ICEPP
The University of Tokyo

Hands-on session 5:

Introduction to superconducting qubits and quantum computers

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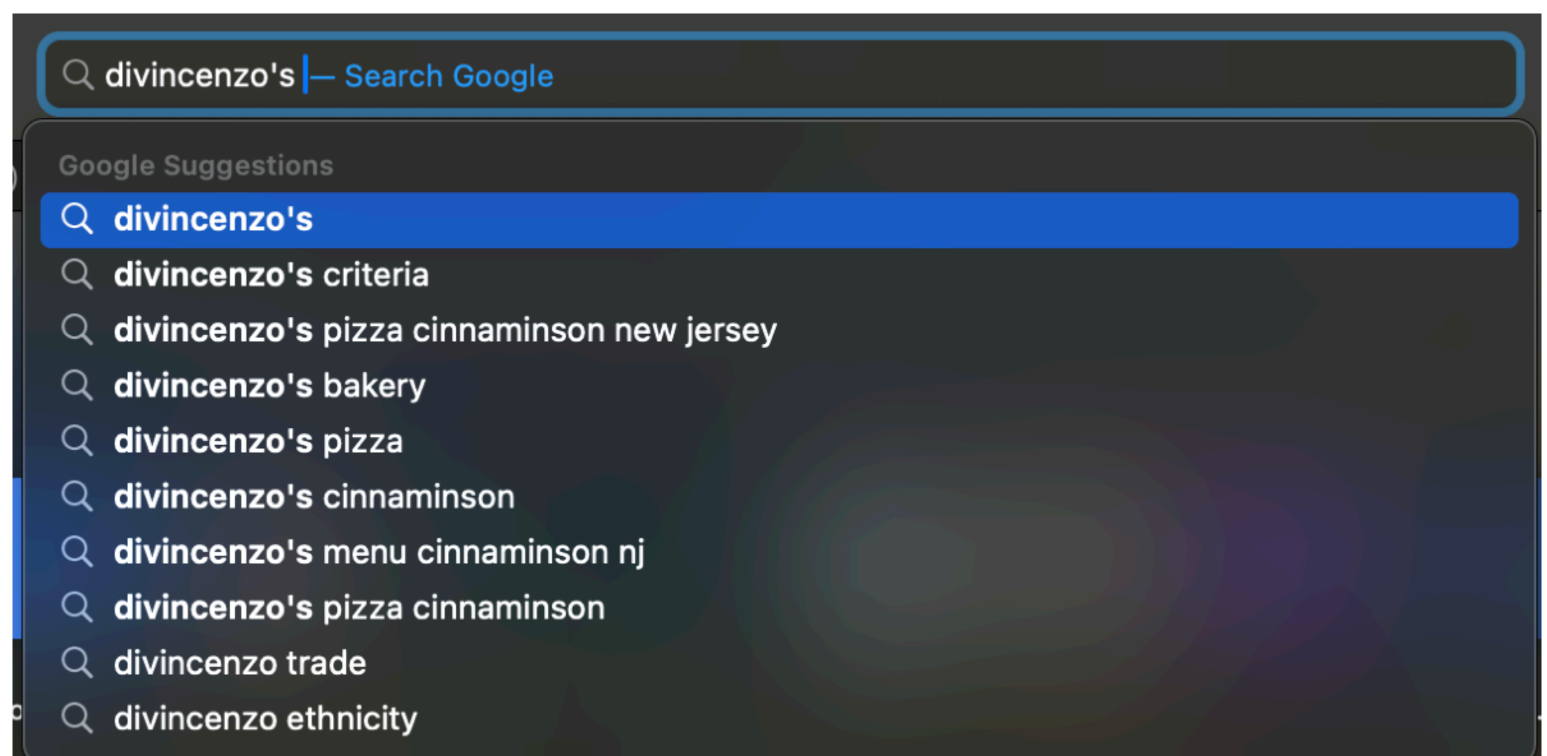
What constitutes a quantum computer

A compute-capable quantum system must incorporate:

1. A scalable physical system with well-characterized qubit*
2. The ability to initialize the state of the qubits to a simple fiducial state
3. Long relevant decoherence times
4. A “universal” set of quantum gates
5. A qubit-specific measurement capability

DiVincenzo’s criteria

Conversely, any quantum system works as long as 1-5 is fulfilled

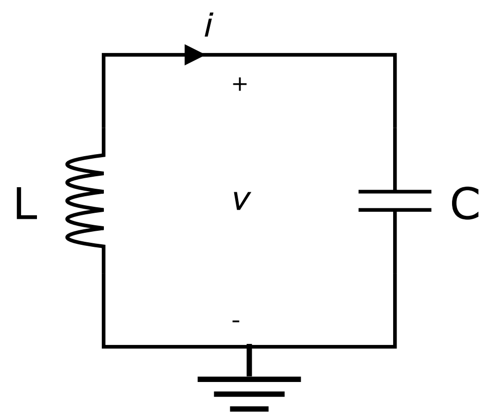


Circuits can be quantized

LC oscillator circuit

- Characteristic frequency ω determined by circuit elements (L and C)
- For typical circuits printed on a silicon chip: ω ~ 2π × (a few GHz)

microwave



$$\frac{1}{C}i(t) + L\frac{d^2}{dt^2}i(t) = 0$$

$$\Rightarrow i(t) = i_0 \cos(\omega t + \phi), \quad \omega = \frac{1}{\sqrt{LC}}$$

Put that in a fridge: Quantum oscillator circuit

Hamiltonian: $H = \frac{1}{2C}q^2 + \frac{1}{2L}\phi^2$ where $q = \int_0^t i(\tau)d\tau$, $\phi = Li(t)$

Canonical quantization: $[\phi, q] = \phi q - q\phi = i\hbar$

→ With ladder operators $a^{(\dagger)} = \sqrt{\frac{C}{2\hbar L}}\phi \pm i\sqrt{\frac{L}{2\hbar C}}q$,

Hamiltonian is $H = \hbar\omega(a^\dagger a + \frac{1}{2})$

Energy eigenstates $|n\rangle$ $(n = 0,1,\ldots)$ →

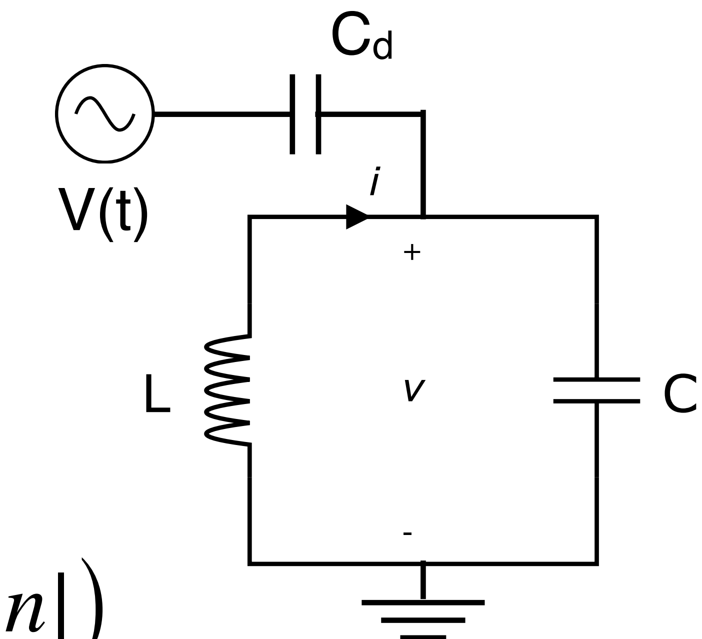
$$a = \sum_n \sqrt{n+1}|n\rangle\langle n+1|$$

$$a^\dagger = \sum_n \sqrt{n+1}|n+1\rangle\langle n|$$

Controlling the quantum LC oscillator

How do we induce level transitions in the oscillator?

One possibility: apply voltage capacitively



$$H = H_0 + \frac{C_d}{C_\Sigma} V(t) q$$

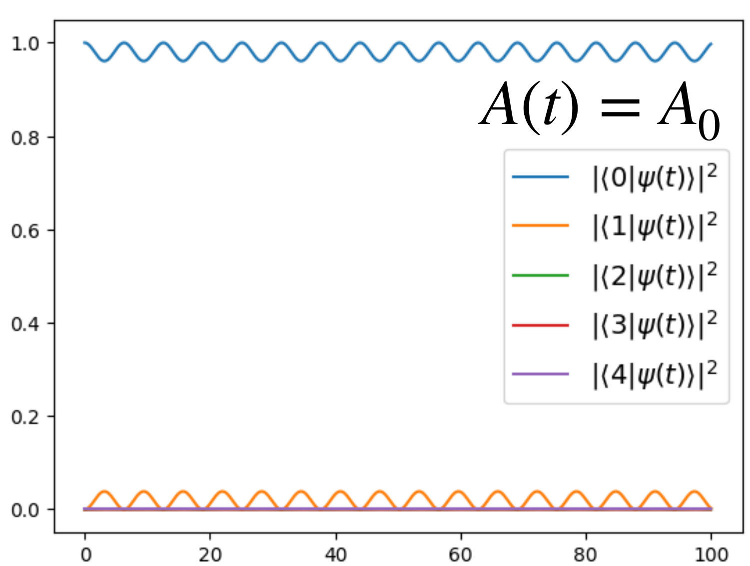
Circuit aggregate capacitance

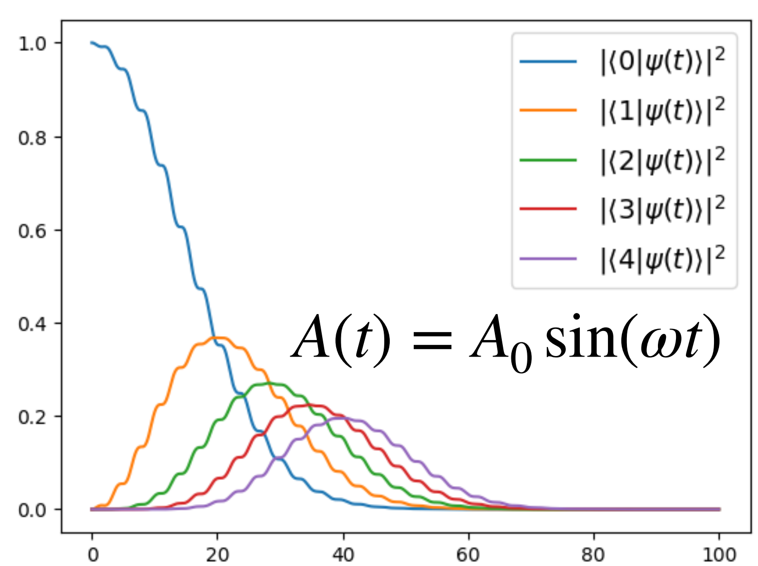
$$= H_0 + iA(t)(a - a^\dagger)$$

$$= H_0 + iA(t) \sum_n \big(\underbrace{|n\rangle\langle n+1| - |n+1\rangle\langle n|} \big)$$

Generator of single-level transitions

A(t) must be a resonant AC signal





Simulation:
$\hbar = 1,\ \omega = 1,\ A_0 = 0.1$

The driven Hamiltonian

$$H(t) = H_0 + iA_0 \sin(\omega_d t)\left(a - a^\dagger\right)$$

Consider a unitary transformation $U_0(t) = e^{iH_0 t}$.

The Schrödinger equation of the transformed state $|\tilde{\psi}(t)\rangle := U_0(t)|\psi(t)\rangle$ is

$$\begin{aligned}
i\frac{d}{dt}|\tilde{\psi}(t)\rangle &= i\left(iH_0 e^{iH_0 t} + e^{iH_0 t}\frac{d}{dt}\right)|\psi(t)\rangle \\
&= -H_0|\tilde{\psi}(t)\rangle + e^{iH_0 t}H|\psi(t)\rangle \\
&= \left(e^{iH_0 t}He^{-iH_0 t} - H_0\right)|\tilde{\psi}(t)\rangle \\
&=: \tilde{H}(t)|\tilde{\psi}(t)\rangle
\end{aligned}$$

where

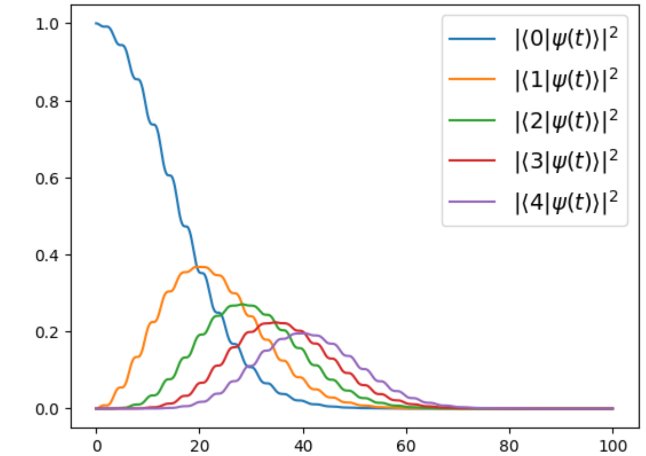
$$e^{i\omega t N}|n\rangle\langle n+1|e^{-i\omega t N} = e^{-i\omega t}|n\rangle\langle n+1|$$

$$\begin{aligned}
\tilde{H}(t) &= iA_0 \sin(\omega_d t)U_0(t)(a - a^\dagger)U_0^\dagger(t) \\
&= \frac{A_0}{2}\left(e^{i\omega_d t} - e^{-i\omega_d t}\right)\left(e^{-i\omega t}a - e^{i\omega t}a^\dagger\right)
\end{aligned}$$

→ Static term arises when $\omega_d = \omega$

With rotating wave approximation (RWA): $\tilde{H}(t) \sim \frac{A_0}{2}\left(a + a^\dagger\right)$

Need for anharmonicity

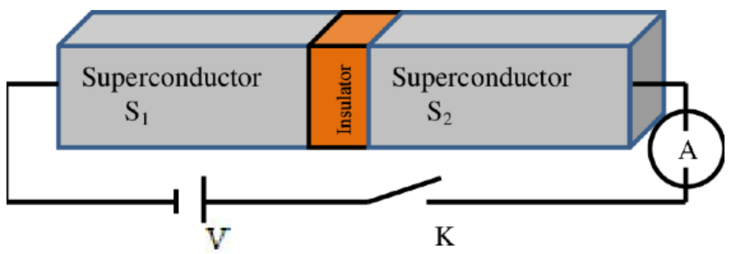


- We can control level transitions of the oscillator with resonant drives
- For harmonic oscillators, $\omega_d = \omega$ excites all levels

→ Can we somehow isolate a two-level system?

Anharmonic oscillator circuit

Josephson junction acts as a nonlinear inductor



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LC circuit: $H = \frac{1}{2C}q^2 + \frac{1}{2L}\phi^2$

Cooper pair box (a.k.a. charge qubit):

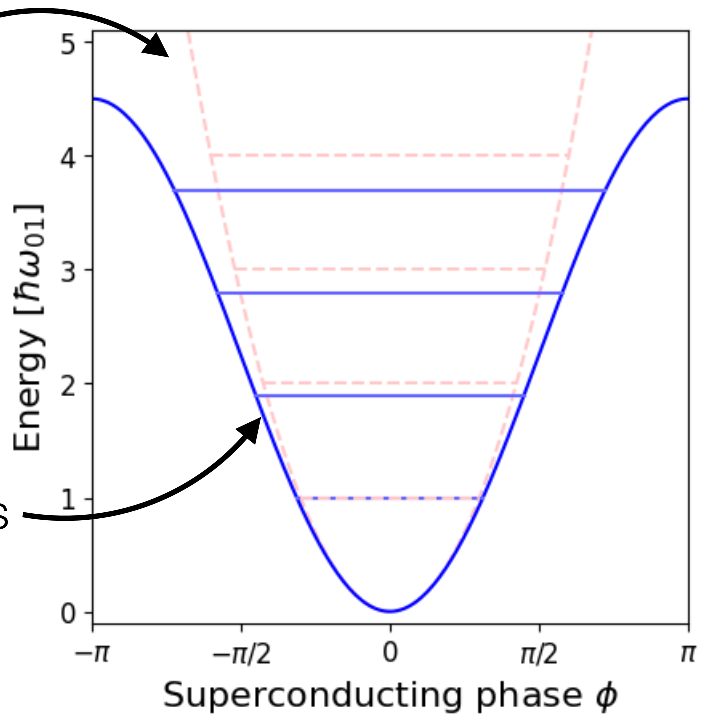
$$H = 4E_C n^2 - E_J \cos(\phi)$$

$$= 4E_C n^2 + E_J\left(\frac{1}{2}\phi^2 - \frac{1}{24}\phi^4\right) + \mathcal{O}(\phi^6)$$

$n = q/2e$ (number of Cooper pairs)

Harmonic oscillator potential and levels

CPB potential and levels



→ Each level gap has distinct frequency

Transmon qubit

$$H = 4E_C n^2 - E_J \cos(\phi)$$

Transmon = CPB with $E_J/E_C \gg 1$

Expanding various quantities in terms of $\epsilon = \sqrt{E_C/E_J}$,

- Energy eigenstates: $|\psi_n\rangle = |n\rangle + \mathcal{O}(\epsilon)$
- Energy eigenvalues: $\frac{\Delta\omega_n}{\omega} := \frac{(E_{n+1} - E_n)}{\hbar\omega} = 1 - \frac{n}{4}\epsilon + \mathcal{O}(\epsilon^2)$
- Creation / annihilation in terms of transmon eigenstates:
  $a = \sum_n \sqrt{n+1}\,|\psi_n\rangle\langle\psi_{n+1}| + \mathcal{O}(\epsilon)$

Driven Hamiltonian is still formally $H(t) = H_0 + iA_0 \sin(\omega_d t)\left(a - a^\dagger\right)$

but with $H_0 = \sum_n E_n |\psi_n\rangle\langle\psi_n|$,

$$\tilde{H}(t) = iA_0 \sin(\omega_d t) U_0(t)(a - a^\dagger)U_0^\dagger(t)$$

$$\sim \frac{A_0}{2}\left(e^{i\omega_d t} - e^{-i\omega_d t}\right) \sum_n \mu_n \left(e^{-i\Delta\omega_n t}|\psi_n\rangle\langle\psi_{n+1}| - e^{i\Delta\omega_n t}|\psi_{n+1}\rangle\langle\psi_n|\right)$$

$$\xrightarrow[\omega_d = \Delta\omega_0]{\text{R.W.A.}} \frac{A_0}{2}\mu_0 \left(\underbrace{|\psi_0\rangle\langle\psi_1| + |\psi_1\rangle\langle\psi_0|}_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}\right) = \frac{A_0}{2}\mu_0 X$$

Shifting the phase of the drive

$$\tilde{H}(t) \sim \frac{A_0}{2}\left(e^{i(\omega_d t+\phi_d)} - e^{-i(\omega_d t+\phi_d)}\right) \sum_n \mu_n \left(e^{-i\Delta\omega_n t}|\psi_n\rangle\langle\psi_{n+1}| - e^{i\Delta\omega_n t}|\psi_{n+1}\rangle\langle\psi_n|\right)$$

$$\xrightarrow[\omega_d=\Delta\omega_0]{\text{R.W.A.}} \frac{A_0}{2}\mu_0 \left(\underbrace{e^{i\phi_d}|\psi_0\rangle\langle\psi_1| + e^{-i\phi_d}|\psi_1\rangle\langle\psi_0|}_{\begin{pmatrix} 0 & e^{i\phi_d} \\ e^{-i\phi_d} & 0 \end{pmatrix}}\right) = \frac{A_0}{2}\mu_0 \left(\cos\phi_d X - \sin\phi_d Y\right)$$

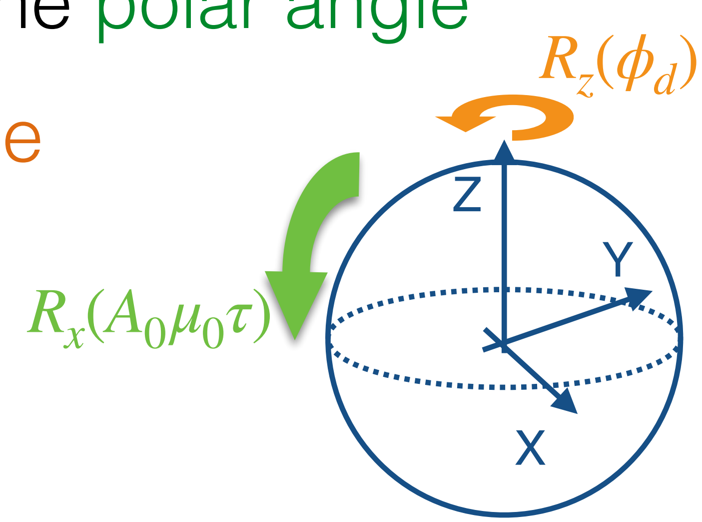
→ Drive of $A_0 \sin(\Delta\omega_0 t + \phi_d)$ for duration τ effects

$$|\psi(t+\tau)\rangle = \exp\left(-\frac{i}{2} A_0 \mu_0 \tau [\cos\phi_d X - \sin\phi_d Y]\right)|\psi(t)\rangle$$

$$= R_z(-\phi_d) R_x(A_0\mu_0\tau) R_z(\phi_d)|\psi(t)\rangle \qquad R_\sigma(\theta) := e^{-i\frac{\theta}{2}\sigma}$$

We learn two things:

- Amplitude and duration of the drive controls the polar angle
- Phase of the drive controls the azimuthal angle



Single-qubit gates in practice

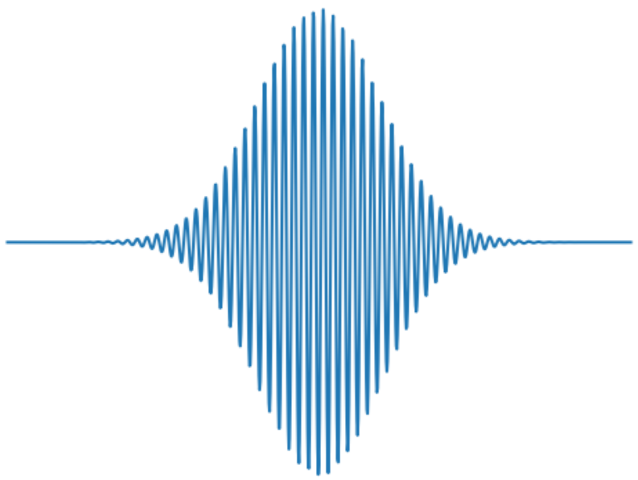
- In practice, finite-duration drive must be applied as *pulses*

$$U = \exp\left(-\frac{i}{2}A_0\mu_0\tau X\right) \rightarrow U = \exp\left(-\frac{i}{2}\left[\int_0^\tau A_0(t)\mu_0 dt\right]X\right)$$

- Previous derivation ignored a lot of small nonlinearities
- We don't have truly continuous control capabilities
(control electronics are digital)

→ Parametric R_x gate is unrealistic

→ Calibrate R_x at specific angles: Typically π (X) and $\pi/2$ (SX)

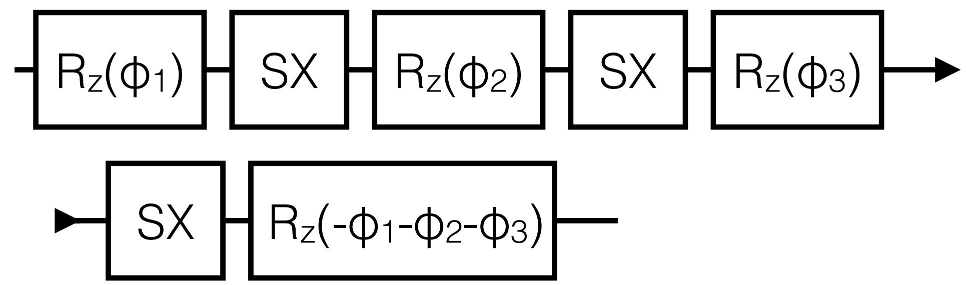


Single-qubit gates in practice

R_z can be parametric:

If we apply SX pulses with phases ϕ_1, $\phi_1+\phi_2$, $\phi_1+\phi_2+\phi_3$, ...

$$U = R_z(-\phi_1 - \phi_2 - \phi_3)\, SX\, R_z(\phi_1 + \phi_2 + \phi_3)$$
$$R_z(-\phi_1 - \phi_2)\, SX\, R_z(\phi_1 + \phi_2)$$
$$R_z(-\phi_1)\, SX\, R_z(\phi_1)$$
$$= R_z(-\phi_1 - \phi_2 - \phi_3)\, SX\, R_z(\phi_3)\, SX\, R_z(\phi_2)\, SX\, R_z(\phi_1)$$



We carry the Z angle over to the final measurement, where it is unobservable:

$$P_{0,1} = |\langle 0,1|\psi\rangle|^2 = |\langle 0,1|R_z(\phi)|\psi\rangle|^2$$

"Virtual Z gate" = phase shift *is* the R_z gate

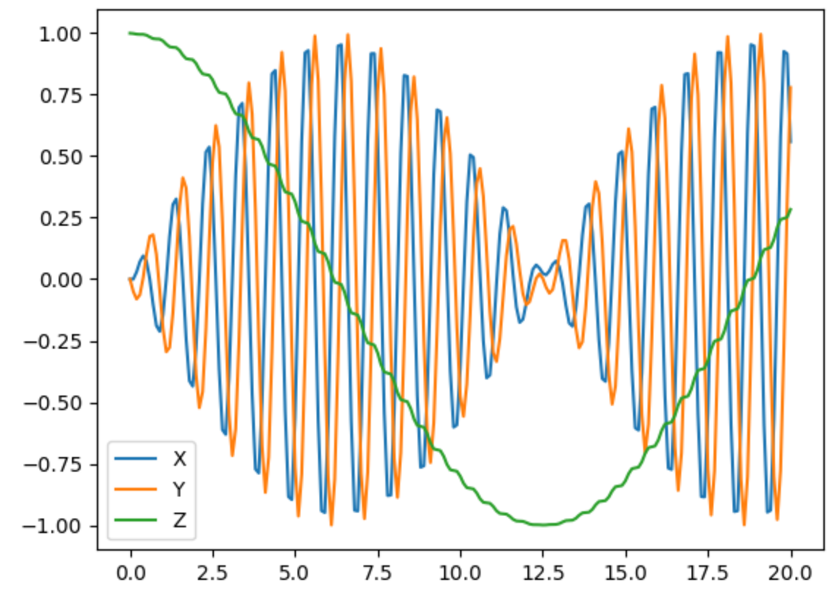
SX and parametric R_z are all we need:

$$U(\theta,\phi,\lambda) = R_z\left(\phi - \frac{\pi}{2}\right) R_x\left(\frac{\pi}{2}\right) R_z(\pi - \theta)\, R_x\left(\frac{\pi}{2}\right) R_z\left(\lambda - \frac{\pi}{2}\right)$$

The drive frame

From the simulation exercise yesterday:

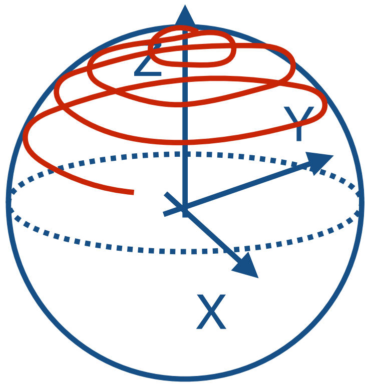
$$H(t) = \omega |\uparrow\rangle\langle\uparrow| + \frac{A}{2}\cos\alpha t \left(|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|\right)$$



- In the "lab frame"

  $$|\psi(t)\rangle = T\left[\exp\left(-\frac{i}{\hbar}\int_0^t H(\tau)d\tau\right)\right]|\psi(0)\rangle$$

  the qubit winds about the Z axis rapidly



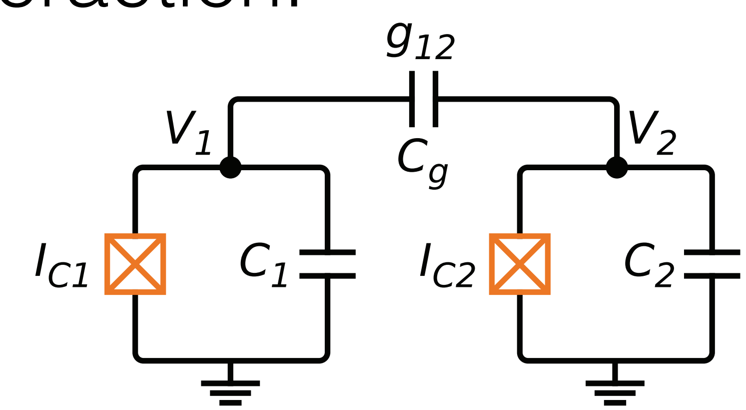
- In the "drive frame"

  $$|\tilde{\psi}(t)\rangle = \exp\left(-\frac{i}{\hbar}\tilde{H}t\right)|\tilde{\psi}(0)\rangle$$

→ Drive physics is revealed by using a frame (unitary transformation) that rotates with the qubit

Coupling the qubits

Capacitative coupling results in an exchange interaction:



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$$H = \sum_{j=1,2} H_0^{(j)} + C_g V_1 V_2$$
$$= \sum_{j=1,2} H_0^{(j)} - g_{12} \left(a^{(1)} - a^{\dagger(1)}\right) \left(a^{(2)} - a^{\dagger(2)}\right)$$

→ XX type, “transverse coupling”

If we diagonalize H:

$$H = \sum_{m,n} E_{m,n} |\psi_m^{(1)}\rangle |\psi_n^{(2)}\rangle \langle\psi_m^{(1)}| \langle\psi_n^{(2)}|$$

we find that $E_{j,n} - E_{k,n} \neq E_{j,m} - E_{k,m}$ in general.

In the qubit space,

$\neq 0$

$$H = E_{00}|00\rangle\langle 00| + E_{01}|01\rangle\langle 01| + E_{10}|10\rangle\langle 10| + E_{11}|11\rangle\langle 11|$$
$$= \frac{E_{00} + E_{01} + E_{10} + E_{11}}{4} II + \frac{E_{00} - E_{01} + E_{10} - E_{11}}{4} IZ + \frac{E_{00} + E_{01} - E_{10} - E_{11}}{4} ZI + \frac{E_{00} - E_{01} - E_{10} + E_{11}}{4} ZZ$$

→ “ZZ interaction” induced!

Entangling gates

Statically coupled qubits have “always-on” entangling interactions

→ To entangle the qubits in a more controlled manner,

- Keep the coupling small and use drive-induced entanglement
- ZZ term depends on the detuning (frequency diff.) of the qubits
  → Make the qubit frequencies tunable, bring the frequencies close only when necessary
- Create dynamical coupling
- etc.

All are highly complicated to implement

→ Entangling gates are the most noise-prone operations in NISQ

Dispersive measurement

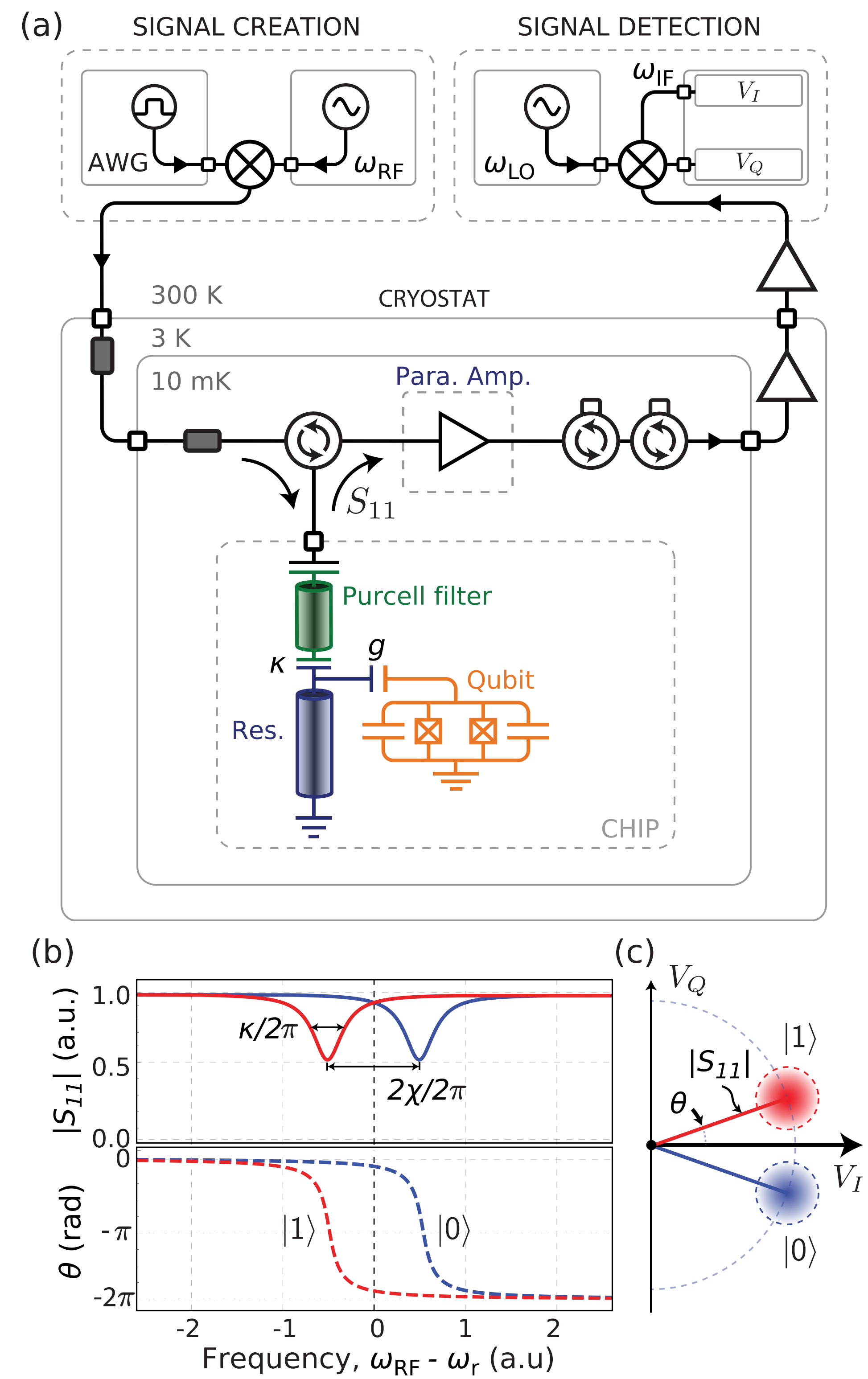
Diagonalize qubit-qubit Hamiltonian

→ ZZ interaction

Diagonalize a qubit-resonator system

→ Z $b^\dagger b$ interaction, i.e., resonator frequency depends on qubit state (dispersive shift)

In other words:
Qubit state can be measured through resonator frequency measurement



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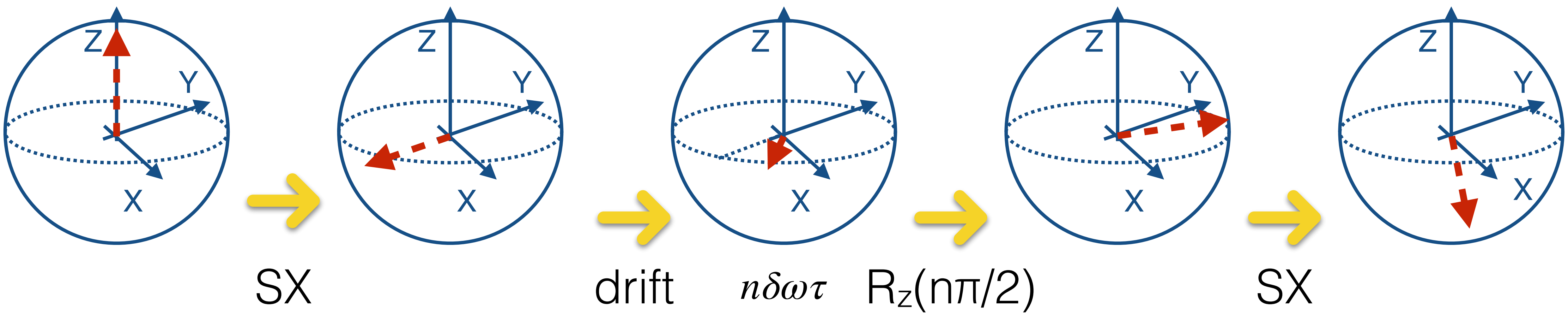
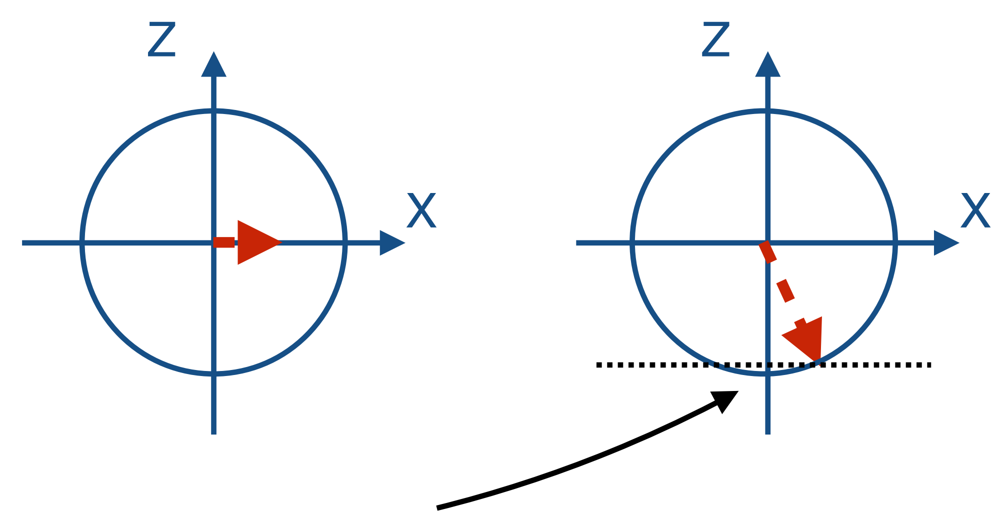
Ramsey interferometry





$$P(1) = \frac{1}{2}\left[\cos\left(n\left(\frac{\pi}{2} + \delta\omega\tau\right)\right) + \delta\omega\tau\right]$$

Summary

- Cooper pair box is formed by replacing the inductor of an LC circuit with a Josephson junction
- Transmon (CPB with $E_J/E_C \gg 1$) has lowest two levels usable as a qubit
- Arbitrary single-qubit gate can be effected by resonantly driving the qubit
- Coupled qubits entangle
- Qubit state can be measured from a coupled resonator
- Qiskit pulse API allows detailed control of drive pulses on supported IBM Quantum backends