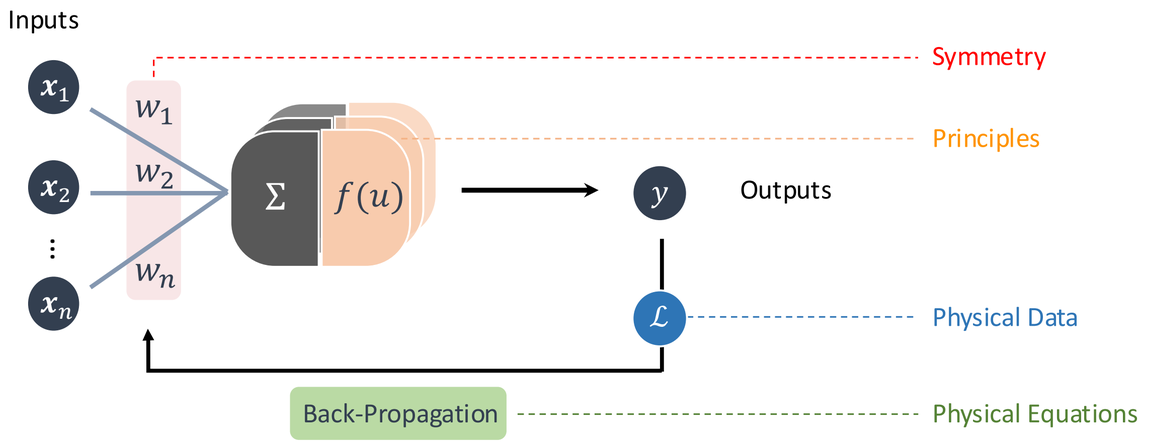
1.Microscopicly stable condition, $\frac{dp}{d\epsilon} \geq 0$

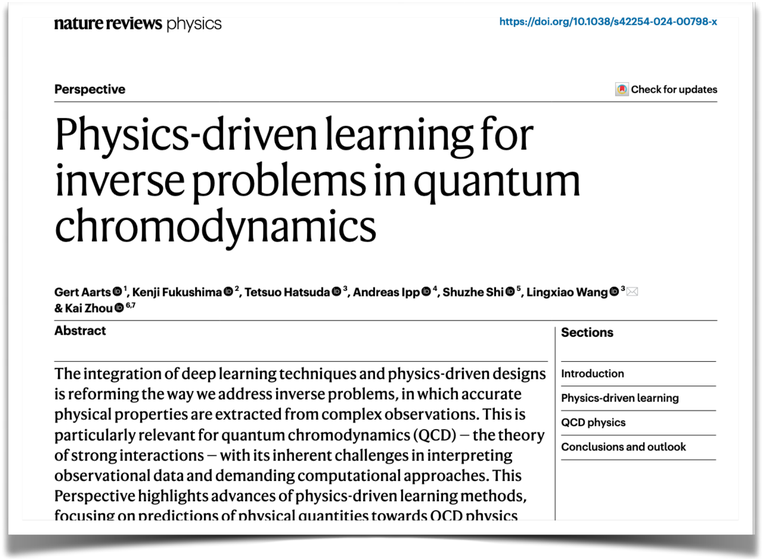
2.Causality condition, $\frac{dp}{d\epsilon} = \frac{c_s^2}{c^2} < 1$

Representing $c_s(\varepsilon) = \sigma(y(\varepsilon))$ with neural netowrks $y(\varepsilon)$, above two conditions can be naturally met with the ***sigmoid*** activation function, $\sigma(x) = 1/(1 + e^{-x})$.

Summary

- **Physics-Driven Desgins**
  - Symmetry
  - Physics Principles
  - Physical Data
  - Physical Equations
- **Inverse Problems**
  - Data-Driven Learning
    - Limited by labelled data
  - Bayesian Inference
    - Limited by finite paramters
  - **Physics-Driven Deep Learning**
    - Neural network representations
    - Embed physics priors explicitly
      - **Continuity** and **Positive Definetion** for SFs
      - **Continuity** and **Causality** for EoSs





nature reviews physics

https://doi.org/10.1038/s42254-024-00798-x

Perspective

Check for updates

Physics-driven learning for inverse problems in quantum chromodynamics

Gert Aarts[1], Kenji Fukushima[2], Tetsuo Hatsuda[3], Andreas Ipp[4], Shuzhe Shi[5], Lingxiao Wang[3] & Kai Zhou[6,7]

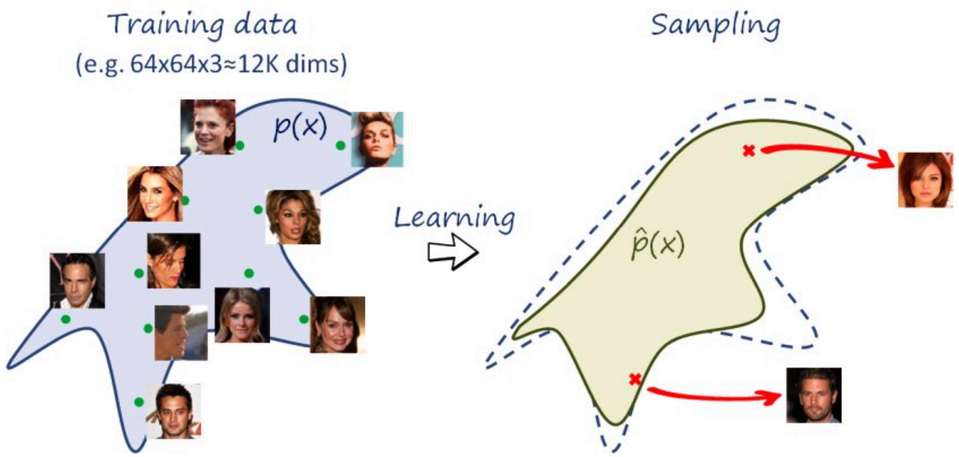
Abstract

The integration of deep learning techniques and physics-driven designs is reforming the way we address inverse problems, in which accurate physical properties are extracted from complex observations. This is particularly relevant for quantum chromodynamics (QCD) – the theory of strong interactions – with its inherent challenges in interpreting observational data and demanding computational approaches. This Perspective highlights advances of physics-driven learning methods, focusing on predictions of physical quantities towards QCD physics

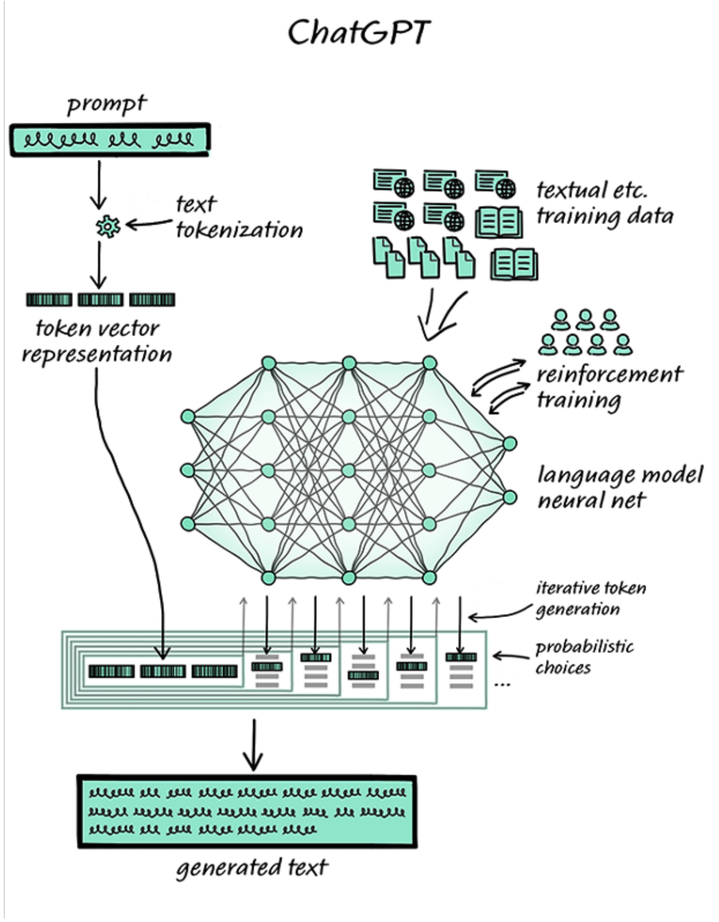
Sections

Introduction
Physics-driven learning
QCD physics
Conclusions and outlook

Generative AI



@blogs of OpenAI

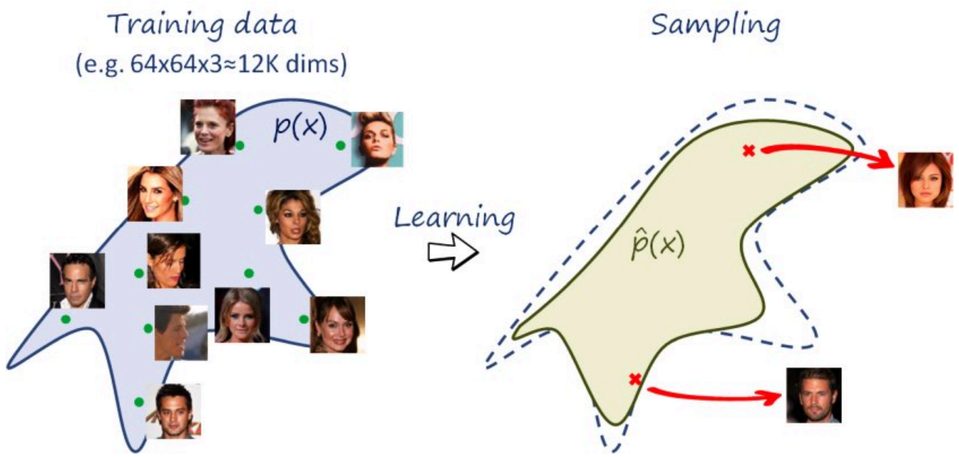


Generative models

→ **Underlying Distributions in Data**

$$\max_{\theta} \prod_{i=1}^{N} p_\theta(\mathbf{x}_i)$$

Probabilistic Models in Physics



@blogs of OpenAI

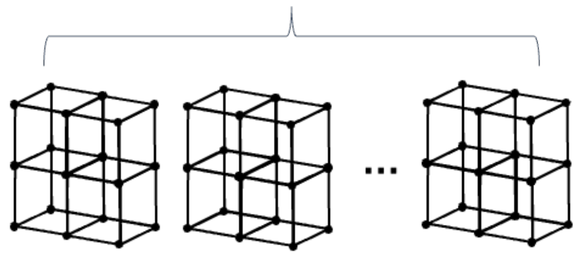
Generative models

→ **Underlying Distributions in Data**

$$p(\phi) = e^{-S(\phi)}/Z$$

$$\langle O \rangle \approx \frac{1}{N} \sum_i O_i$$

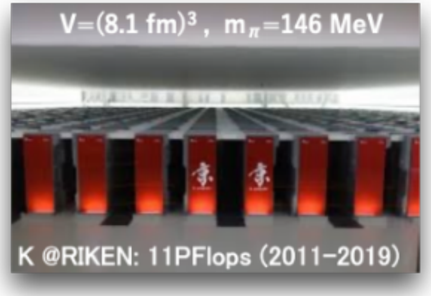
Ensemble

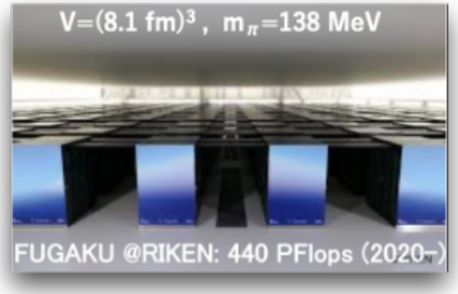


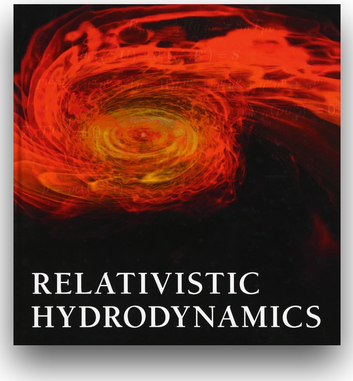
Lattice calculations

→ **physical distributions, sampling**

Large-Scale Simulations







Simulations	**Lattice QCD**	**Relativistic Viscous Hydrodynamics**
Size	96×96×96×96	201x201x201 spatial grid
Supercomputer	Fugaku	GPU Clusters
Resource Consumption	**512 cores**	**1 GPU (NVIDIA 3080)**
Time Consumption	**~ 1 day/configuration**	**~30 min/event**

Learn to Sample

$$p(\phi) = e^{-S(\phi)}/Z$$

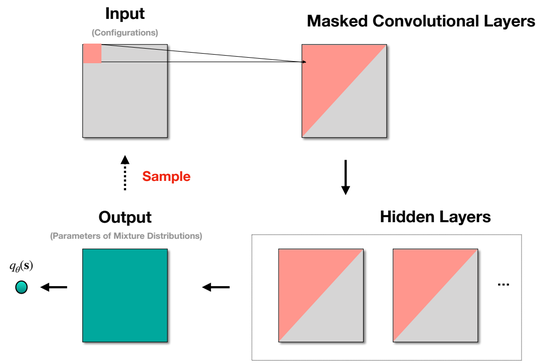
$$\langle O \rangle \approx \frac{1}{N} \sum_i O_i$$

→ **Physical Distribution, Sampling**
via Generative Models

Global Sampling

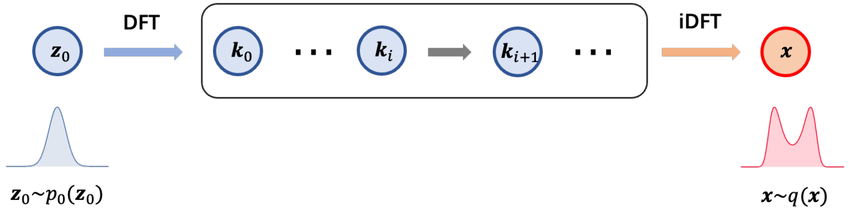
Fast and Independent Sampler

Prog.Part.Nucl.Phys. 104084(2023)



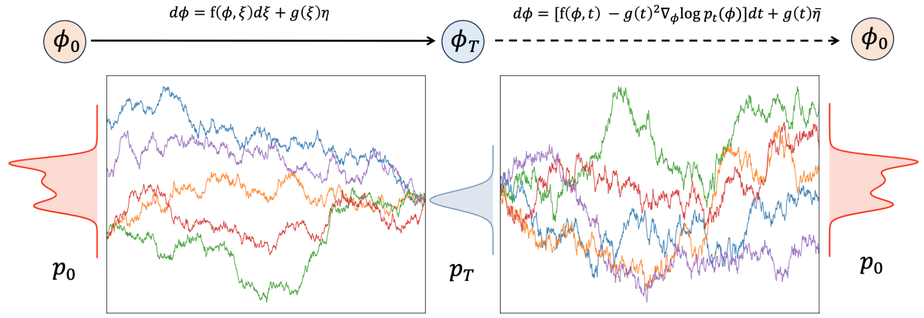
Continuous Autoregressive Models

Chinese Phys. Lett. 39, 120502 (2022)



Fourier Flow-based Model

Phys. Rev. D 107, 056001

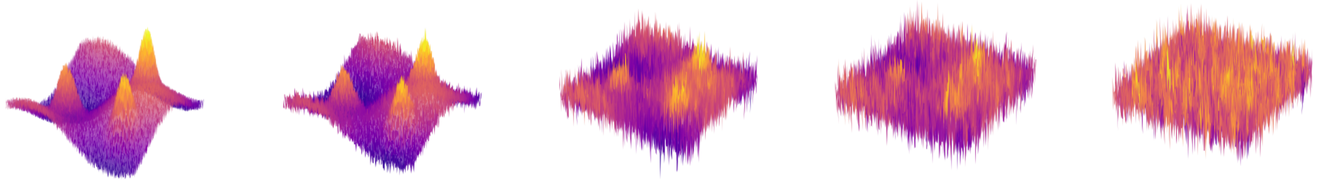


Diffusion Models

JHEP 05(2024)060

Diffusion Models

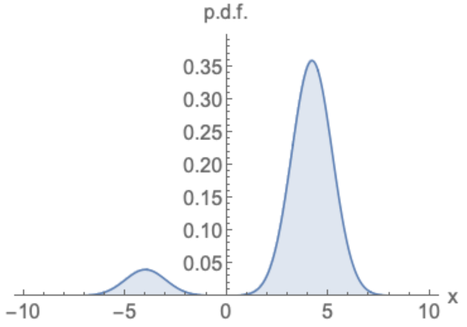
$$\mathbf{x} \sim p_{data}(\mathbf{x})$$

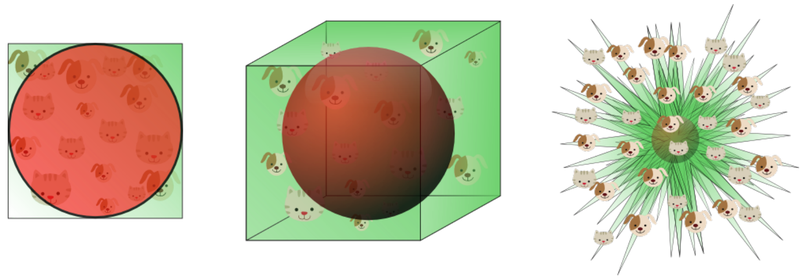


$$\mathbf{z}_0 \sim p_0(\mathbf{z_0})$$

$$p_\theta(\mathbf{x}) = \frac{e^{-f_\theta(\mathbf{x})}}{Z_\theta}$$

from a **statistical phsyics** perspective

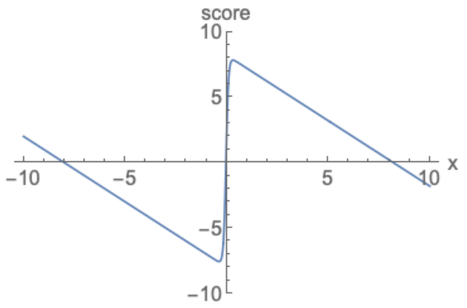




Curse of Dimensionality

$$\nabla_{\mathbf{x}} \log p(\mathbf{x})$$

$$\mathbf{s}_\theta(\mathbf{x}) \to \nabla_{\mathbf{x}} \log p(\mathbf{x})$$



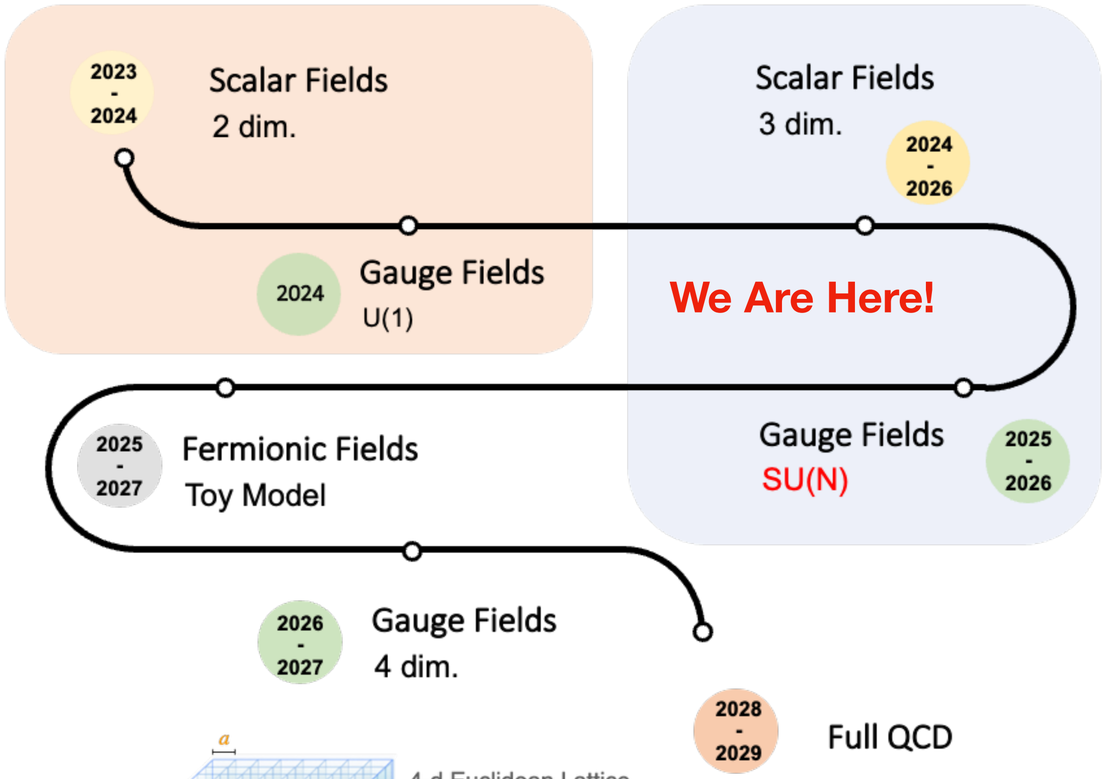
Approaching **Score Function**

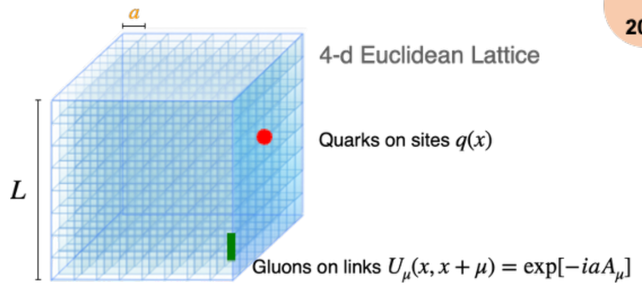
$$\mathbf{s}_\theta(\mathbf{x}) = \nabla_{\mathbf{x}} \log p_\theta(\mathbf{x})$$

$$= -\nabla_{\mathbf{x}} f_\theta(\mathbf{x}) - \underbrace{\nabla_{\mathbf{x}} \log Z_\theta}_{=0} = -\nabla_{\mathbf{x}} f_\theta(\mathbf{x})$$

Diffusion Models toward LQCD

BOOST 国家戦略分野の若手研究者及び博士後期課程学生の育成事業 次世代AI人材育成プログラム（若手研究者支援）





To be continued

JHEP 05 (2024) 060, 2309.17082 [hep-lat]
NeurIPS 2023, 2311.03578 [hep-lat]
NeurIPS 2024, 2410.19602 [hep-lat]
Mach.Learn.Sci.Tech. 6 (2025) 2, 025004, 2410.21212 [hep-lat]
JHEP 12 (2025) 160, 2510.01328 [hep-lat]
JHEP 03 (2026) 111, 2502.05504 [hep-lat]
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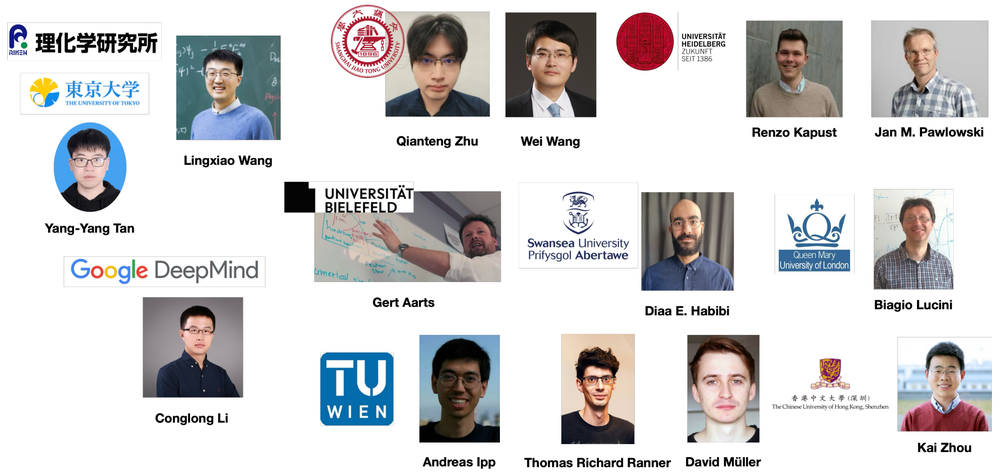
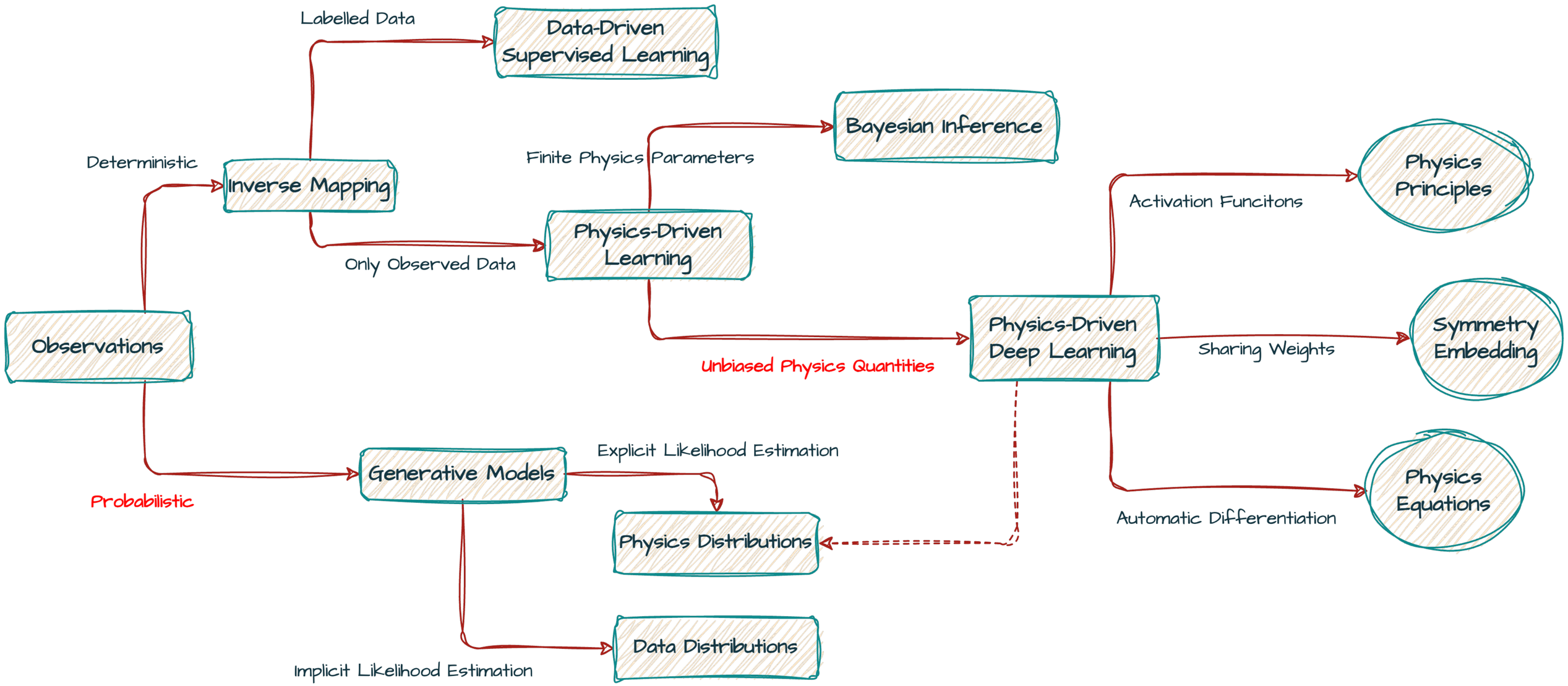


DM-QFT

Inverse Modeling



Thank You !

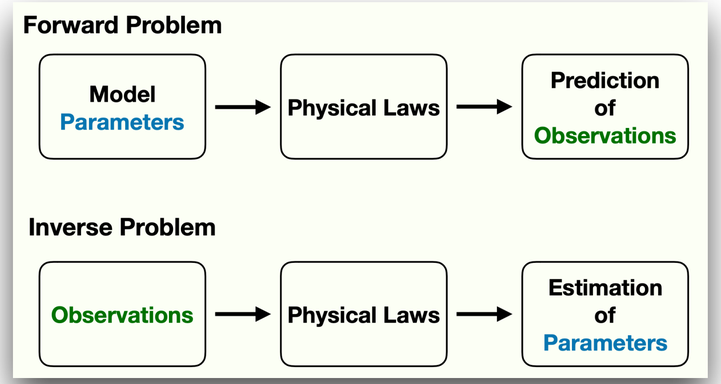
ML meets Physics, Opportunities and Challenges



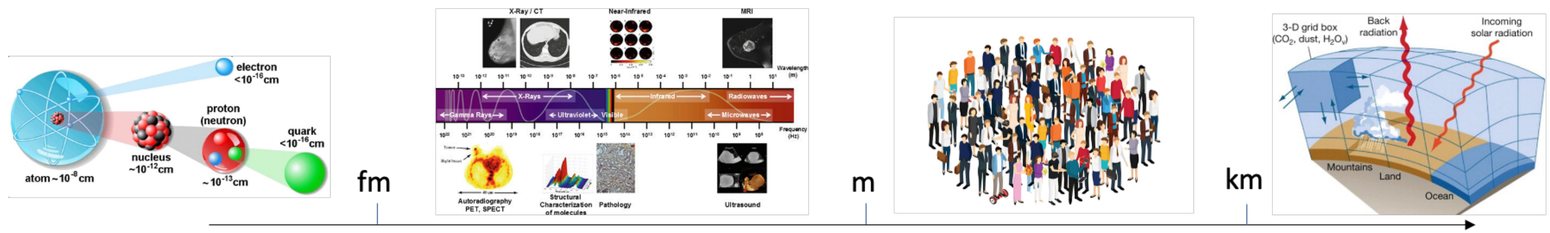
Kouzou Sakai @Quanta Magazine

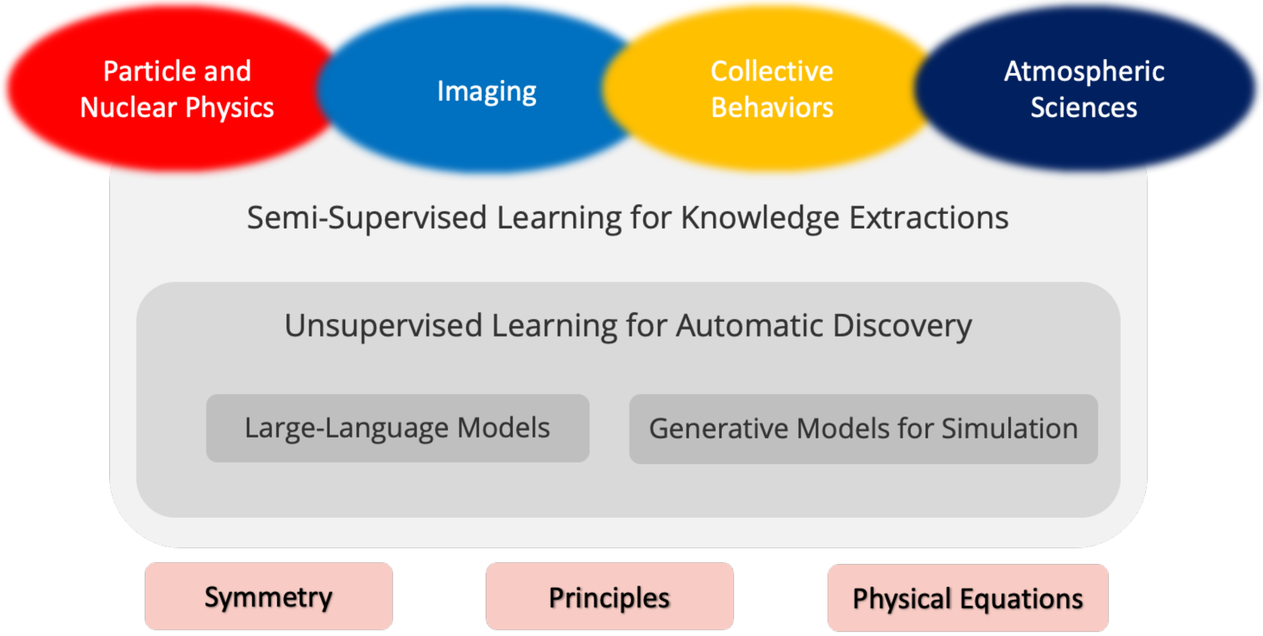
DEEP-IN Working Group











https://sites.google.com/view/deep-in-wg/homepage

ill-posed Inverse Problem

Reconstructing Spectral Function

Comput. Phys. Commun. 282, 108547

- In practice, the Euclidean correlations have **finite number of points** and **with finite precision**;
- The ill-posedness of the spectral reconstruction **fundamentally exists even for** *continuous correlation functions(infinite observations)*;
- It's caused by the **numerical inaccuracy** of the correlation measurements (induced high degeneracy in solution space).

$$K_{ij}, i \in N_x, j \in N_\omega, N_x < N_\omega$$

$$\vec{D} \equiv K\vec{\rho}$$

highly rectangular

vectorization

$$D(x) \equiv \int_0^\infty K(x,\omega)\rho(\omega)\mathrm{d}\omega$$

eigenvalue problem

$$\int_0^\infty \psi_s(\omega)\, K(x,\omega)\, \mathrm{d}\omega = \lambda_s \psi_s(x)$$

J. Phys. A: Math. Gen., Vol. 11, No. 9, 1978. Printed in Great Britain.

Why NN Can Help?

- **Maximizing Bayesian Posterior**

$$P(\rho\,|\,D,I) = \frac{P(D\,|\,\rho,I)P(\rho\,|\,I)}{P(D\,|\,I)}$$

  - Likelihood, $P(D\,|\,\rho,I) = e^{-\chi^2/2}$
  - Prior, $P(\rho\,|\,I) = e^{\mathcal{S}[\rho]}$

- **Minimization the loss function**

$$J \equiv \frac{\chi^2}{2} - \mathcal{S}[\rho]$$

  - **Chi-square term**

$$\chi^2 \equiv \sum_{i,j=1}^{N} C_{ij}^{-1}(D_i^{\rm obs} - D_i)(D_j^{\rm obs} - D_j)$$

the inverse covariance matrix, C^{-1}

$$\Delta_i \equiv -\frac{\delta\chi^2/2}{\delta D(k_i)} = \sum_j C_{ij}^{-1}(D_j^{\rm obs} - D(k_j))$$

  - **"Entropy" term serves as a regulator**

Three typical "Entropy" terms

$$\mathcal{S}_{\rm TK} = -\frac{\alpha}{2}\sum_{a=1}^{N_\omega}(\rho_a - {\rm DM}_a)^2\Delta\omega\,,$$

$$\mathcal{S}_{\rm MEM} = \alpha\sum_{a=1}^{N_\omega}\left(\rho_a - {\rm DM}_a - \rho_a \ln\frac{\rho_a}{{\rm DM}_a}\right)\Delta\omega\,,$$

$$\mathcal{S}_{\rm BR} = \alpha\sum_{a=1}^{N_\omega}\left(1 - \frac{\rho_a}{{\rm DM}_a} + \ln\frac{\rho_a}{{\rm DM}_a}\right)\Delta\omega\,.$$

α, a hyper parameter; defaulted model (DM); $\Delta\omega$, step length

$$\frac{\delta J}{\delta\rho(\omega)} = 0$$

$$\rho_a^{\rm TK} - {\rm DM}_a = \frac{1}{\alpha}\sum_i \Delta_i^{\rm TK} K_{ia}\,,$$

$$\ln\frac{\rho_a^{\rm MEM}}{{\rm DM}_a} = \frac{1}{\alpha}\sum_i \Delta_i^{\rm MEM} K_{ia}\,,$$

$$\frac{1}{{\rm DM}_a} - \frac{1}{\rho_a^{\rm BR}} = \frac{1}{\alpha}\sum_i \Delta_i^{\rm BR} K_{ia}\,.$$

the optimal solution exists

Comput. Phys. Commun. 282, 108547

- **Neural Networks (e.g., NN representation)**

$$\rho_a \equiv \rho(\omega_a)$$

  - Output layer, $\rho_a = {\rm DM}_a\,\sigma^{(l)}(f_a^{(l)})$
  - Activation functions, $f_a^{(n)} = \sigma^{(n)}(x_a^{(n)})$
  - Hidden layers, $x_a^{(n)} = \sum_b W_{ab}^{(n)} f_b^{(n-1)}$

$$a = 1,2,\cdots,N^{(n)};\ n = 1,2,\cdots,l$$

- **Set-ups**

$$N^{(0)} = 1, N^{(l)} = N_\omega$$

  - **Input layer,** $a_1^{(0)} = 1$
  - **Hidden layer, no activation functions**
  - **Output layer,** $\sigma^{(l)}(x) = \sigma(x),\ f_a \equiv f_a^{(l)}$
  - **L2 regulation,** $L_2 \equiv \alpha\Delta\omega\sum_{l,a,b}\left(W_{ab}^{(l)}\right)^2$

Regularized Invere Problems

$$\frac{\delta J}{\delta \rho(\omega)} = 0$$

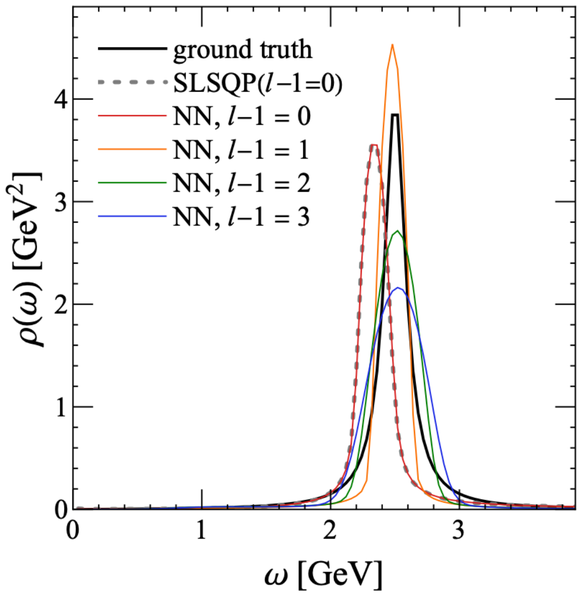
the optimal solution exists!

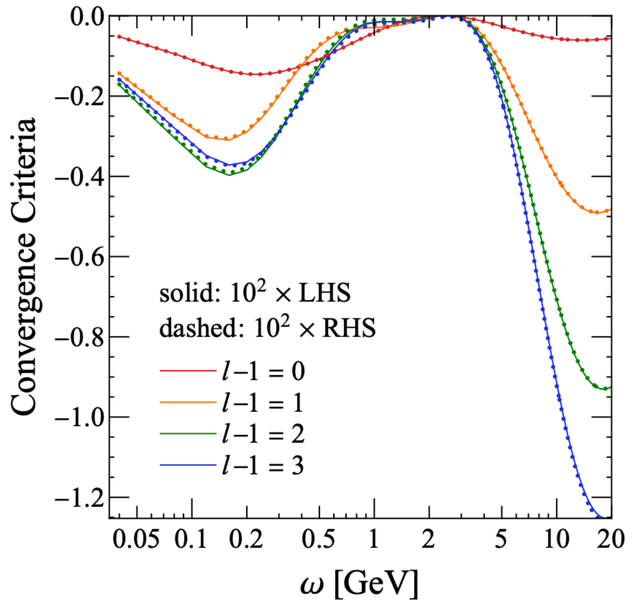
$$\frac{f_a/\sigma'(f_a)}{\left(\sum_b f_b^2\right)^{\frac{l-1}{l}}} = \frac{\mathrm{DM}_a}{\alpha}\sum_i \Delta_i K_{ia}$$

$\frac{(1+e^{-f_a})f_a}{\left(\sum_b f_b^2\right)^{\frac{l-1}{l}}} = \frac{\mathrm{DM}_a}{\alpha}\sum_i \Delta_i K_{ia}$ for *Softplus* activation function

$$\alpha f_a(1+e^{-f_a}) \equiv \mathrm{DM}_a\left(\sum_b f_b^2\right)^{\frac{l-1}{l}}\sum_i \Delta_i K_{ia}$$

non-local constraints from NN!





Comput. Phys. Commun. 282, 108547

- **Neural Networks (e.g., NN representation)**

$$\rho_a \equiv \rho(\omega_a)$$

  - Output layer, $\rho_a = \mathrm{DM}_a\,\sigma^{(l)}(f_a^{(l)})$
  - Activation functions, $f_a^{(n)} = \sigma^{(n)}(x_a^{(n)})$
  - Hidden layers, $x_a^{(n)} = \sum_b W_{ab}^{(n)} f_b^{(n-1)}$

$$a = 1,2,\cdots,N^{(n)};\ n = 1,2,\cdots,l$$

- **Set-ups**

$$N^{(0)} = 1, N^{(l)} = N_\omega$$

  - **Input layer,** $a_1^{(0)} = 1$
  - **Hidden layer, no activation functions**
  - **Output layer,** $\sigma^{(l)}(x) = \sigma(x),\ f_a \equiv f_a^{(l)}$
  - **L2 regulation,** $L_2 \equiv \alpha\Delta\omega\sum_{l,a,b}\left(W_{ab}^{(l)}\right)^2$

Uncertainty Estimation for EoSs

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- x : reconstructed EoSs given a sample
- $O(x)$: observables, M, R, P

Variance $\sigma(O)^2 = \langle \hat{O}^2 \rangle - \bar{O}^2$

Mean $\bar{O} = \langle \hat{O} \rangle = \sum_j^N w^{(j)} O^{(j)}$

Recall: Importance Sampling

- x : random variables
- $f(x)$: observables
- n : number of samples
- $p(x)$: original(true) distribution
- $q(x)$: reference distribution

$$E[f(x)] = \int f(x)p(x)dx = \int f(x)\frac{p(x)}{q(x)}q(x)dx$$

Weights

$$w^{(j)} = \frac{\text{Posterior}(\boldsymbol{\theta}^{(j)}_{\text{EoS}} \,|\, \text{data})}{\text{Proposal}(\boldsymbol{\theta}^{(j)}_{\text{EoS}})}$$

$$\propto \frac{p(\text{data} \,|\, \boldsymbol{\theta}^{(j)}_{\text{EoS}}) \cdot \text{Prior}(\boldsymbol{\theta}^{(j)}_{\text{EoS}})}{p(\boldsymbol{\theta}^{(j)}_{\text{EoS}} \,|\, \text{samples}^{(j)}) \cdot p(\text{samples}^{(j)} \,|\, \text{data}) \cdot \text{Prior(data)}}$$

L2 : $exp(-\lambda\,||\theta||_2^2) \sim 1, \quad \lambda = 10^{-8}$

$$p(\text{data} \,|\, \boldsymbol{\theta}^{(j)}_{\text{EoS}}) \propto \exp(-\chi^2(M_{\boldsymbol{\theta}^{(j)}_{\text{EoS}}}, R_{\boldsymbol{\theta}^{(j)}_{\text{EoS}}}))$$

$$\tilde{w}^{(j)} = w^{(j)} / \sum_j w^{(j)}$$

normalization

$$p(\boldsymbol{\theta}^{(j)}_{\text{EoS}} \,|\, \text{samples}^{(j)}) = 1$$

deterministic

$$p(\text{samples} \,|\, \text{data}) = \mathcal{N}(M_{\text{obs}}, \Delta M^2)\mathcal{N}(R_{\text{obs}}, \Delta R^2)$$