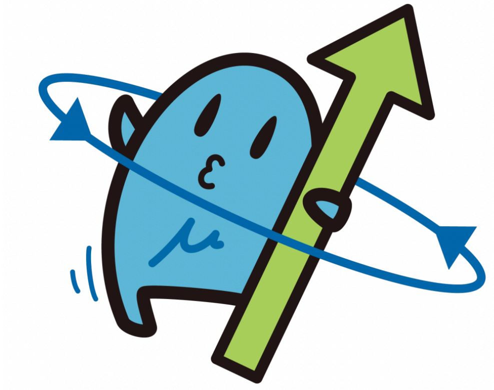
New physics and muon g-2

Kei Yamamoto

Hiroshima Institute of Technology
→ Iwate University

International Physics School :
Simon Eidelman School on Muon Dipole Moments and Hadronic Effects
Sep 2-6, 2024



Preface

Though all phenomena seems to be well described by **the Standard Model**, it should be regarded as an effective theory of more fundamental theory

Flavor puzzle, Neutrino, Hierarchy problem, DM, BAU,,

Indirect searches are complementary to direct searches at the LHC and probe New physics at high energy scale which is not accessible at collider

Energy frontier	Intensity frontier
LHC at high-pT	**Flavor physics**

Flavor physics play important role of probing NP

Muon g-2 anomaly provides the most longstanding hint of New Physics

Outline

Introduction of muon g-2

New physics interpretation of muon g-2

Flavor symmetry and muon g-2

Summary

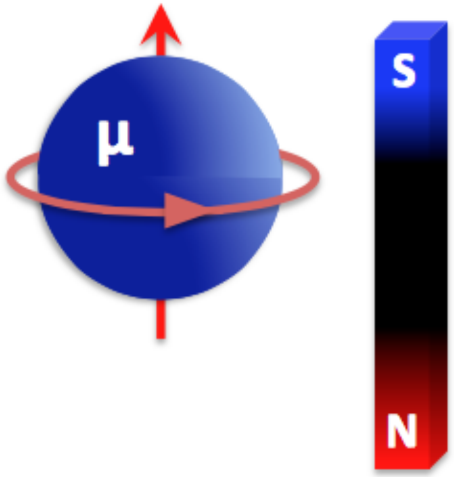
Outline

- Introduction of muon g-2
- New physics interpretation of muon g-2
- Flavor symmetry and muon g-2
- Summary

Anomalous magnetic moment in classical description

Interaction of spin $\overrightarrow{S}$ with magnetic field $\overrightarrow{B}$

$$\mathcal{H} = -\vec{\mu}_\ell \cdot \vec{B} \qquad (\ell = e, \mu, \tau)$$



magnetic moment (magnetic dipole moment) $\overrightarrow{\mu} \propto \overrightarrow{S}$

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{S}$$

g-factor

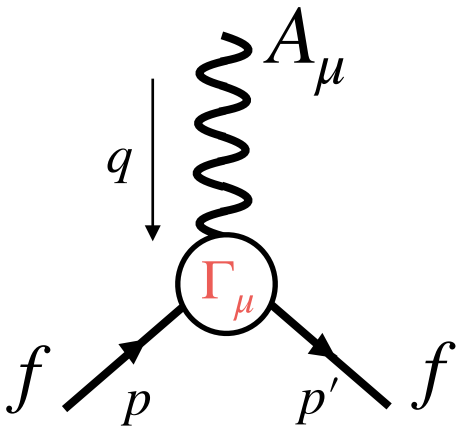
from Dirac eq. $g_\ell = 2$

+ Radiative corrections $g_\ell \neq 2 \longrightarrow$

$$a_\ell \equiv \frac{g_\ell - 2}{2}$$

“anomalous” magnetic moment

Anomalous magnetic moment in QFT



Scattering amplitude of fermion f and electromagnetic field A_μ

$$i\mathcal{M}\,(2\pi)\delta(p^{0\prime} - p^0) = -ie\bar{u}(p')\,\Gamma^\mu(p', p)\,u(p) \cdot \tilde{A}^{\mathrm{cl}}_\mu(p' - p)$$

Gamma structure

※ in Parity conserving

$$\Gamma^\mu \;=\; \gamma^\mu \cdot A \;+\; \underline{(p'^\mu + p^\mu)} \cdot B \;+\; (p'^\mu - p^\mu) \cdot \underline{C}$$

→decomposed

by Gordon identity

→0

by Ward identity

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{p'^\mu + p^\mu}{2m} + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

$$\Big\downarrow$$

$$\Gamma^\mu(p', p) = \gamma^\mu \underline{F_1(q^2)} + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\underline{F_2(q^2)}$$

Tree level

Loop level

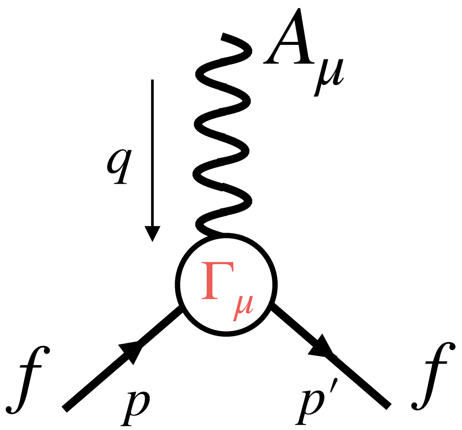
F_1, F_2 : form factors

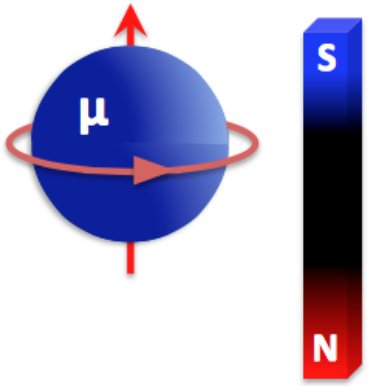
at lowest order (tree level), $\Gamma^\mu = \gamma^\mu$ → $F_1 = 1,\ F_2 = 0$

$Q = -eF_1(0)$ → charge renormalization condition, $F_1(q^2 = 0) = 1$

Anomalous magnetic moment in QFT

Relation between g-factor ↔ form factors F_1, F_2





$$i\mathcal{M} = +ie\Big[\bar{u}(p')\Big(\gamma^i F_1 + \frac{i\sigma^{i\nu}q_\nu}{2m}F_2\Big)u(p)\Big]\tilde{A}^i_{\text{cl}}(\mathbf{q})$$

non-relativistic limit,
$q^2 \to 0$

$$\mathcal{H} = -\frac{e}{2m_\ell}2[F_1(0)+F_2(0)]\vec{S}\cdot\vec{B}$$

$$\mathcal{H} = -g_\ell\frac{e}{2m_\ell}\vec{S}\cdot\vec{B}$$

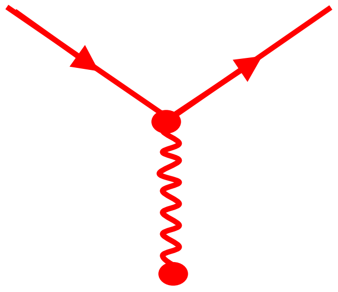
$$g_\ell = 2[F_1(0)+F_2(0)] \overset{F_1(0)=1}{=} 2+2F_2(0)$$

Anomalous magnetic moment in QFT

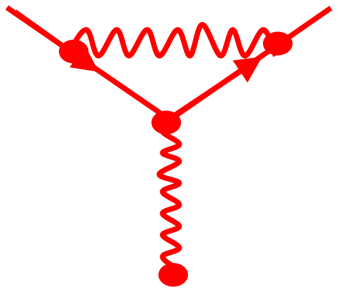
Relation between g-factor ↔ form factors F_1, F_2

$$\boxed{g_\ell = 2 + 2F_2(0)}$$

at leading order + QED correction + corrections,,,



+



+ $\cdots$

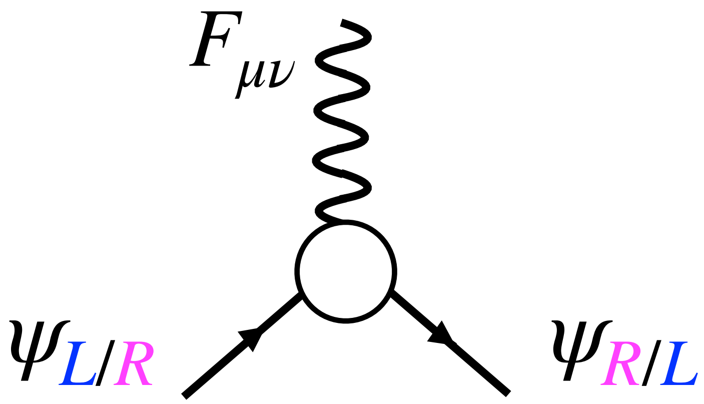
$$g_\ell = 2 + \underbrace{\mathcal{O}(\alpha) + \cdots}$$

$$\boxed{a_\ell \equiv \frac{g_\ell - 2}{2} = F_2(0)}$$

anomalous magnetic moment

quantum loop effects

Effective Lagrangian for anomalous magnetic moment



$$\mathcal{L}_{eff} = -\frac{ea_f}{4m_f}\bar{\psi}_f\sigma^{\mu\nu}\psi_f F_{\mu\nu}$$

features of anomalous magnetic moment

Loop induced

Chirality flip

$\sigma_{\mu\nu}$ dipole interaction → induces a chirality flip

$$\bar{\psi}\sigma^{\mu\nu}\psi \xrightarrow{\psi = \psi_L + \psi_R} \overline{\psi_L}\sigma^{\mu\nu}\psi_R + \overline{\psi_R}\sigma^{\mu\nu}\psi_L$$

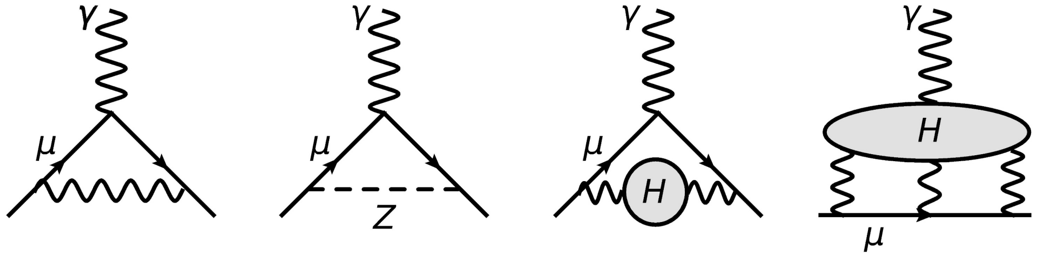
Spontaneously breaking of EW gauge interaction

ψ_L, ψ_R , no gauge invariant → higgs in the SM

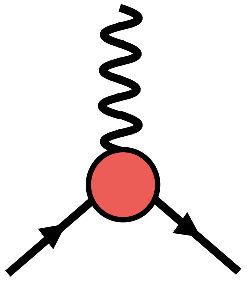
muon g-2 anomaly

SM $$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm HVP} + a_\mu^{\rm HLbL}$$

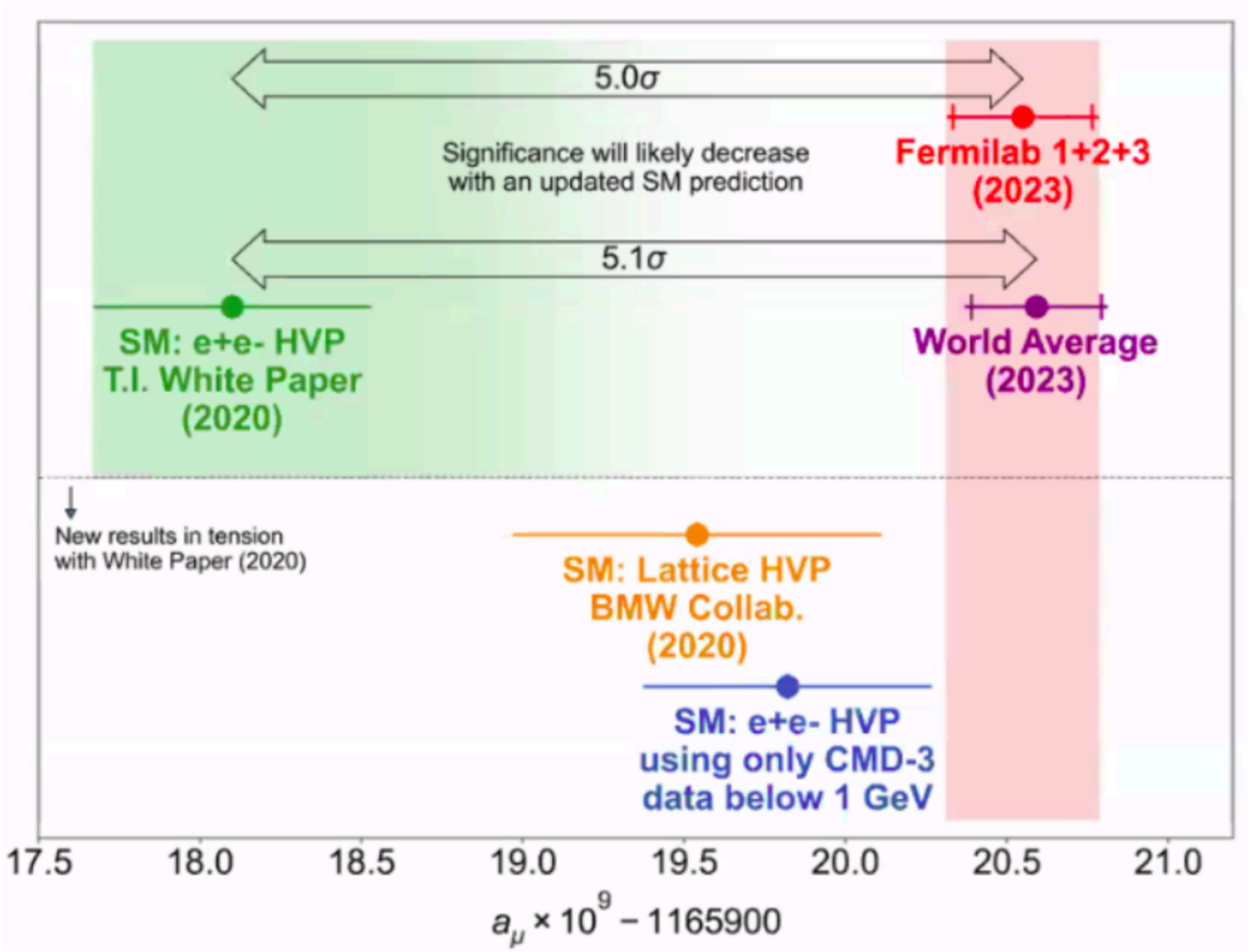
New physics (NP) ?



+



muon g-2 anomaly?



Outline

Introduction of muon g-2

New physics interpretation of muon g-2

Flavor symmetry and muon g-2

Summary

Review
P.Athron, C.Balázs,D.H.J. Jacob,W. Kotlarski, D.Stöckinger, H. Stöckinger-Kim *2104.03691*

Crivellin, Hoferichter *1905.03789*

New physics interpretation

Large positive NP effect is needed: deviation is larger than EW correction

$$\Delta a_\mu \equiv \underset{\text{2023 world ave.}}{a_\mu^{\rm exp}} - \underset{\text{White Paper}}{a_\mu^{\rm SM}} = (24.9 \pm 4.8) \times 10^{-10} \quad > \quad a_\mu^{\rm EW} \simeq 15.4 \times 10^{-10}$$

5.1 σ discrepancy

No new particles have been discovered in EW scale

→ Need mechanism to enhance contribution to muon g-2

enhancement mechanism or Light NP particle

New physics interpretation

NP effect

$$a_\mu^{NP} \sim \frac{g_{NP}^2}{16\pi^2}\frac{m_\mu^2}{M_{NP}^2}$$

g_{NP} : NP coupling

M_{NP}: mass of new particle

Large g_{NP} or Light M_{NP}

→ Need mechanism to enhance contribution to muon g-2

enhancement mechanism or Light NP particle

Heavy NP particle

Large coupling

Light NP particle

tiny coupling

New physics interpretation

Typical NP scale and coupling

$$a_\mu^{NP} \sim \frac{g_{NP}^2}{16\pi^2}\frac{m_\mu^2}{M_{NP}^2} \quad = \quad \Delta a_\mu = 24.9 \times 10^{-10}$$

$$\to M_{NP} \sim g_{NP} \times 150 \text{ GeV}$$

$M_{NP} \sim \mathcal{O}(1)$ TeV

$g_{NP} \sim \mathcal{O}(10)$

Heavy NP particle

Large coupling

$M_{NP} \sim \mathcal{O}(100)$ MeV

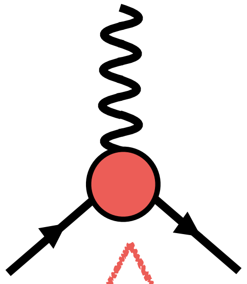
$g_{NP} \sim \mathcal{O}(10^{-3})$

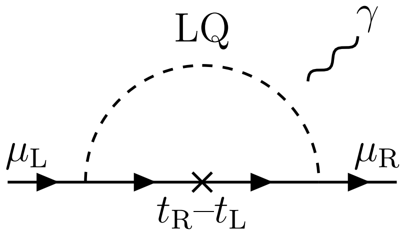
Light NP particle

tiny coupling

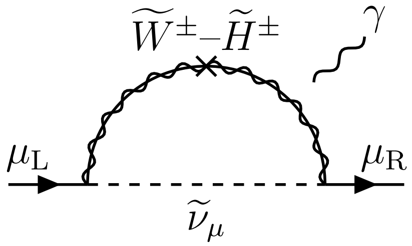
New physics interpretation

New physics (NP)

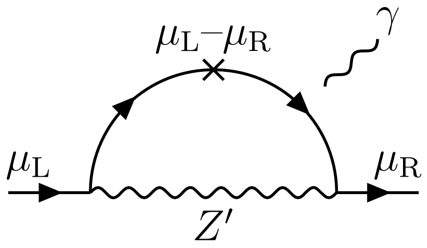




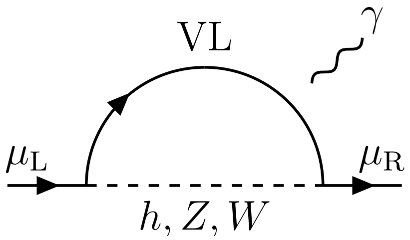
leptoquarks(LQ)



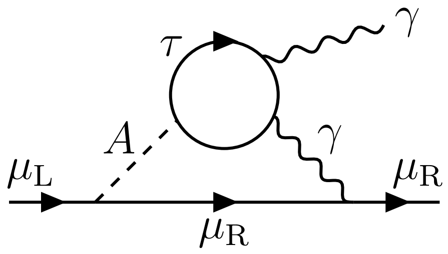
SUSY



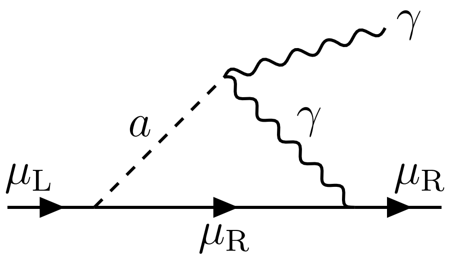
Z' new gauge boson



Vectorlike lepton(VL)



Scalar particle(A)



axion

Heavy NP particle
Large coupling

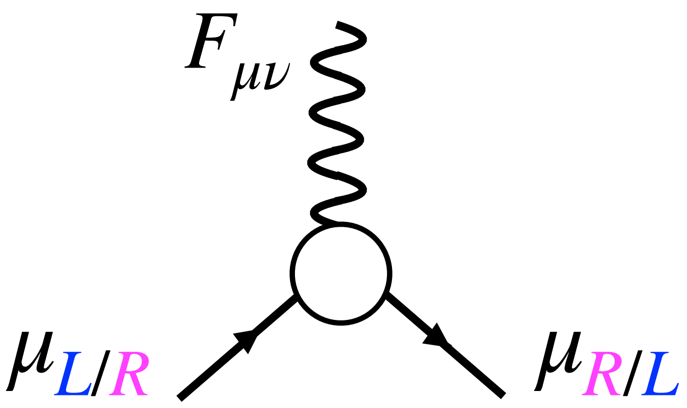
Light NP particle
tiny coupling

Many models relevant for muon g-2

Fine tuning in the muon mass

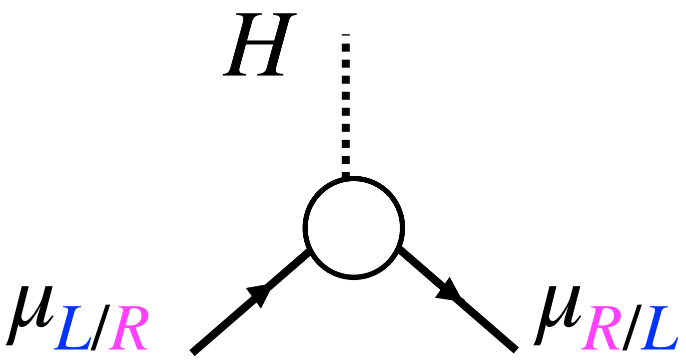
NP on muon g-2 also contribute to muon mass in similar loops

Dipole operator



muon g-2

Yukawa



muon mass m_μ

$$\Delta a_\mu^{\mathrm{BSM}} = C_{\mathrm{BSM}} \frac{m_\mu^2}{M_{\mathrm{BSM}}^2}$$

$$\Big\downarrow$$

$$\Delta a_\mu^{\mathrm{BSM}} \lesssim \mathcal{O}(1) \frac{m_\mu^2}{M_{\mathrm{BSM}}^2}$$

$$M_{\mathrm{BSM}} \lesssim 2\ \mathrm{TeV}$$

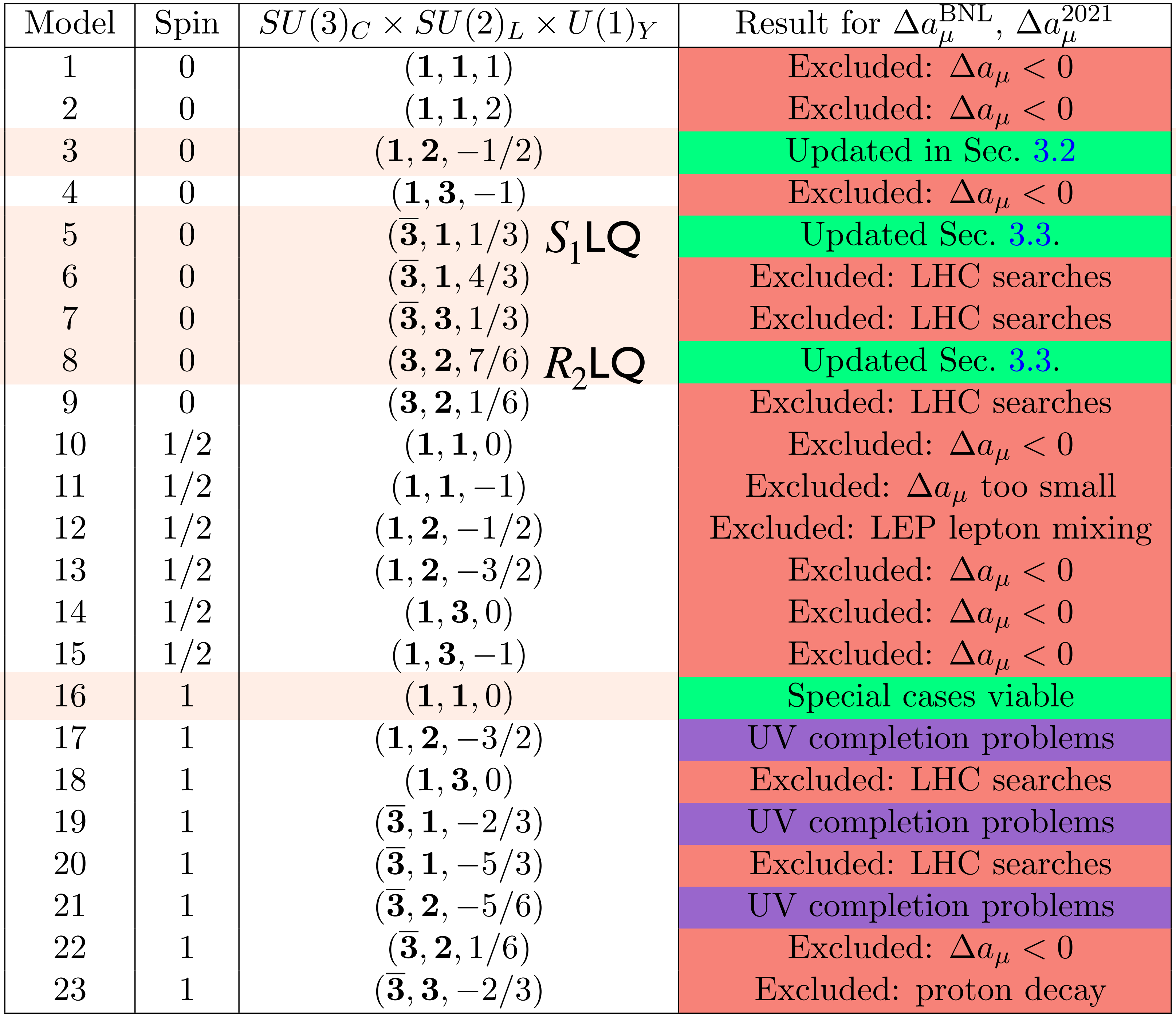
$$\frac{\Delta m_\mu^{\mathrm{BSM}}}{m_\mu} = \mathcal{O}(C_{\mathrm{BSM}}) \quad \lesssim \mathcal{O}(1)$$

To avoid fine tuning in the muon mass, i.e., do not exceed the actual muon mass

New physics possibility

P.Athron, C.Balázs,D.H.J. Jacob,W. Kotlarski, D. Stöckinger, H. Stöckinger-Kim2104.03691

single field extension



	Model	Spin	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Result for $\Delta a_\mu^{\rm BNL}$, Δa_μ^{2021}
	1	0	$(\mathbf{1}, \mathbf{1}, 1)$	Excluded: $\Delta a_\mu < 0$
	2	0	$(\mathbf{1}, \mathbf{1}, 2)$	Excluded: $\Delta a_\mu < 0$
2HDM	3	0	$(\mathbf{1}, \mathbf{2}, -1/2)$	Updated in Sec. 3.2
	4	0	$(\mathbf{1}, \mathbf{3}, -1)$	Excluded: $\Delta a_\mu < 0$
Scalar leptoquarks(LQ)	5	0	$(\overline{\mathbf{3}}, \mathbf{1}, 1/3)$ S_1LQ	Updated Sec. 3.3.
	6	0	$(\overline{\mathbf{3}}, \mathbf{1}, 4/3)$	Excluded: LHC searches
	7	0	$(\overline{\mathbf{3}}, \mathbf{3}, 1/3)$	Excluded: LHC searches
	8	0	$(\mathbf{3}, \mathbf{2}, 7/6)$ R_2LQ	Updated Sec. 3.3.
	9	0	$(\mathbf{3}, \mathbf{2}, 1/6)$	Excluded: LHC searches
	10	1/2	$(\mathbf{1}, \mathbf{1}, 0)$	Excluded: $\Delta a_\mu < 0$
	11	1/2	$(\mathbf{1}, \mathbf{1}, -1)$	Excluded: Δa_μ too small
	12	1/2	$(\mathbf{1}, \mathbf{2}, -1/2)$	Excluded: LEP lepton mixing
	13	1/2	$(\mathbf{1}, \mathbf{2}, -3/2)$	Excluded: $\Delta a_\mu < 0$
	14	1/2	$(\mathbf{1}, \mathbf{3}, 0)$	Excluded: $\Delta a_\mu < 0$
	15	1/2	$(\mathbf{1}, \mathbf{3}, -1)$	Excluded: $\Delta a_\mu < 0$
Dark photon and dark Z	16	1	$(\mathbf{1}, \mathbf{1}, 0)$	Special cases viable
	17	1	$(\mathbf{1}, \mathbf{2}, -3/2)$	UV completion problems
	18	1	$(\mathbf{1}, \mathbf{3}, 0)$	Excluded: LHC searches
	19	1	$(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$	UV completion problems
	20	1	$(\overline{\mathbf{3}}, \mathbf{1}, -5/3)$	Excluded: LHC searches
	21	1	$(\overline{\mathbf{3}}, \mathbf{2}, -5/6)$	UV completion problems
	22	1	$(\overline{\mathbf{3}}, \mathbf{2}, 1/6)$	Excluded: $\Delta a_\mu < 0$
	23	1	$(\overline{\mathbf{3}}, \mathbf{3}, -2/3)$	Excluded: proton decay

Difficulty of g-2 explanation

I) negative contribution $a_\mu^{NP} < 0$: corrections only decrease muon g-2

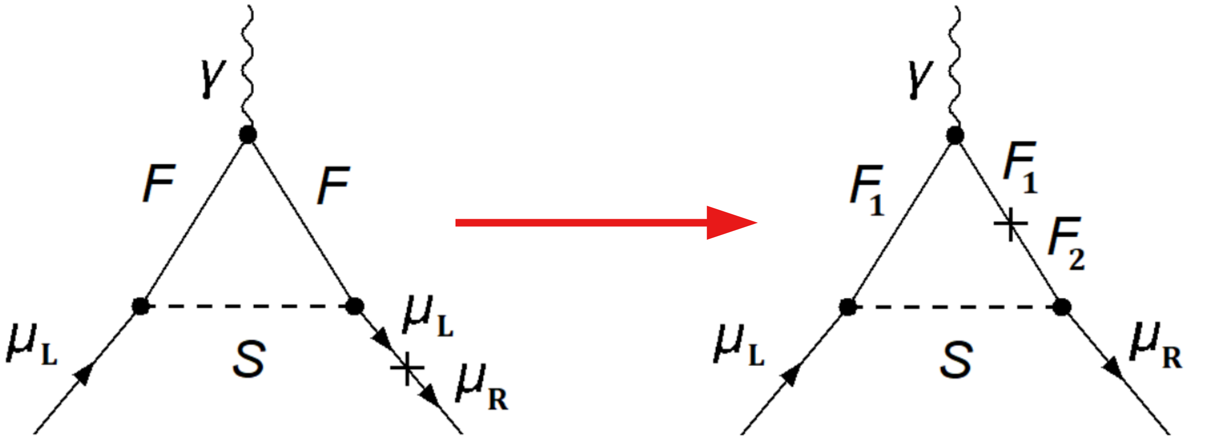
II) Tension with collider experiments

Leptoquarks

Leptoquarks couple to both lepton and quark together

Introduced in lepton and quark unified model e.g. Pati-Salam model

S_1 and R_2 LQ have both left- and right-handed couplings → chirality flip enhancement



$$a_\mu^{NP} \sim \frac{g_{NP}^2}{16\pi^2} \frac{m_\mu^2}{M_{NP}^2}$$

$$a_\mu^{NP} \sim \frac{g_{NP}^2}{16\pi^2} \frac{m_\mu m_F}{M_{NP}^2}$$

$\frac{m_F}{m_\mu}$ enhancement $\frac{m_t}{m_\mu} \sim 1600$

Leptoquarks

P.Athron, C.Balázs,D.H.J. Jacob,W. Kotlarski,D. Stöckinger, H. Stöckinger-Kim 2104.03691

Chirality flipping enhancement → can explain muon g-2 with large masses

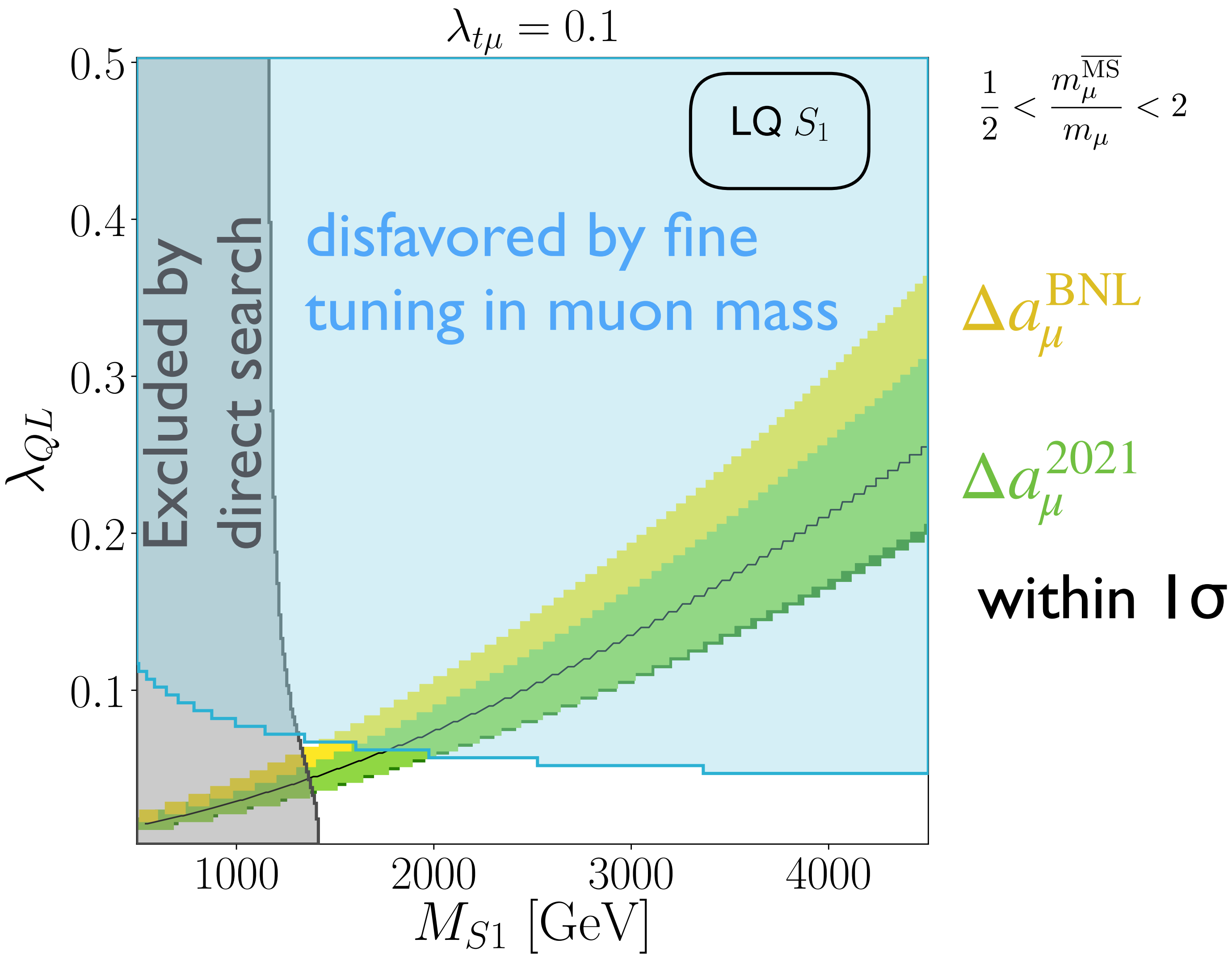
But hard to avoid fine tuning in the muon mass :
NP contribute to muon mass in similar loops

$$\Delta m_\mu^{\mathrm{BSM}}/m_\mu \sim \mathcal{O}(C_{\mathrm{BSM}})$$

$$\Delta a_\mu^{\mathrm{BSM}} \sim \mathcal{O}(\Delta m_\mu^{\mathrm{BSM}}/m_\mu) \times \frac{m_\mu^2}{M_{\mathrm{BSM}}^2}$$

Testable with

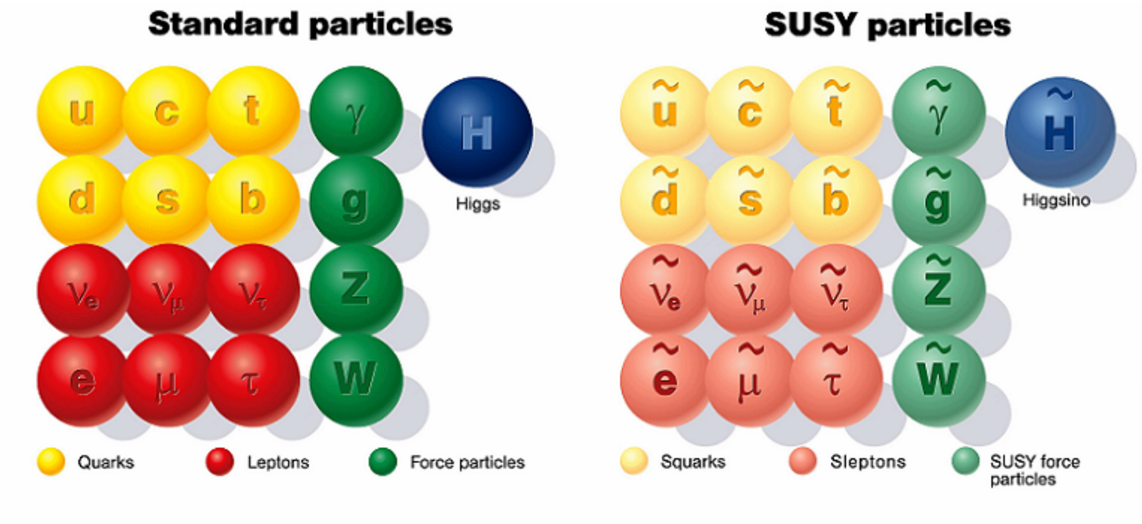
$$p\bar{p} \rightarrow \mathrm{LQ}\overline{\mathrm{LQ}} \qquad Z \rightarrow \mu^+\mu^-$$



Supersymmetry

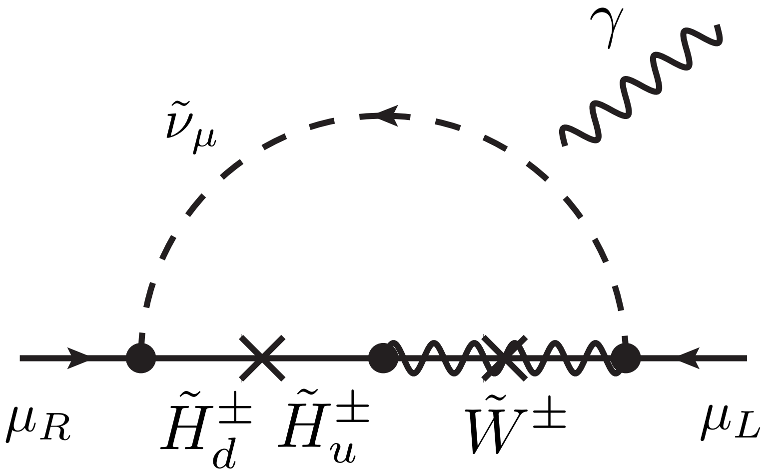
Minimal Supersymmetric Standard Model (MSSM) is one of attractive candidate of NP

supersymmetry
boson ↔ fermion



Δa_μ could be explained with chirality flip $tan\beta$ enhancement

$$a_\mu^{SUSY} \sim \frac{g_{EW}^2}{16\pi^2}\frac{m_\mu^2}{M_{SUSY}^2}\tan\beta$$



Ratio of vevs of two Higgs doublets $H_{u,d}$

$$\frac{v_u}{v_d} = \tan\beta$$

$$y_\mu^{\rm SUSY} = y_\mu^{\rm SM}\frac{v}{v_d} \simeq y_\mu^{\rm SM}\tan\beta$$

$$\sqrt{v_{\rm u}^2 + v_{\rm d}^2} = v$$

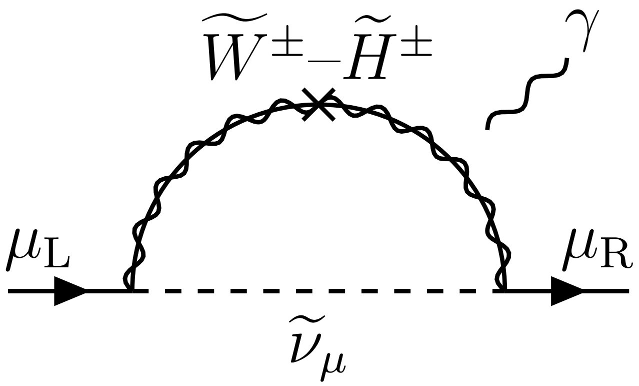
Supersymmetry

Endo,Hamaguchi,Iwamoto,Kitahara
2104.03217

SUSY contributions to the muon g−2 can be sizable when at least three SUSY multiplets are as light as O(100) GeV

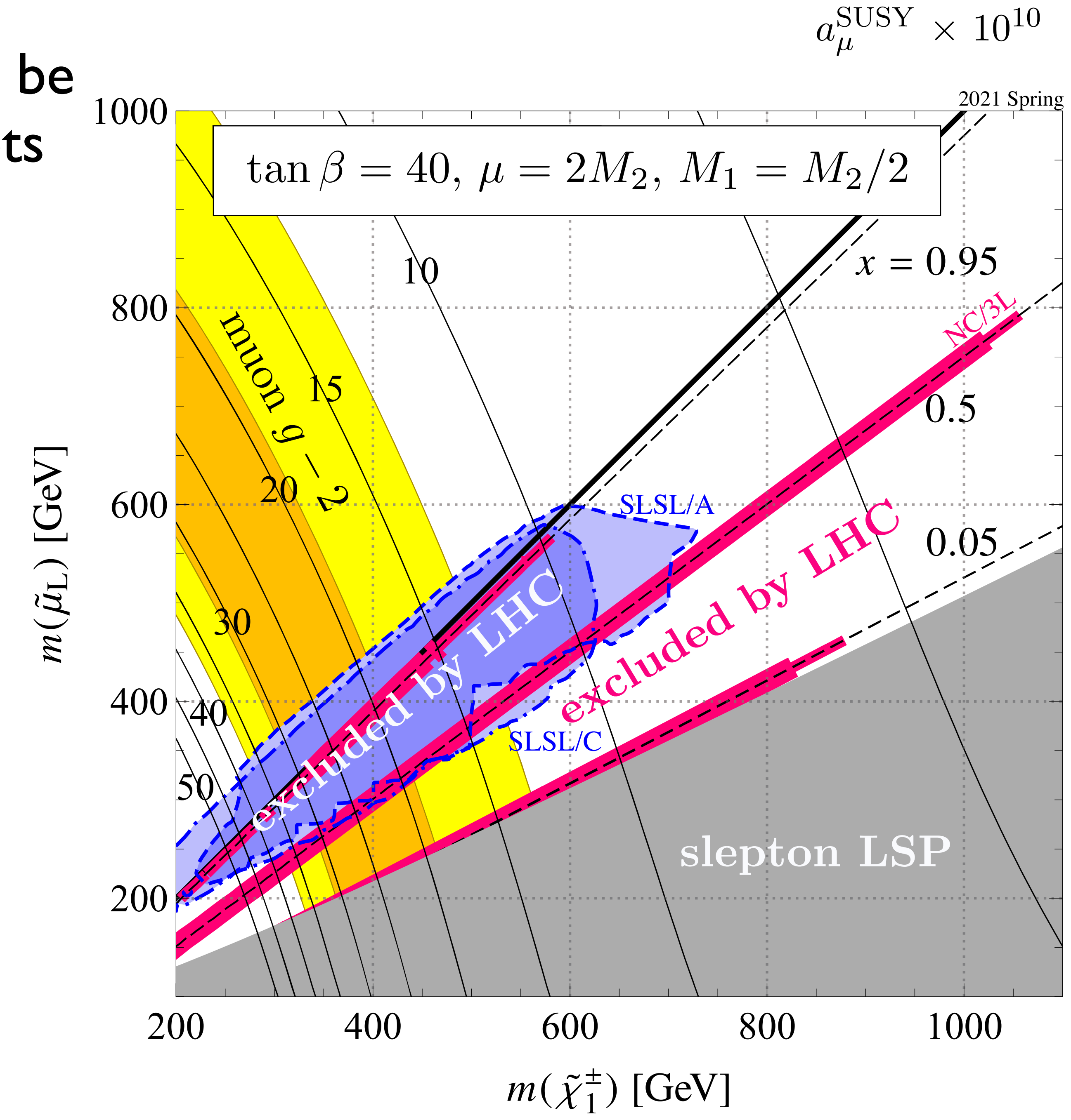
Light Wino, Higgsino, smuon scenario

Large tanβ



g-2 favors ~100-1000GeV

→ LHC search



$$x = \frac{m_{\widetilde{\mu}_L} - m_{\widetilde{\chi}_1^0}}{m_{\widetilde{\chi}_1^{\pm}} - m_{\widetilde{\chi}_1^0}}$$

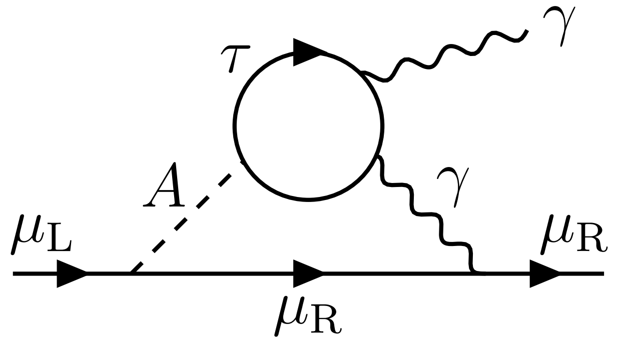
2HDM

Iguro, Kitahara, Lang Takeuchi
2304.09887

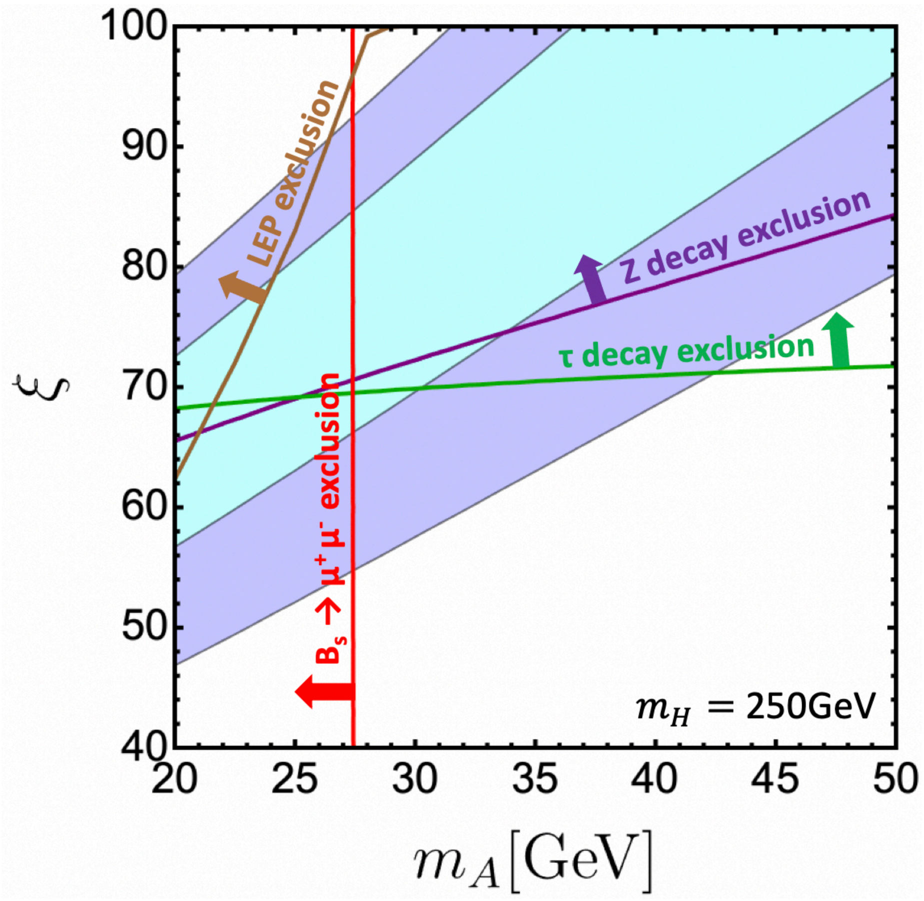
Two Higgs doublet model (2HDM) : Neutral H, A and charged $H^{\pm}$ higgs

To avoid tight constraints form flavor observables,
need specific yukawa structure → Type-X, flavor aligned

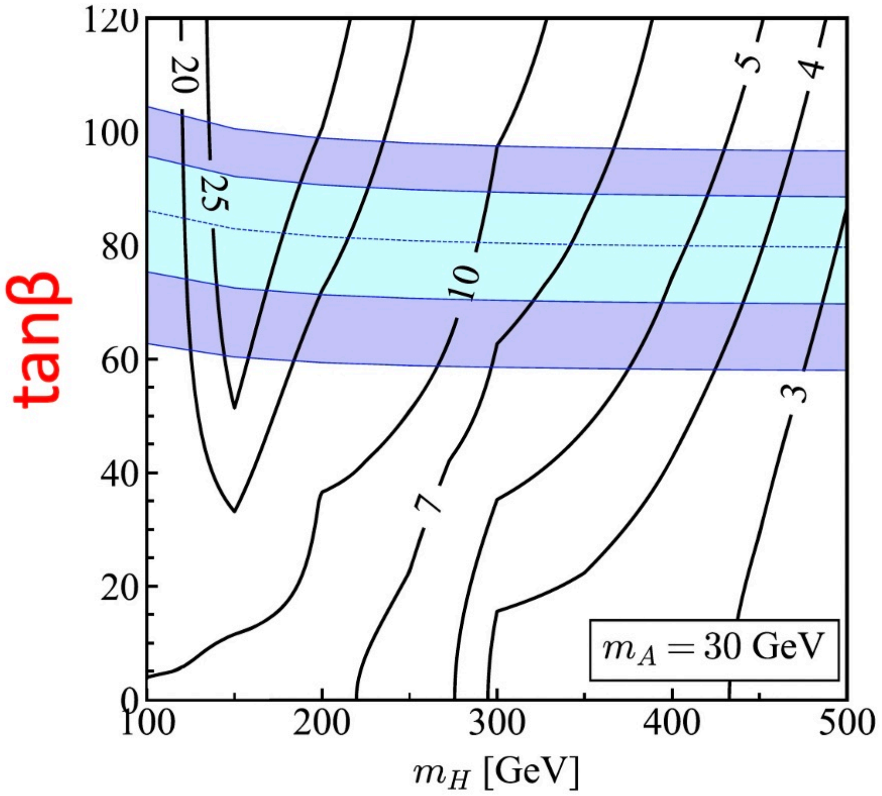
Light neutral pseudo scalar A $(tan\beta)^2$ enhancement



2 loop Barr-Zee diagram



$pp \to HA, H^{\pm}A, H^{\pm}H, H^{\pm}H^{\mp}(\to \text{multi}-\tau)$,



larger than 1 is expected to be excluded in Run2 data

excluded by flavor and collider bounds

Z' new gauge boson

W.Altmannshofer, S.Gori, M.Pospelov, I.Yavin

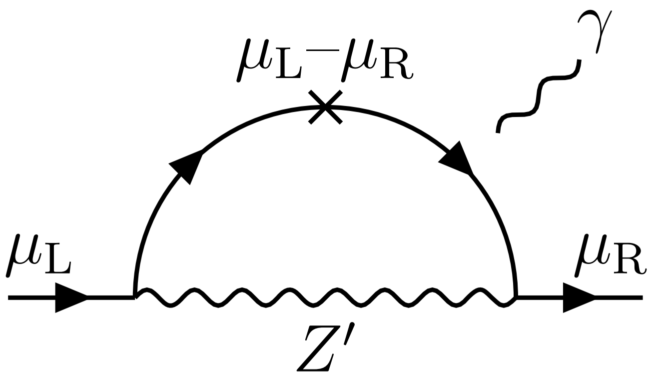
1406.2332

Additional $U(1)_X$ gauge symmetry with Z' boson

- anomaly free $X=B-L, B-3L_e, B-3L_\mu, B-3L_\tau, L_e-L_\mu, L_e-L_\tau, L_\mu-L_\tau$

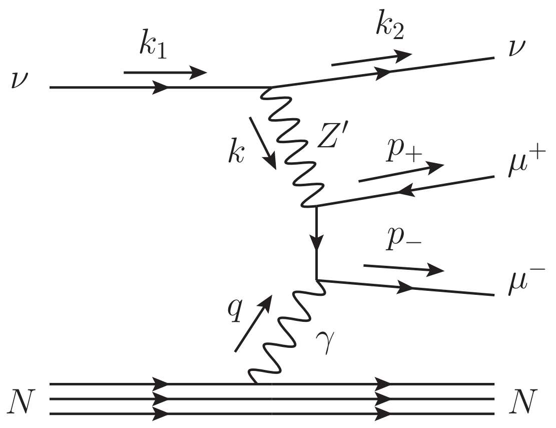
for muon g-2, couple to muon → $L_\mu - L_\tau$ model

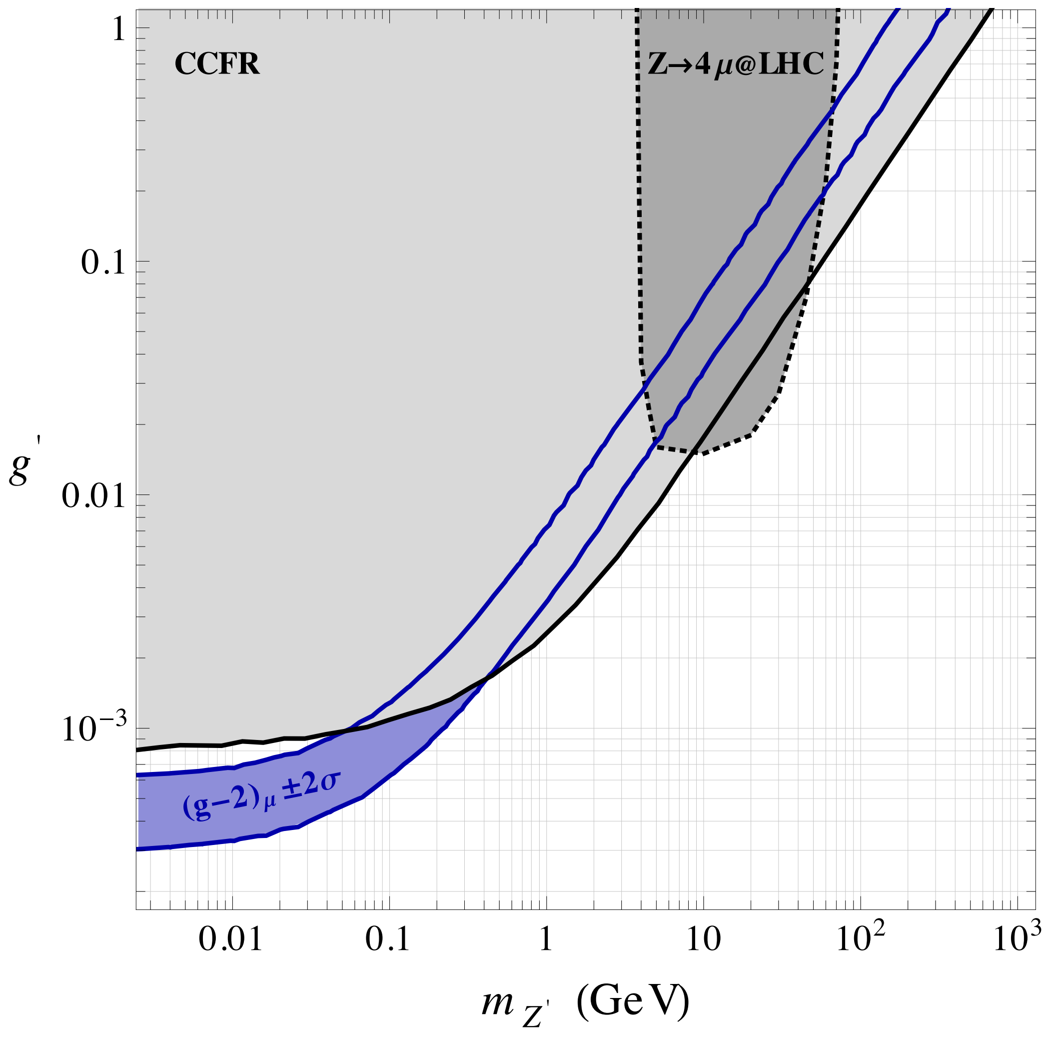
not couple to electron



interact only with 2nd & 3rd generation leptons

Severely constrained by neutrino trident production



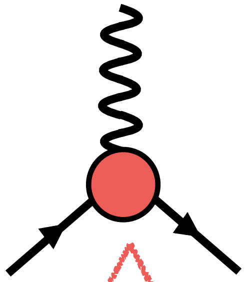


Light Z' boson can explain muon g-2

$$10 \text{ MeV} \lesssim m_{Z'} \lesssim 200 \text{ MeV}$$

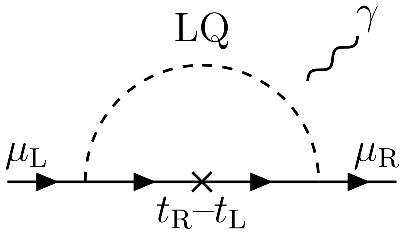
BBN

From NP models to EFT

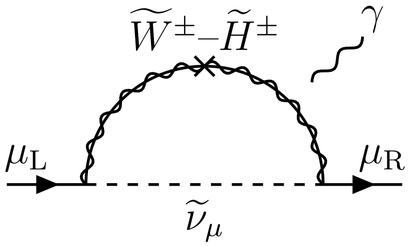


New physics (NP)

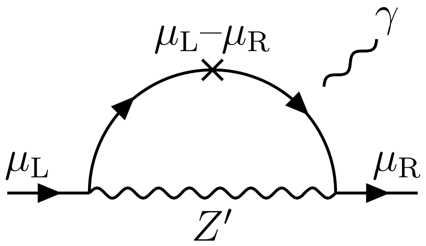
Effective field theory (EFT)



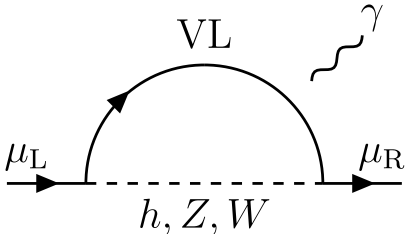
leptoquarks(LQ)



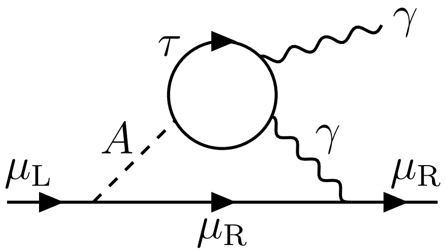
SUSY



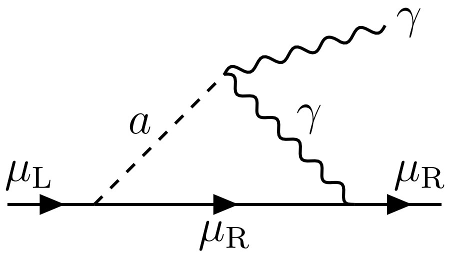
Z’ new gauge boson



Vectorlike lepton(VL)



Scalar particle(A)

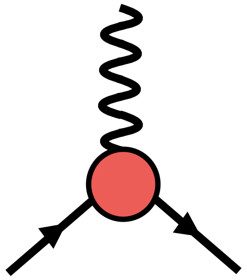


axion

Heavy NP particle
Large coupling

Light NP particle
tiny coupling

From NP models to EFT



New physics (NP)

Effective field theory (EFT)

No new particles have been observed

Importance of approaches in effective theory that do not rely on model details

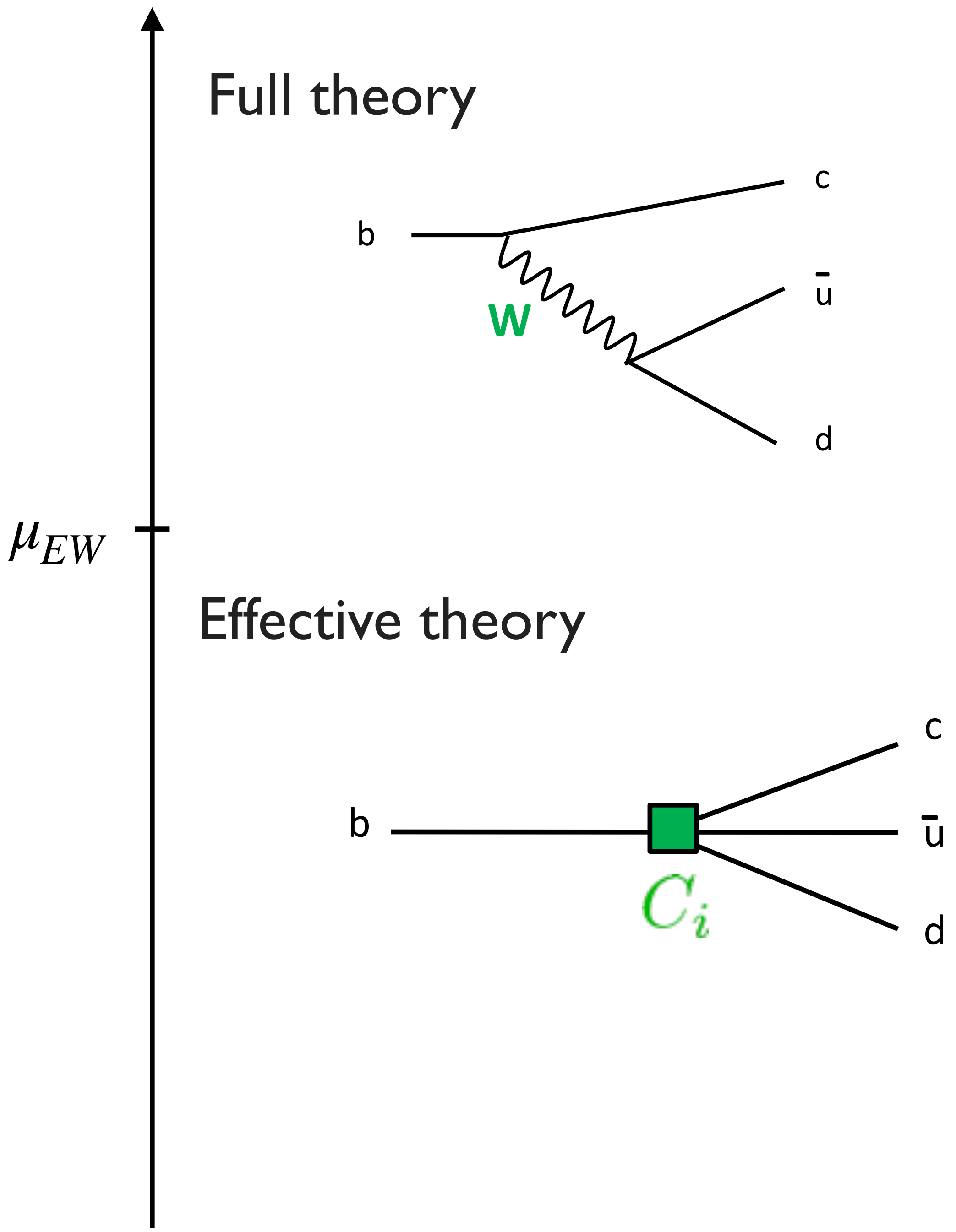
SM Effective Field Theory (SMEFT) enables parametrization of high-scale NP using SM fields

※ for light particle study, e.g. axion and g-2 in low-energy EFT

Galda and Neubert 2308.01338

Effective field theory

c.g. b→cud decay



$$\mathcal{M} \sim g^2 V_{cb}^* V_{ud} \frac{1}{q^2 - M_W^2} (\bar{c}\gamma^\mu b)_L (\bar{d}\gamma^\mu u)_L$$

↓ Integrate heavy particle

$$\simeq \boxed{-\frac{g^2}{M_W^2} V_{cb}^* V_{ud}} \boxed{(\bar{c}\gamma^\mu b)_L (\bar{d}\gamma^\mu u)_L}$$

$$C_i \times O_i$$

Wilson coefficients

Local operator (dim6)

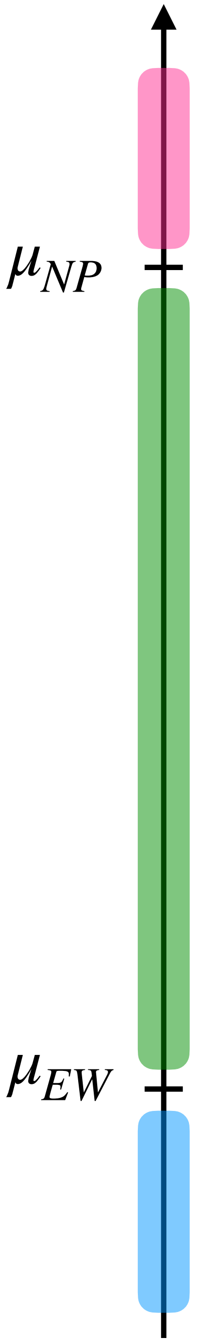
$$\mathcal{H}_{\text{eff}} \sim \sum_i C_i O_i$$

SM Effective Field Theory (SMEFT)

Grzadkowski, Iskrzynski, Misiak and Rosiek 1008.4884

SMEFT is a effective theory based on $SU(3)_c \times SU(2)_L \times U(1)_Y$ at scale $\mu_{\rm EW} < \mu < \mu_{\rm NP}$

Complete non-redundant classification of baryon- and lepton-number conserving dim6 operators (Warsaw basis)



New physics

SMEFT

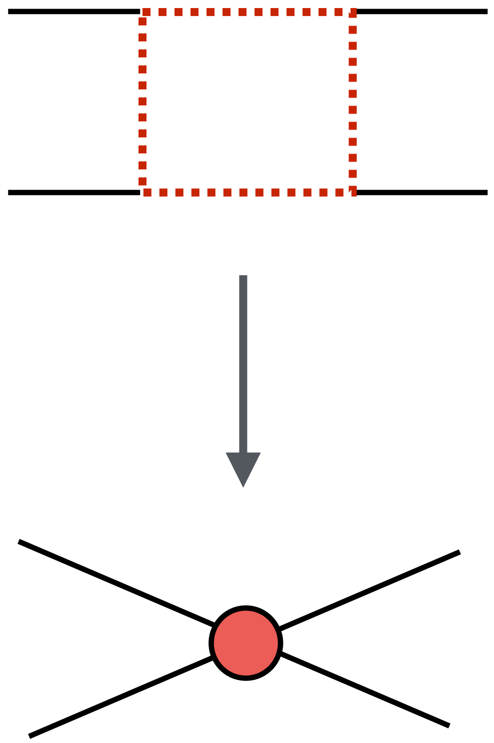
$$\mathscr{L}_{eff} = \mathscr{L}_{SM} + \frac{C_{NP}}{\Lambda_{NP}}\mathcal{O}^{\rm dim5} + \boxed{\sum_i \frac{C_{NP}}{\Lambda_{NP}^2}\mathcal{O}^{\rm dim6}} + \sum_i \frac{C_{NP}}{\Lambda_{NP}^3}\mathcal{O}^{\rm dim7} + \cdots$$

w/o flavor index **59** dim six operators in SMEFT

w/ flavor index ($n_g = 3$) **2499** free couplings in the SMEFT

1350 CP-even and **1149** CP-odd

Low energy effective theory (LEFT)

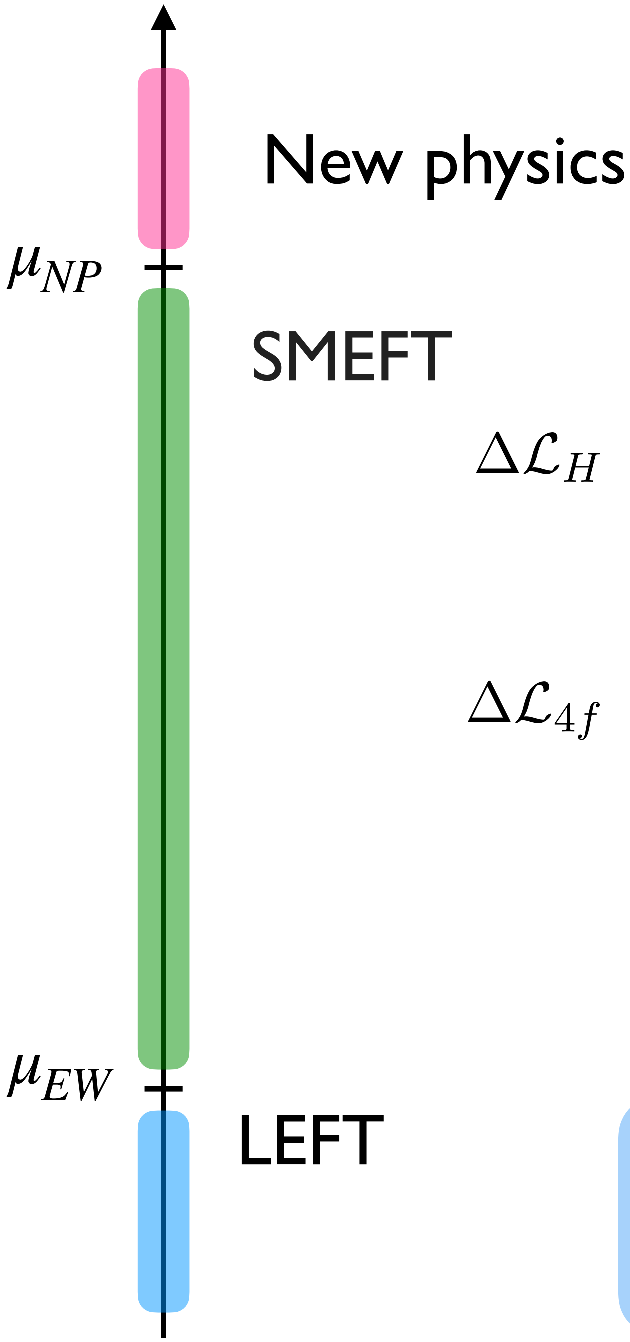


EFT and g-2

Aebischer, Dekens, Jenkins, Manohar, Sengupta, Stoffer, 2102.08954

Isidori, Pages and Wilsch 2111.13724

For g-2

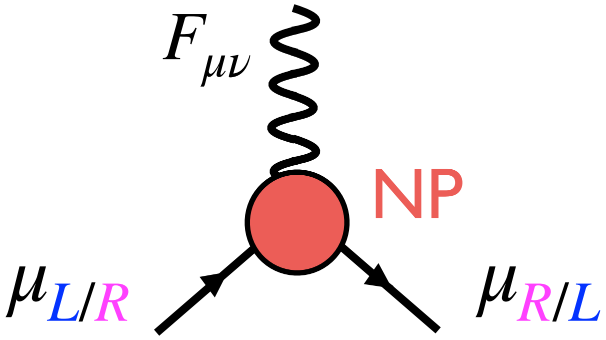


$$\begin{aligned}\Delta\mathcal{L}_H &= -[Y_e]_{pr}(\bar{\ell}_p e_r)H + C_{\underset{pr}{eH}}(\bar{\ell}_p e_r)H(H^\dagger H)\\ &\quad + C_{\underset{pr}{eB}}(\bar{\ell}_p \sigma^{\mu\nu} e_r)HB_{\mu\nu} + C_{\underset{pr}{eW}}(\bar{\ell}_p \sigma^{\mu\nu} e_r)\tau^I H W^I_{\mu\nu}\,,\\ \Delta\mathcal{L}_{4f} &= C^{(3)}_{\underset{prst}{lequ}}(\bar{\ell}^j_p \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}^k_s \sigma^{\mu\nu} u_t) + C^{(1)}_{\underset{prst}{lequ}}(\bar{\ell}^j_p e_r)\epsilon_{jk}(\bar{q}^k_s u_t)\\ &\quad + C_{\underset{prst}{ledq}}(\bar{\ell}^j_p e_r)(\bar{d}_s q_{tj})\,.\end{aligned}$$

↓

$$\mathcal{O}_{\underset{LR}{e\gamma}} = \frac{v}{\sqrt{2}}\overline{E}_L \sigma^{\mu\nu} E_R F_{\mu\nu}$$

Dipole operator

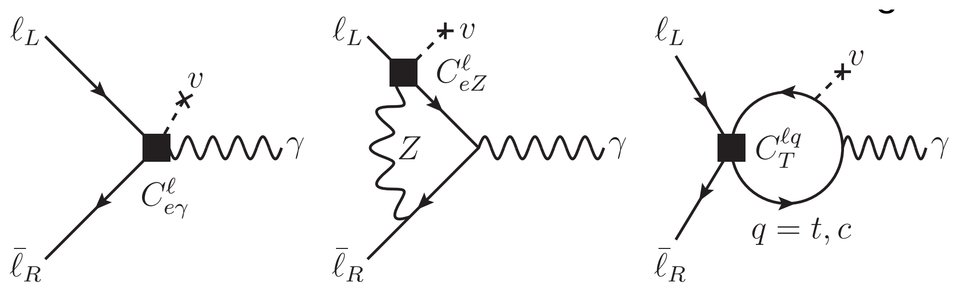


EFT and g-2

Buttazzo and Paradisi
2012.02769

I loop effect study

$$\mathscr{L} = \frac{C_{eB}}{\Lambda^2}(\bar{\ell}_L\sigma^{\mu\nu}e_R)\,H\,B_{\mu\nu} + \frac{C_{eW}}{\Lambda^2}(\bar{\ell}_L\sigma^{\mu\nu}e_R)\tau^I\,H\,W^I_{\mu\nu} + \frac{C_{qT}}{\Lambda^2}(\bar{\ell}_L\sigma^{\mu\nu}e_R)\,\epsilon\,(\bar{q}_L\sigma_{\mu\nu}u_R)$$



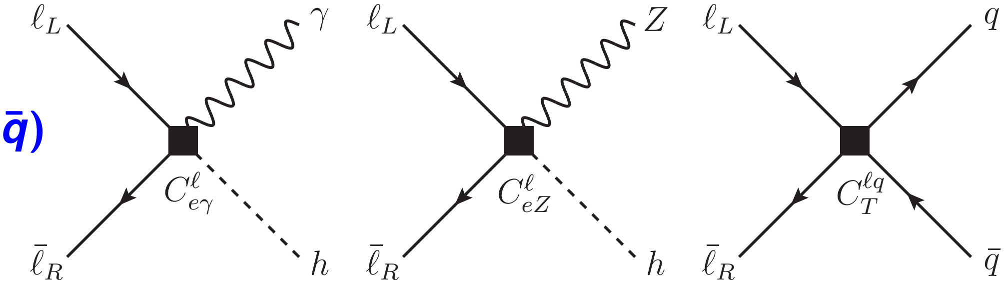
$$\Delta a_\mu \simeq \frac{4m_\mu v}{e\Lambda^2}\left(C_{e\gamma}(m_\mu) - \frac{3\alpha}{2\pi}\frac{c_W^2 - s_W^2}{s_W c_W}C_{eZ}\log\frac{\Lambda}{m_Z}\right) - \sum_{q=c,t}\frac{4m_\mu m_q}{\pi^2}\frac{C_{Tq}}{\Lambda^2}\log\frac{\Lambda}{m_q}$$

$$\approx \left(\frac{250\,\mathrm{TeV}}{\Lambda^2}\right)^2 (C_{e\gamma} - 0.2C_{Tt} - 0.001C_{Tc} - 0.05C_{eZ})$$

Connection with Muon collider

- **Strongly coupled NP:** $C^{\mu}_{e\gamma}, C^{\mu t}_{T} \sim g^2_{\mathrm{NP}}/16\pi^2 \lesssim 1$ implying $\Lambda \lesssim$ *few* x 100 TeV, beyond the direct production reach of any foreseen collider.
- **Weakly coupled NP:** $C^{\mu}_{e\gamma}, C^{\mu t}_{T} \lesssim 1/16\pi^2$ implying $\Lambda \lesssim 20$ TeV maybe within the direct production reach of a very high-energy Muon Collider

$$\Delta a_\mu \sim \frac{m_\mu v}{\Lambda^2} C_{eV,T} \quad \Longleftrightarrow \quad \sigma_{\mu\mu\to f} \sim \frac{s}{\Lambda^4}|C_{eV,T}|^2 \quad (f = e\gamma, eZ, q\bar{q})$$



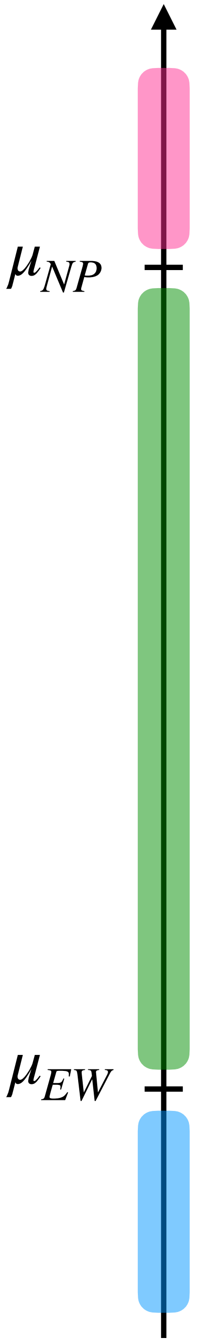
At high energy $\sigma_{\mu\mu\to f}$ can compete with Δa_μ to test the very same NP

EFT and g-2

Aebischer, Dekens, Jenkins, Manohar, Sengupta, Stoffer, 2102.08954

Isidori, Pages and Wilsch 2111.13724

From here on, the discussion is EFT-based and focus on LEFT dipole operator

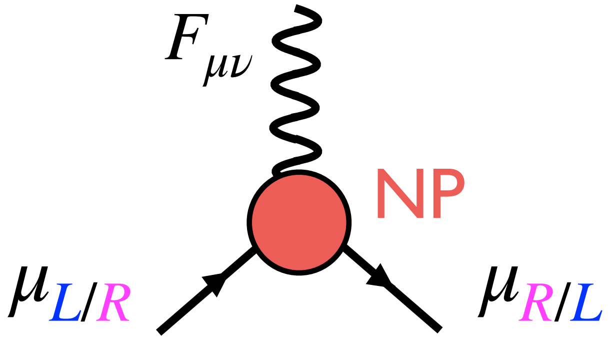


$$\begin{aligned}\Delta\mathcal{L}_H &= -[Y_e]_{pr}(\bar{\ell}_p e_r)H + C_{\underset{pr}{eH}}(\bar{\ell}_p e_r)H(H^\dagger H)\\ &\quad + C_{\underset{pr}{eB}}(\bar{\ell}_p \sigma^{\mu\nu} e_r)HB_{\mu\nu} + C_{\underset{pr}{eW}}(\bar{\ell}_p \sigma^{\mu\nu} e_r)\tau^I H W^I_{\mu\nu}\,,\\ \Delta\mathcal{L}_{4f} &= C^{(3)}_{\underset{prst}{lequ}}(\bar{\ell}^j_p \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}^k_s \sigma^{\mu\nu} u_t) + C^{(1)}_{\underset{prst}{lequ}}(\bar{\ell}^j_p e_r)\epsilon_{jk}(\bar{q}^k_s u_t)\\ &\quad + C_{\underset{prst}{ledq}}(\bar{\ell}^j_p e_r)(\bar{d}_s q_{tj})\,.\end{aligned}$$

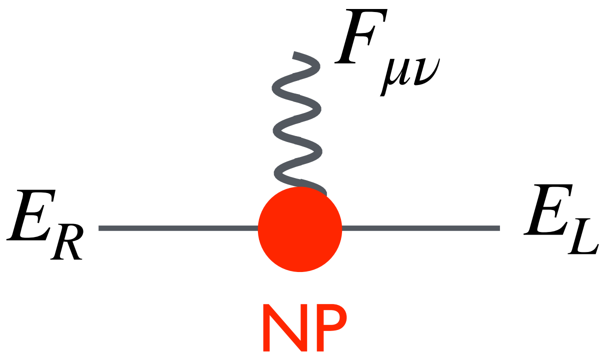
↓

$$\mathcal{O}_{\underset{LR}{e\gamma}} = \frac{v}{\sqrt{2}}\overline{E}_L \sigma^{\mu\nu} E_R F_{\mu\nu}$$

Dipole operator



Lepton flavor structure from $(g-2)_\ell$, LFV and EDM



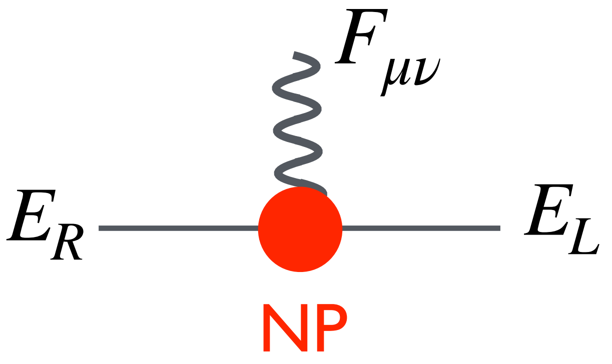
Dipole operator

$$\mathcal{O}_{\underset{LR}{e\gamma}} = \frac{v}{\sqrt{2}}\overline{E}_L\sigma^{\mu\nu}E_R F_{\mu\nu}$$

$$\mathcal{L}_{\text{dipole}} = \frac{1}{\Lambda^2}\left(\mathcal{C}'_{\underset{LR}{e\gamma}}\mathcal{O}_{\underset{LR}{e\gamma}} + \mathcal{C}'_{\underset{RL}{e\gamma}}\mathcal{O}_{\underset{RL}{e\gamma}}\right)$$

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{array}{cc} \begin{array}{ccc} e & \mu & \tau \end{array} & \\ \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix} & \begin{array}{c} e \\ \mu \\ \tau \end{array} \end{array}$$

Lepton flavor structure from $(g-2)_\ell$, LFV and EDM



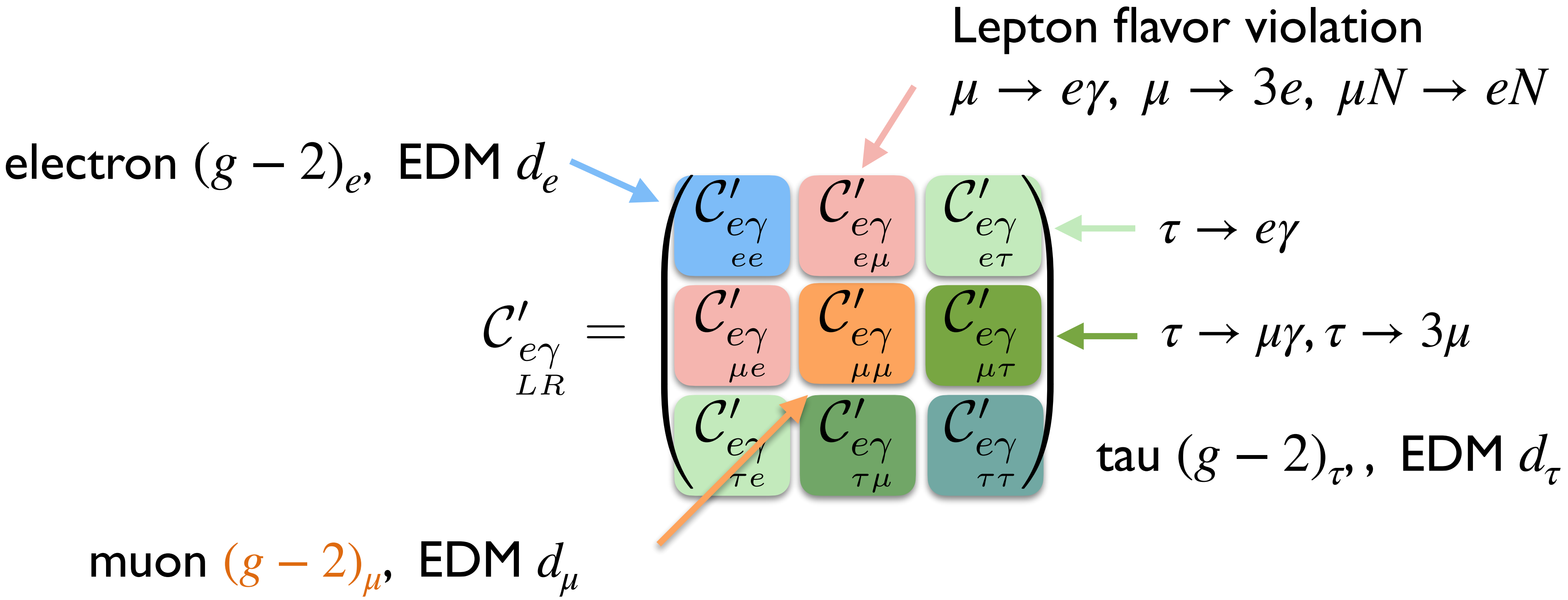
Dipole operator

$$\mathcal{O}_{\underset{LR}{e\gamma}} = \frac{v}{\sqrt{2}} \overline{E}_L \sigma^{\mu\nu} E_R F_{\mu\nu} \qquad \mathcal{L}_{\text{dipole}} = \frac{1}{\Lambda^2}\left(\mathcal{C}'_{\underset{LR}{e\gamma}} \mathcal{O}_{\underset{LR}{e\gamma}} + \mathcal{C}'_{\underset{RL}{e\gamma}} \mathcal{O}_{\underset{RL}{e\gamma}}\right)$$

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{array}{cc} \begin{matrix} \quad e \quad & \quad \mu \quad & \quad \tau \quad \end{matrix} & \\ \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \boxed{\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix} & \begin{matrix} e \\ \mu \\ \tau \end{matrix} \end{array}$$

muon $(g-2)_\mu$

Lepton flavor structure from $(g-2)_\ell$, LFV and EDM



Lepton flavor violation

$\mu \to e\gamma,\ \mu \to 3e,\ \mu N \to eN$

electron $(g-2)_e$, EDM d_e

$$\mathcal{C}'_{\substack{e\gamma\\LR}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma\\ee}} & \mathcal{C}'_{\substack{e\gamma\\e\mu}} & \mathcal{C}'_{\substack{e\gamma\\e\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\mu e}} & \mathcal{C}'_{\substack{e\gamma\\\mu\mu}} & \mathcal{C}'_{\substack{e\gamma\\\mu\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\tau e}} & \mathcal{C}'_{\substack{e\gamma\\\tau\mu}} & \mathcal{C}'_{\substack{e\gamma\\\tau\tau}} \end{pmatrix}$$

$\tau \to e\gamma$

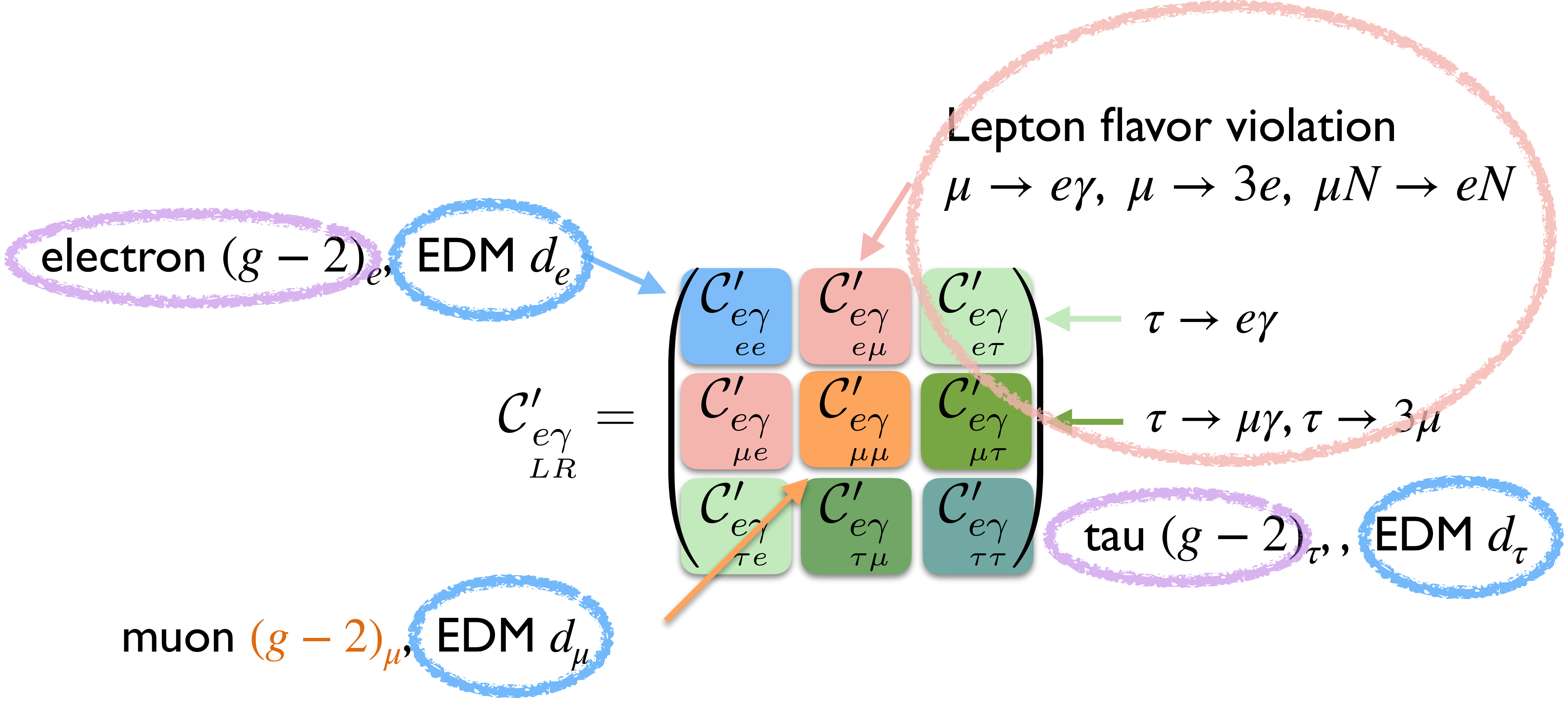
$\tau \to \mu\gamma, \tau \to 3\mu$

tau $(g-2)_\tau$, , EDM d_τ

muon $(g-2)_\mu$, EDM d_μ

once we introduce NP operator with flavor index, other flavor observables are also introduced

Lepton flavor structure from $(g-2)_\ell$, LFV and EDM



Lepton flavor violation

$\mu \to e\gamma,\ \mu \to 3e,\ \mu N \to eN$

electron $(g-2)_e$, EDM d_e

$$\mathcal{C}'_{\substack{e\gamma\\LR}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma\\ee}} & \mathcal{C}'_{\substack{e\gamma\\e\mu}} & \mathcal{C}'_{\substack{e\gamma\\e\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\mu e}} & \mathcal{C}'_{\substack{e\gamma\\\mu\mu}} & \mathcal{C}'_{\substack{e\gamma\\\mu\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\tau e}} & \mathcal{C}'_{\substack{e\gamma\\\tau\mu}} & \mathcal{C}'_{\substack{e\gamma\\\tau\tau}} \end{pmatrix}$$

$\tau \to e\gamma$

$\tau \to \mu\gamma, \tau \to 3\mu$

tau $(g-2)_\tau$, , EDM d_τ

muon $(g-2)_\mu$, EDM d_μ

once we introduce NP operator with flavor index, other flavor observables are also introduced

diagonal elements → g-2 and EDM

off diagonal elements → Lepton flavor violation

electron/tau g-2

In broad class of NP, contributions to a_ℓ scale as

$$a_\mu^{NP} \sim \frac{g_{NP}^2}{16\pi^2}\frac{m_\mu^2}{M_{NP}^2}$$

$$\frac{\Delta a_\ell}{\Delta a_{\ell'}} = \left(\frac{m_\ell}{m_{\ell'}}\right)^2$$

Naive scaling

$$\Delta a_e \approx \left(\frac{\Delta a_\mu}{2.5\times10^{-9}}\right)6.3\times10^{-14} \qquad \Delta a_\tau \approx \left(\frac{\Delta a_\mu}{2.5\times10^{-9}}\right)0.7\times10^{-6}$$

Exp

$$\Delta a_e^{Cs} = a_e^{\mathrm{Exp}} - a_e^{\mathrm{SM,CS}} = (-8.8\pm3.6)\times10^{-13}$$
$$\Delta a_e^{Rb} = a_e^{\mathrm{Exp}} - a_e^{\mathrm{SM,Rb}} = (4.8\pm3.0)\times10^{-13} \quad .$$

$$a_\tau = -0.018\pm0.017$$

DELPHI J.Abdallah et al

Magnetic & Electric dipole moment

$$\mathscr{H} = -\overrightarrow{\mu_\ell}\cdot\overrightarrow{B} - \overrightarrow{d_\ell}\cdot\overrightarrow{E}$$

magnetic dipole moment

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell}\vec{S}$$

electric dipole moment

$$\overrightarrow{d_\ell} = \eta_\ell \frac{e}{2m_\ell}\overrightarrow{S}$$

Magnetic & Electric dipole moment

$$\mathscr{H} = -\overset{\text{CP even}}{\boxed{\overrightarrow{\mu_\ell} \cdot \overrightarrow{B}}} - \overset{\text{CP odd}}{\boxed{\overrightarrow{d_\ell} \cdot \overrightarrow{E}}}$$

Time reversal $-$ $-$ $-$ $+$

Time reversal

$\overrightarrow{S}$ $-$

$\overrightarrow{B}$ $-$

$\overrightarrow{E}$ $+$

magnetic dipole moment

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{S}$$

electric dipole moment

$$\overrightarrow{d_\ell} = \eta_\ell \frac{e}{2m_\ell} \overrightarrow{S}$$

Magnetic & Electric dipole moment

$$\mathscr{H} = -\overbrace{\boxed{\vec{\mu}_\ell \cdot \vec{B}}}^{\text{CP even}} - \overbrace{\boxed{\vec{d}_\ell \cdot \vec{E}}}^{\text{CP odd}}$$

Time reversal $\quad - \quad - \quad - \quad +$

Time reversal

$\vec{S}$ $-$

$\vec{B}$ $-$

$\vec{E}$ $+$

magnetic dipole moment

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{S}$$

electric dipole moment

$$\vec{d}_\ell = \eta_\ell \frac{e}{2m_\ell} \vec{S}$$

In QFT

$$\mathcal{L}_{\text{eff}}^{\text{DM}} = -\frac{1}{2} \left\{ \bar{\psi}\, \sigma^{\mu\nu} \left[D_\mu \frac{1+\gamma_5}{2} + D_\mu^* \frac{1-\gamma_5}{2} \right] \psi \right\} F_{\mu\nu}$$

$\propto \sigma_{\mu\nu}$ $\qquad$ $\propto \sigma_{\mu\nu}\gamma_5$

$$\text{Re}\, D_\mu = a_\mu \frac{e}{2m_\mu} \quad , \quad \text{Im}\, D_\mu = d_\mu = \frac{\eta_\mu}{2} \frac{e}{2m_\mu}$$

Magnetic & Electric dipole moment

$$\begin{tikzcd} & (g-2)_\mu \\ \left[\mathcal{C}'_{e\gamma}\right]_{\mu\mu} \arrow[ur, "\mathrm{Re}"] \arrow[dr, "\mathrm{Im}"'] & \\ & \text{EDM } d_\mu \end{tikzcd}$$

EDM d_e

$$d_e = -\sqrt{2}\,\frac{v}{\Lambda^2}\,\mathrm{Im}\,\left[\mathcal{C}'_{e\gamma}\right]_{ee}$$

$$|d_e/e| \lesssim 1.1 \times 10^{-29}\,\mathrm{cm} = 5.6 \times 10^{-13}\,\mathrm{TeV}^{-1}$$

ACME

$$\frac{1}{\Lambda^2}\mathrm{Im}\,\left[\mathcal{C}'_{e\gamma}\right]_{ee} < 1.6 \times 10^{-12}\,\mathrm{TeV}^{-2}$$

EDM d_μ

$$d_\mu = -\sqrt{2}\,\frac{v}{\Lambda^2}\,\mathrm{Im}\,\left[\mathcal{C}'_{e\gamma}\right]_{\mu\mu}$$

$$|d_\mu/e| < 1.8 \times 10^{-19}\ \mathrm{cm}$$

BNL

$$\longrightarrow \quad \frac{1}{\Lambda^2}\mathrm{Im}\,\left[\mathcal{C}'_{e\gamma}\right]_{\mu\mu} < 2.7 \times 10^{-2}\,\mathrm{TeV}^{-2}$$

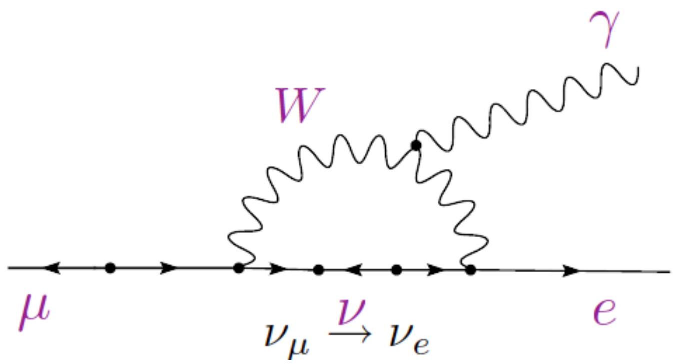
$$\frac{d_\ell}{d_{\ell'}} = \frac{m_\ell}{m_{\ell'}}$$

Naive scaling

$$\frac{m_e}{m_\mu} \sim 5 \times 10^{-3}$$

Lepton flavor violation

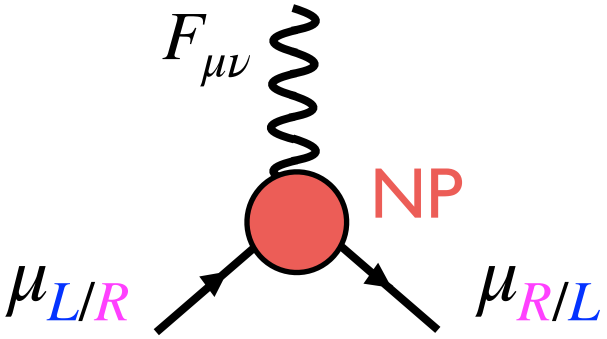
Highly suppressed in SM+m_ν by GIM mechanism due to the smallness of m_ν



$$\mathcal{B}(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_l (V_{\mathrm{MNS}})^*_{\mu l}(V_{\mathrm{MNS}})_{el}\frac{\Delta m^2_{\nu l}}{M_W^2}\right|^2$$

$$\sim \mathcal{O}(10^{-54})$$

→ Good probe for NP



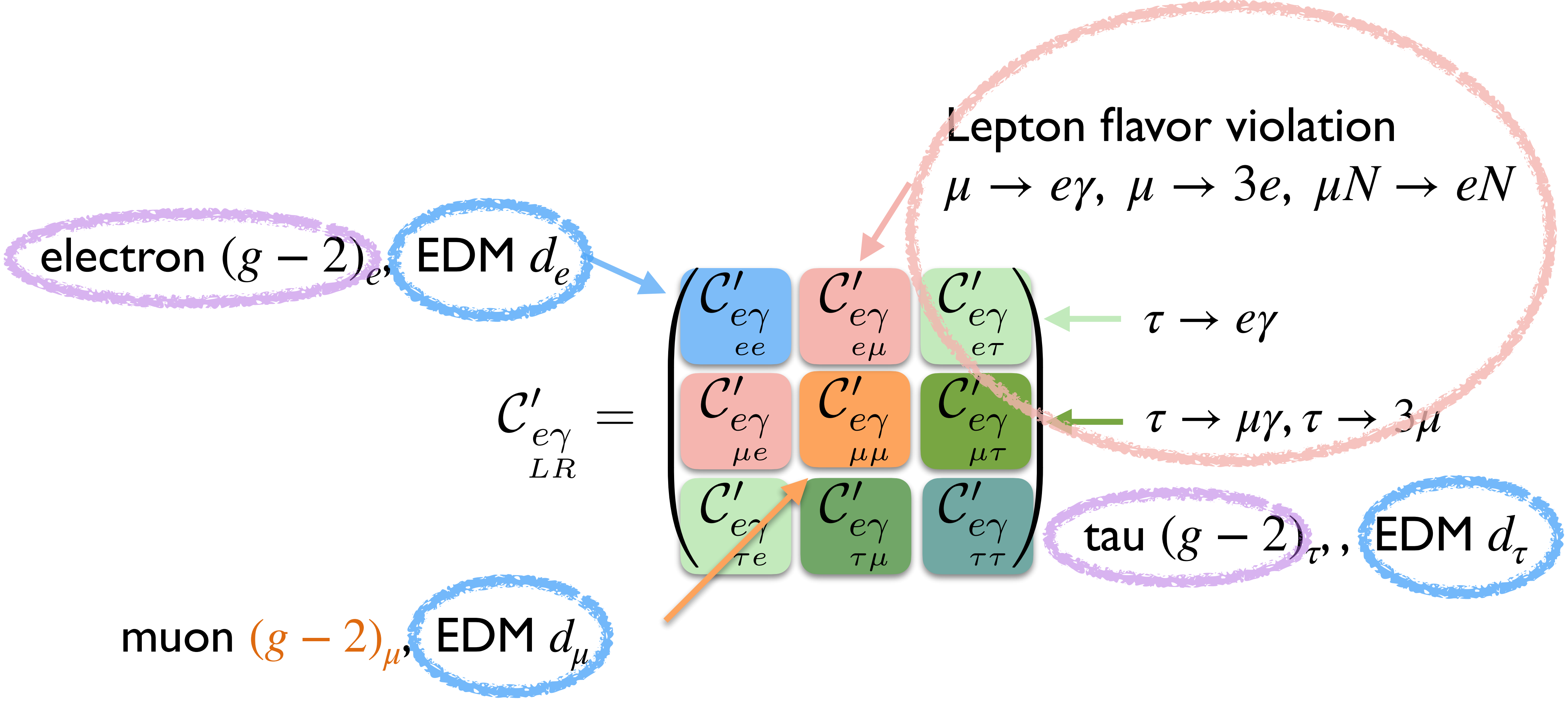
$$\mathcal{B}(\ell_r \to \ell_s\gamma) = \frac{m^3_{\ell_r}v^2}{8\pi\Gamma_{\ell_r}}\frac{1}{\Lambda^4}\left(|\mathcal{C}'_{\substack{e\gamma\\ rs}}|^2 + |\mathcal{C}'_{\substack{e\gamma\\ sr}}|^2\right)$$

$\mu \to e\gamma$ searched by MEG, $\tau \to \ell\gamma$ searched by Belle II

$$\begin{array}{lll}
\mathcal{B}(\mu^+ \to e^+\gamma) & < 4.2\times 10^{-13} & |\mathcal{C}'_{\substack{e\gamma\\ e\mu(\mu e)}}| < 2.1\times 10^{-10} \\
\mathcal{B}(\tau \to \mu\gamma) & < 4.2\times 10^{-8} & |\mathcal{C}'_{\substack{e\gamma\\ \mu\tau(\tau\mu)}}| < 2.65\times 10^{-6} \\
\mathcal{B}(\tau \to e\gamma) & < 3.3\times 10^{-8} & |\mathcal{C}'_{\substack{e\gamma\\ e\tau(\tau e)}}| < 2.35\times 10^{-6}
\end{array}$$

$\mu \to e\gamma$ gives tight constraint on NP

Lepton flavor structure from $(g-2)_\ell$, LFV and EDM



Lepton flavor violation

$\mu \to e\gamma,\ \mu \to 3e,\ \mu N \to eN$

electron $(g-2)_e$, EDM d_e

$$\mathcal{C}'_{\substack{e\gamma\\LR}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma\\ee}} & \mathcal{C}'_{\substack{e\gamma\\e\mu}} & \mathcal{C}'_{\substack{e\gamma\\e\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\mu e}} & \mathcal{C}'_{\substack{e\gamma\\\mu\mu}} & \mathcal{C}'_{\substack{e\gamma\\\mu\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\tau e}} & \mathcal{C}'_{\substack{e\gamma\\\tau\mu}} & \mathcal{C}'_{\substack{e\gamma\\\tau\tau}} \end{pmatrix}$$

$\tau \to e\gamma$

$\tau \to \mu\gamma, \tau \to 3\mu$

tau $(g-2)_\tau$, , EDM d_τ

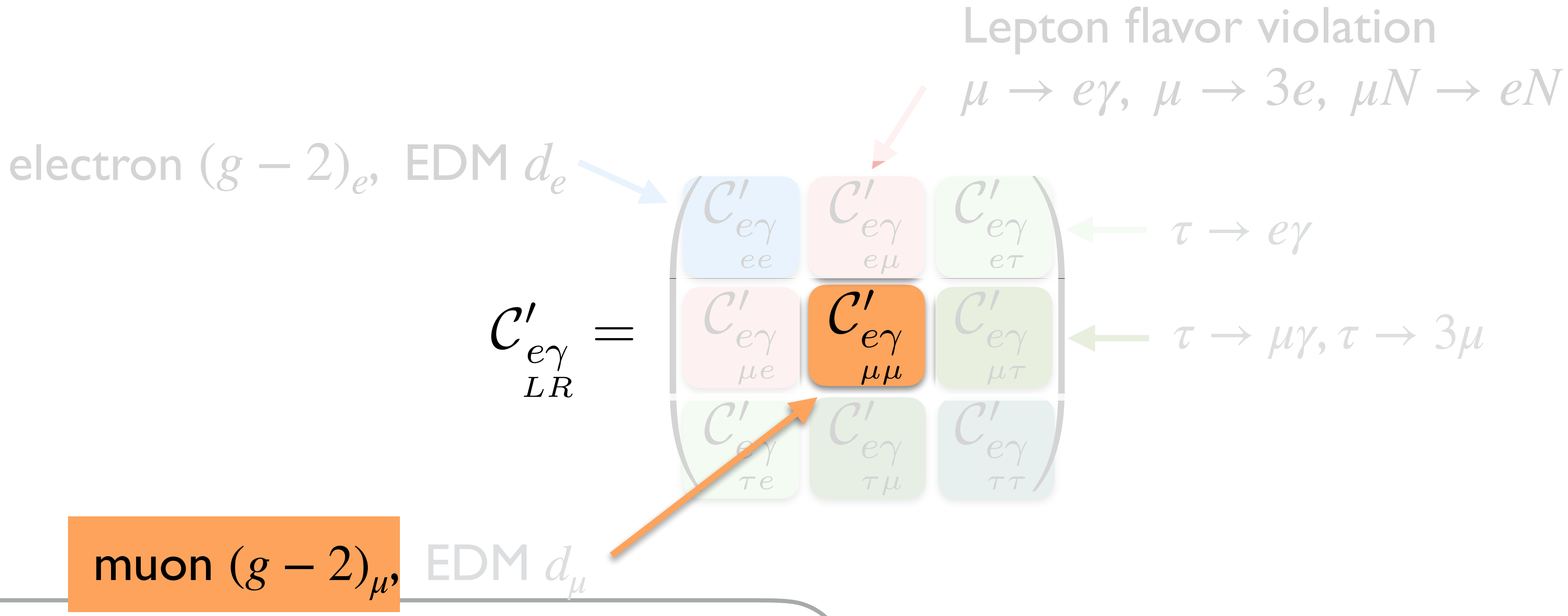
muon $(g-2)_\mu$, EDM d_μ

once we introduce NP operator with flavor index, other flavor observables are also introduced

diagonal elements → g-2 and EDM

off diagonal elements → Lepton flavor violation

Lepton flavor structure from $(g-2)_\ell$, LFV and EDM



Lepton flavor violation

$\mu \to e\gamma,\ \mu \to 3e,\ \mu N \to eN$

electron $(g-2)_e$, EDM d_e

$\tau \to e\gamma$

$\tau \to \mu\gamma, \tau \to 3\mu$

$$\mathcal{C}'_{\substack{e\gamma\\LR}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma\\ee}} & \mathcal{C}'_{\substack{e\gamma\\e\mu}} & \mathcal{C}'_{\substack{e\gamma\\e\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\mu e}} & \mathcal{C}'_{\substack{e\gamma\\\mu\mu}} & \mathcal{C}'_{\substack{e\gamma\\\mu\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\tau e}} & \mathcal{C}'_{\substack{e\gamma\\\tau\mu}} & \mathcal{C}'_{\substack{e\gamma\\\tau\tau}} \end{pmatrix}$$

muon $(g-2)_\mu$, EDM d_μ

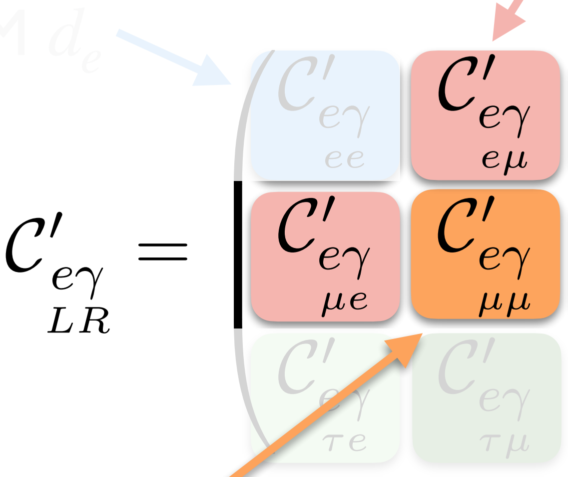
$$\Delta a_\mu = \frac{4m_\mu}{e}\frac{v}{\sqrt{2}}\frac{1}{\Lambda^2}\mathrm{Re}\left[\mathcal{C}'_{\substack{e\gamma\\\mu\mu}}\right]$$

$$\Delta a_\mu = a_\mu^{\mathrm{Exp}} - a_\mu^{\mathrm{SM}} = (251 \pm 59) \times 10^{-11}$$

FNAL, BNL

$$\longrightarrow \quad \frac{1}{\Lambda^2}\mathrm{Re}\left[\mathcal{C}'_{\substack{e\gamma\\\mu\mu}}\right] \approx 1.0 \times 10^{-5}\ \mathrm{TeV}^{-2}$$

Lepton flavor structure from $(g-2)_\ell$, LFV and EDM



electron $(g-2)_e$, EDM d_e

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} \end{pmatrix}$$

Lepton flavor violation

$\mu \to e\gamma$, $\mu \to 3e$, $\mu N \to eN$

$$\mathcal{B}(\ell_r \to \ell_s\gamma) = \frac{m_{\ell_r}^3 v^2}{8\pi\Gamma_{\ell_r}} \frac{1}{\Lambda^4} \left(|\mathcal{C}'_{\underset{rs}{e\gamma}}|^2 + |\mathcal{C}'_{\underset{sr}{e\gamma}}|^2 \right)$$

$$\mathcal{B}(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13} \ (90\% \text{ C.L.})$$

MEG

$$\longrightarrow \quad \frac{1}{\Lambda^2} |\mathcal{C}'_{\underset{e\mu(\mu e)}{e\gamma}}| < 2.1 \times 10^{-10}\,\text{TeV}^{-2}$$

muon $(g-2)_\mu$, EDM d_μ

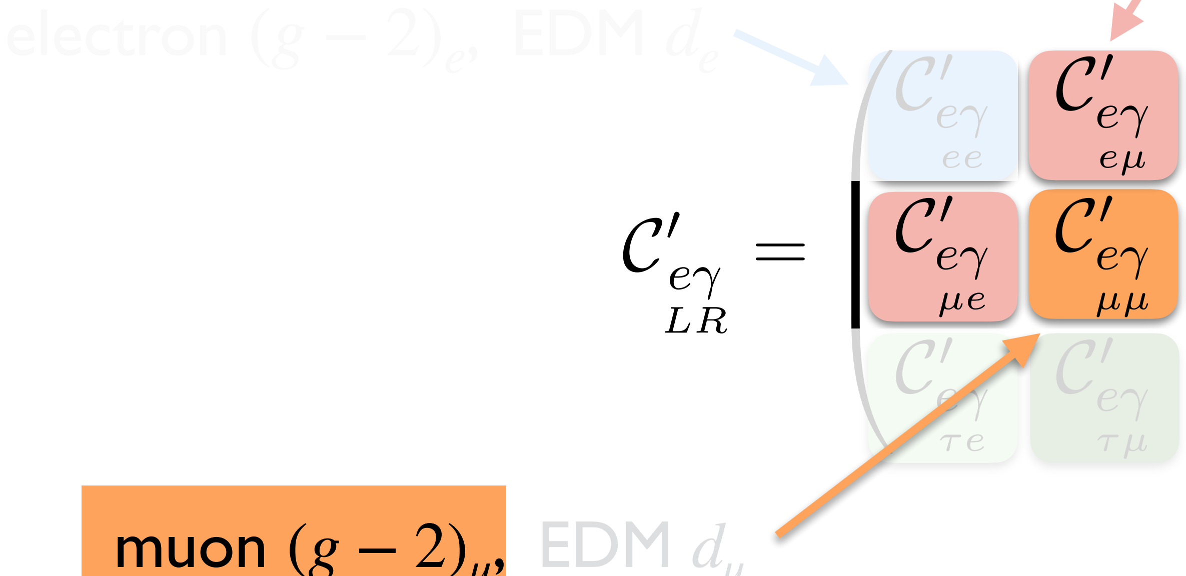
$$\Delta a_\mu = \frac{4m_\mu}{e} \frac{v}{\sqrt{2}} \frac{1}{\Lambda^2} \text{Re}\,[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}]$$

$$\Delta a_\mu = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$$

FNAL, BNL

$$\longrightarrow \quad \frac{1}{\Lambda^2} \text{Re}\,[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}] \approx 1.0 \times 10^{-5}\,\text{TeV}^{-2}$$

Lepton flavor structure from $(g-2)_\ell$, LFV and EDM



electron $(g-2)_e$, EDM d_e

$$\mathcal{C}'_{\substack{e\gamma\\LR}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma\\ee}} & \mathcal{C}'_{\substack{e\gamma\\e\mu}} \\ \mathcal{C}'_{\substack{e\gamma\\\mu e}} & \mathcal{C}'_{\substack{e\gamma\\\mu\mu}} \\ \mathcal{C}'_{\substack{e\gamma\\\tau e}} & \mathcal{C}'_{\substack{e\gamma\\\tau\mu}} \end{pmatrix}$$

Lepton flavor violation

$\mu \to e\gamma$, $\mu \to 3e$, $\mu N \to eN$

$$\mathcal{B}(\ell_r \to \ell_s\gamma) = \frac{m_{\ell_r}^3 v^2}{8\pi\Gamma_{\ell_r}} \frac{1}{\Lambda^4} \left(\left|\mathcal{C}'_{\substack{e\gamma\\rs}}\right|^2 + \left|\mathcal{C}'_{\substack{e\gamma\\sr}}\right|^2 \right)$$

$$\mathcal{B}(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13} \ (90\% \ \text{C.L.})$$

MEG

$$\longrightarrow \quad \frac{1}{\Lambda^2}\left|\mathcal{C}'_{\substack{e\gamma\\e\mu(\mu e)}}\right| < 2.1 \times 10^{-10}\,\text{TeV}^{-2}$$

muon $(g-2)_\mu$, EDM d_μ

$$\Delta a_\mu = \frac{4m_\mu}{e} \frac{v}{\sqrt{2}} \frac{1}{\Lambda^2} \text{Re}\left[\mathcal{C}'_{\substack{e\gamma\\\mu\mu}}\right]$$

$$\Delta a_\mu = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$$

FNAL, BNL

$$\longrightarrow \quad \frac{1}{\Lambda^2}\text{Re}\left[\mathcal{C}'_{\substack{e\gamma\\\mu\mu}}\right] \approx 1.0 \times 10^{-5}\,\text{TeV}^{-2}$$

Strong flavor alignment

$$\left| \frac{\mathcal{C}'_{\substack{e\gamma\\e\mu(\mu e)}}}{\mathcal{C}'_{\substack{e\gamma\\\mu\mu}}} \right| < 2.1 \times 10^{-5}$$

$$\mathcal{C}'_{\substack{e\gamma\\LR}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma\\ee}} & \mathcal{C}'_{\substack{e\gamma\\e\mu}} & \mathcal{C}'_{\substack{e\gamma\\e\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\mu e}} & \mathcal{C}'_{\substack{e\gamma\\\mu\mu}} & \mathcal{C}'_{\substack{e\gamma\\\mu\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\tau e}} & \mathcal{C}'_{\substack{e\gamma\\\tau\mu}} & \mathcal{C}'_{\substack{e\gamma\\\tau\tau}} \end{pmatrix}$$

Lepton flavor structure from $(g-2)_\ell$, LFV and EDM

$$\mathcal{B}(\ell_r \to \ell_s \gamma) = \frac{m_{\ell_r}^3 v^2}{8\pi\Gamma_{\ell_r}} \frac{1}{\Lambda^4} \left(|\mathcal{C}'_{\underset{rs}{e\gamma}}|^2 + |\mathcal{C}'_{\underset{sr}{e\gamma}}|^2 \right)$$

$$\mathcal{B}(\tau^\pm \to \mu^\pm \gamma) < 4.4 \times 10^{-8} \;\; (90\% \;\, \mathrm{CL})$$

BaBar

$$\frac{1}{\Lambda^2} |\mathcal{C}'_{\underset{\mu\tau(\tau\mu)}{e\gamma}}| < 2.7 \times 10^{-6} \, \mathrm{TeV}^{-2}$$

electron $(g-2)_e$, EDM d_e

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$\tau \to \mu\gamma, \tau \to 3\mu$

tau $(g-2)_\tau$, EDM d_τ

muon $(g-2)_\mu$, EDM d_μ

$$\Delta a_\mu = \frac{4m_\mu}{e} \frac{v}{\sqrt{2}} \frac{1}{\Lambda^2} \mathrm{Re} \left[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}} \right]$$

$$\Delta a_\mu = a_\mu^{\mathrm{Exp}} - a_\mu^{\mathrm{SM}} = (251 \pm 59) \times 10^{-11}$$

FNAL, BNL

$$\frac{1}{\Lambda^2} \mathrm{Re} \left[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}} \right] \approx 1.0 \times 10^{-5} \, \mathrm{TeV}^{-2}$$

$$\left| \frac{\mathcal{C}'_{\underset{\mu\tau(\tau\mu)}{e\gamma}}}{\mathcal{C}'_{\underset{\tau\tau}{e\gamma}}} \right| < 1.6 \times 10^{-2} \times \left| \frac{y_\tau \, \mathcal{C}'_{\underset{\mu\mu}{e\gamma}}}{y_\mu \, \mathcal{C}'_{\underset{\tau\tau}{e\gamma}}} \right|$$

with natural expectation $|\mathcal{C}'_{\underset{\tau\tau}{e\gamma}}|/y_\tau \sim |\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}|/y_\mu$

Isidori, Pages and Wilsch[2111.13724]

$$\Delta a_e = \frac{4m_e}{e}\frac{v}{\sqrt{2}}\frac{1}{\Lambda^2}\mathrm{Re}\,[\mathcal{C}'_{\underset{ee}{e\gamma}}]$$

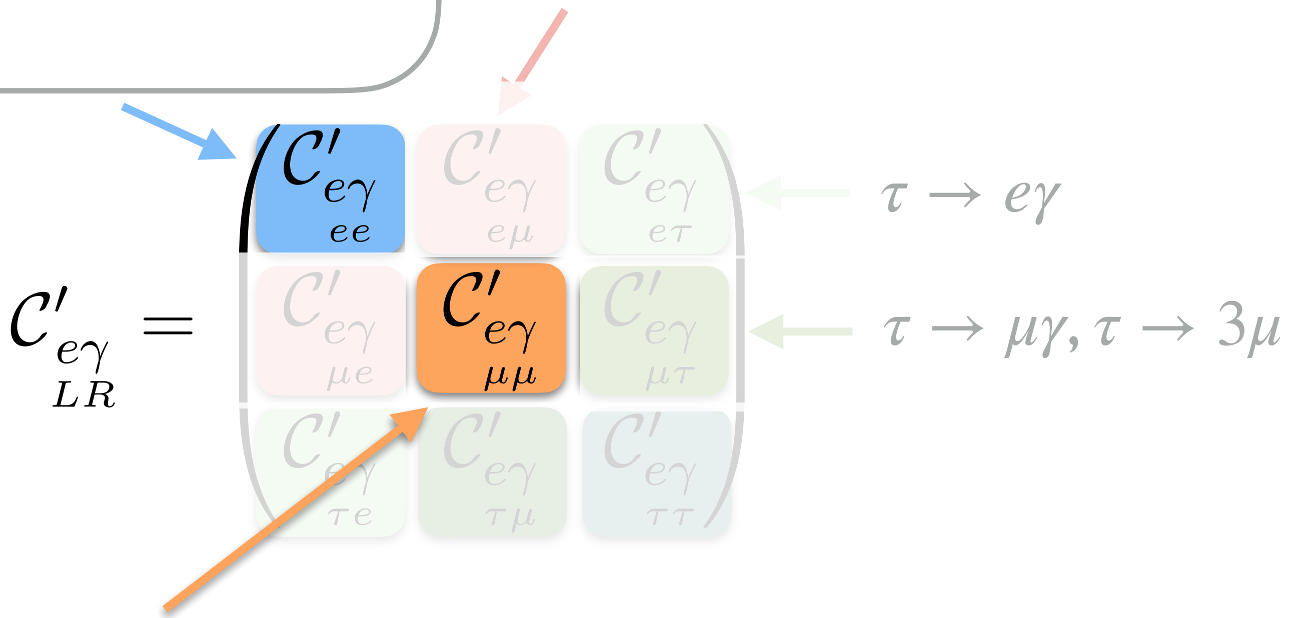
$$\Delta a_e^{Cs} = a_e^{\mathrm{Exp}} - a_e^{\mathrm{SM,CS}} = (-8.8 \pm 3.6) \times 10^{-13}$$
$$\Delta a_e^{Rb} = a_e^{\mathrm{Exp}} - a_e^{\mathrm{SM,Rb}} = (4.8 \pm 3.0) \times 10^{-13} \; .$$

electron $(g-2)_e$

Two determination of the fine structure constant

11 vs 22

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$



$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$\tau \to e\gamma$

$\tau \to \mu\gamma, \tau \to 3\mu$

muon $(g-2)_\mu$

$$\Delta a_\mu = \frac{4m_\mu}{e}\frac{v}{\sqrt{2}}\frac{1}{\Lambda^2}\mathrm{Re}\,[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}]$$

$$\Delta a_\mu = a_\mu^{\mathrm{Exp}} - a_\mu^{\mathrm{SM}} = (251 \pm 59) \times 10^{-11}$$

FNAL, BNL

$$\longrightarrow \quad \frac{1}{\Lambda^2}\mathrm{Re}\,[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}] \approx 1.0 \times 10^{-5}\ \mathrm{TeV}^{-2}$$

$$\Delta a_e = \frac{4m_e}{e}\frac{v}{\sqrt{2}}\frac{1}{\Lambda^2}\,\mathrm{Re}\,[\mathcal{C}'_{e\gamma}]_{ee}$$

$$\Delta a_e^{Cs} = a_e^{\rm Exp} - a_e^{\rm SM,CS} = (-8.8 \pm 3.6)\times 10^{-13}$$
$$\Delta a_e^{Rb} = a_e^{\rm Exp} - a_e^{\rm SM,Rb} = (4.8 \pm 3.0)\times 10^{-13} \quad .$$

electron $(g-2)_e$

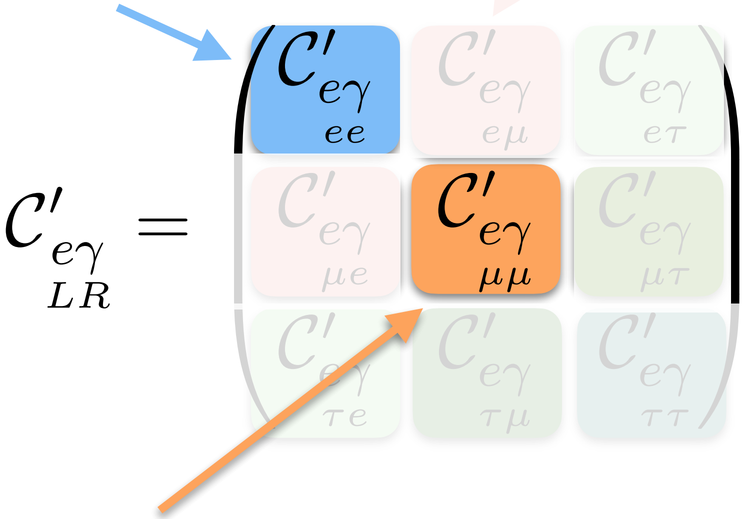
$$d_e = -\sqrt{2}\,\frac{v}{\Lambda^2}\,\mathrm{Im}\,[\mathcal{C}'_{e\gamma}]_{ee}$$

$$|d_e/e| \lesssim 1.1\times 10^{-29}\,\mathrm{cm} = 5.6\times 10^{-13}\,\mathrm{TeV}^{-1}$$

ACME

$$\frac{1}{\Lambda^2}\,\mathrm{Im}\,[\mathcal{C}'_{e\gamma}]_{ee} < 1.6\times 10^{-12}\,\mathrm{TeV}^{-2}$$

EDM d_e



$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} [\mathcal{C}'_{e\gamma}]_{ee} & [\mathcal{C}'_{e\gamma}]_{e\mu} & [\mathcal{C}'_{e\gamma}]_{e\tau} \\ [\mathcal{C}'_{e\gamma}]_{\mu e} & [\mathcal{C}'_{e\gamma}]_{\mu\mu} & [\mathcal{C}'_{e\gamma}]_{\mu\tau} \\ [\mathcal{C}'_{e\gamma}]_{\tau e} & [\mathcal{C}'_{e\gamma}]_{\tau\mu} & [\mathcal{C}'_{e\gamma}]_{\tau\tau} \end{pmatrix}$$

$$\begin{tikzcd} & (g-2)_\mu \\ {[\mathcal{C}'_{e\gamma}]_{\mu\mu}} \arrow[ur, "\mathrm{Re}"] \arrow[dr, "\mathrm{Im}"'] & \\ & \text{EDM } d_\mu \end{tikzcd}$$

muon $(g-2)_\mu$

$$\Delta a_\mu = \frac{4m_\mu}{e}\frac{v}{\sqrt{2}}\frac{1}{\Lambda^2}\,\mathrm{Re}\,[\mathcal{C}'_{e\gamma}]_{\mu\mu}$$

$$\Delta a_\mu = a_\mu^{\rm Exp} - a_\mu^{\rm SM} = (251 \pm 59)\times 10^{-11}$$

FNAL, BNL

$$\longrightarrow \quad \frac{1}{\Lambda^2}\,\mathrm{Re}\,[\mathcal{C}'_{e\gamma}]_{\mu\mu} \approx 1.0\times 10^{-5}\,\mathrm{TeV}^{-2}$$

EDM d_μ

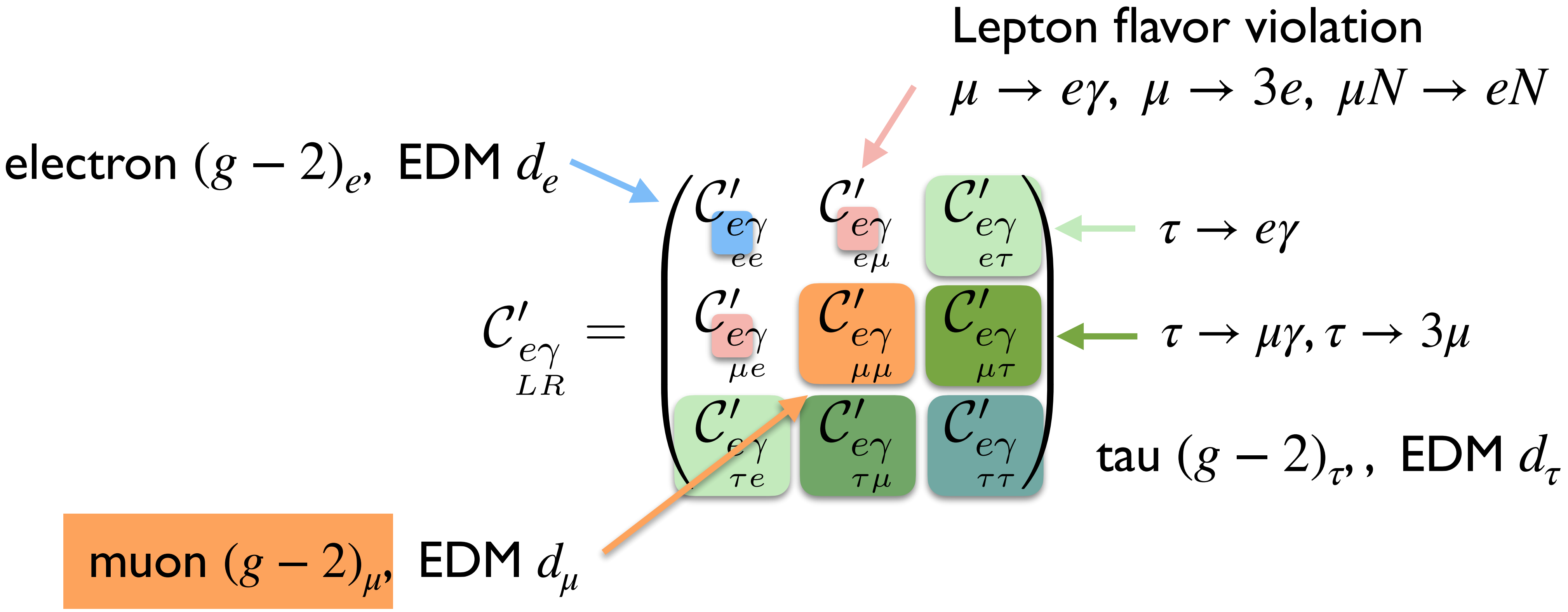
$$d_\mu = -\sqrt{2}\,\frac{v}{\Lambda^2}\,\mathrm{Im}\,[\mathcal{C}'_{e\gamma}]_{\mu\mu}$$

$$|d_\mu/e| < 1.8\times 10^{-19}\,\mathrm{cm}$$

BNL

$$\longrightarrow \quad \frac{1}{\Lambda^2}\,\mathrm{Im}\,[\mathcal{C}'_{e\gamma}]_{\mu\mu} < 2.7\times 10^{-2}\,\mathrm{TeV}^{-2}$$

Lepton flavor structure from $(g-2)_\ell$, LFV and EDM



Lepton flavor violation

$\mu \to e\gamma,\ \mu \to 3e,\ \mu N \to eN$

electron $(g-2)_e$, EDM d_e

$$\mathcal{C}'_{\substack{e\gamma\\LR}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma\\ee}} & \mathcal{C}'_{\substack{e\gamma\\e\mu}} & \mathcal{C}'_{\substack{e\gamma\\e\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\mu e}} & \mathcal{C}'_{\substack{e\gamma\\\mu\mu}} & \mathcal{C}'_{\substack{e\gamma\\\mu\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\\tau e}} & \mathcal{C}'_{\substack{e\gamma\\\tau\mu}} & \mathcal{C}'_{\substack{e\gamma\\\tau\tau}} \end{pmatrix}$$

$\tau \to e\gamma$

$\tau \to \mu\gamma, \tau \to 3\mu$

tau $(g-2)_\tau$, , EDM d_τ

muon $(g-2)_\mu$, EDM d_μ

NP (realize muon $(g-2)_\mu$ anomaly

satisfy constraint from LFV) need strong flavor alignment

→ might be controlled by flavor symmetry

Outline

Introduction of muon g-2

New physics interpretation of muon g-2

Flavor symmetry and muon g-2

Summary

Flavor symmetry

Flavor physics play a role of

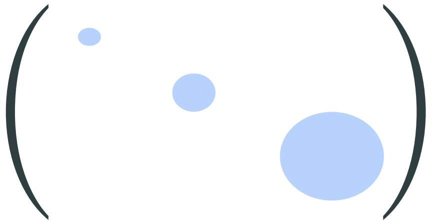
identify origin of flavor puzzle

probing NP

The SM flavor problem

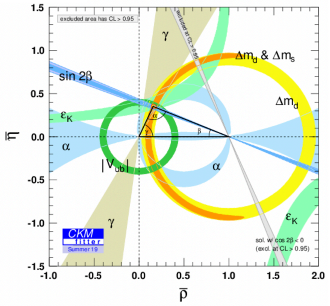
The origin of flavor
3 generations
hierarchical structure

$M_{u,d,e} \sim$



NP flavor problem

No significant NP signal
→ NP have highly non-generic flavor structure



see next

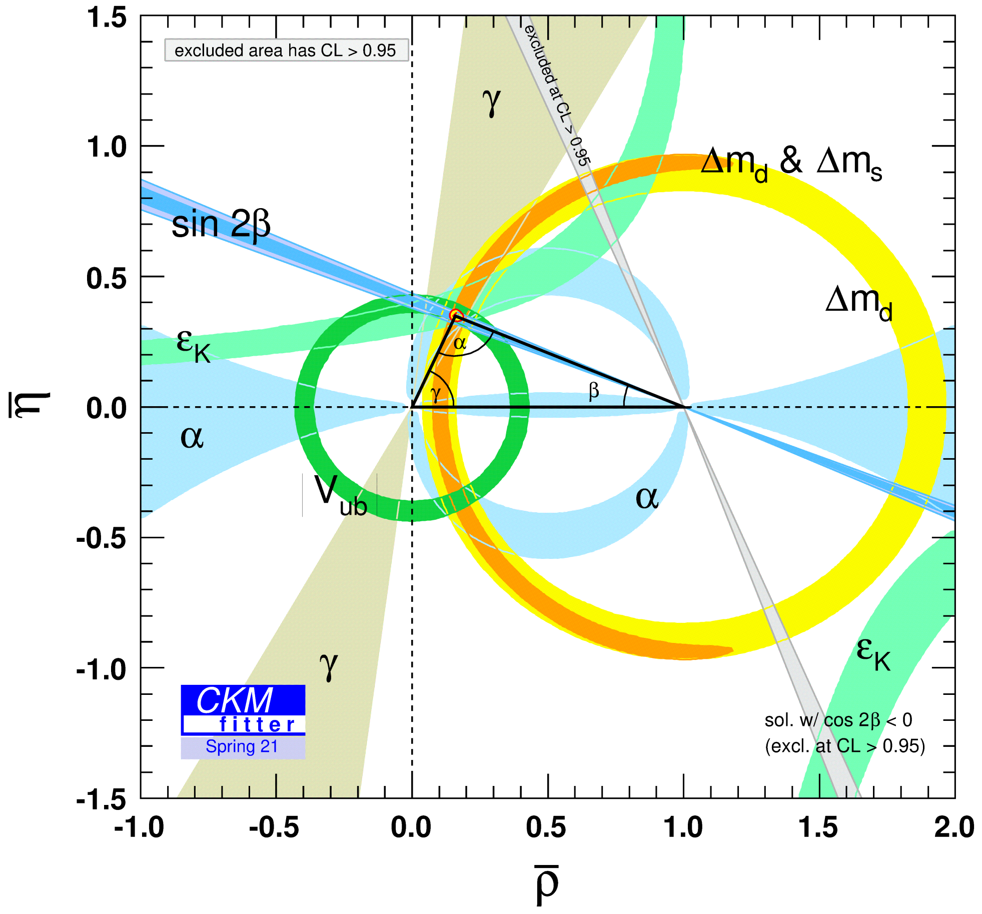
Flavor symmetry

Flavor symmetry would play an important role both in the SM and NP
Connect SM mistery

→ flavor symmetry

NP Flavor Problem

- Theoretical arguments based on the hierarchy problem → TeV scale NP



- The measurements of quark flavor-violating observables show a remarkable overall success of the SM

- New flavor-breaking sources of O(1) at the TeV scale are definitely excluded

$$\mathscr{L}_{eff} = \mathscr{L}_{SM} + \sum_i \frac{C_i}{\Lambda^2} \mathscr{O}_i^{d=6} \qquad \text{(NP)}$$

$$|C_{NP}| \sim 1 \longrightarrow \Lambda_{NP} \sim \begin{cases} 500\ \text{TeV} & : \quad B_s \\ 2000\ \text{TeV} & : \quad B_d \\ 10^4 - 10^5\ \text{TeV} & : \quad K^0 \end{cases}$$

- if we insist with the theoretical prejudice that NP has to emerge in the TeV region, we have to conclude that NP have a highly non-generic flavor structure

→ Flavor symmetry

From MFV to $U(2)^5$

$U(3)^5 = U(3)_{Q_L} \times U(3)_{u_R} \times U(3)_{d_R} \times U(3)_{L_L} \times U(3)_{e_R}$ flavor symmetry

- Largest flavor symmetry group compatible with the SM gauge symmetry
- MFV = minimal breaking of $U(3)^5$ by SM Yukawa couplings

MFV virtue

Naturally small effects in FCNC observables assuming TeV-scale NP

MFV main problem

No explanation for Yukawa hierarchies (masses and mixing angles)

$U(2)^5 = U(2)_{Q_L} \times U(2)_{u_R} \times U(2)_{d_R} \times U(2)_{L_L} \times U(2)_{e_R}$ flavor symmetry

- acting on 1st & 2nd generations only
- The exact symmetry limit is good starting point for the SM quark spectrum $(m_u, m_d, m_c, m_s = 0$ & $V_{CKM} = 1)$ $\Rightarrow$ we only need small breaking terms

$U(2)^5$ flavor symmetry

Barbieri, Isidori, Jones-Perez, Lodone, Straub [1105.2296]

- $U(2)^5$ symmetry gives "natural" explanation of why 3rd Yukawa couplings are large

$$\psi = (\underbrace{\psi_1, \psi_2}_{\text{SU(2) doublet}}, \underbrace{\psi_3}_{\text{singlet}})$$

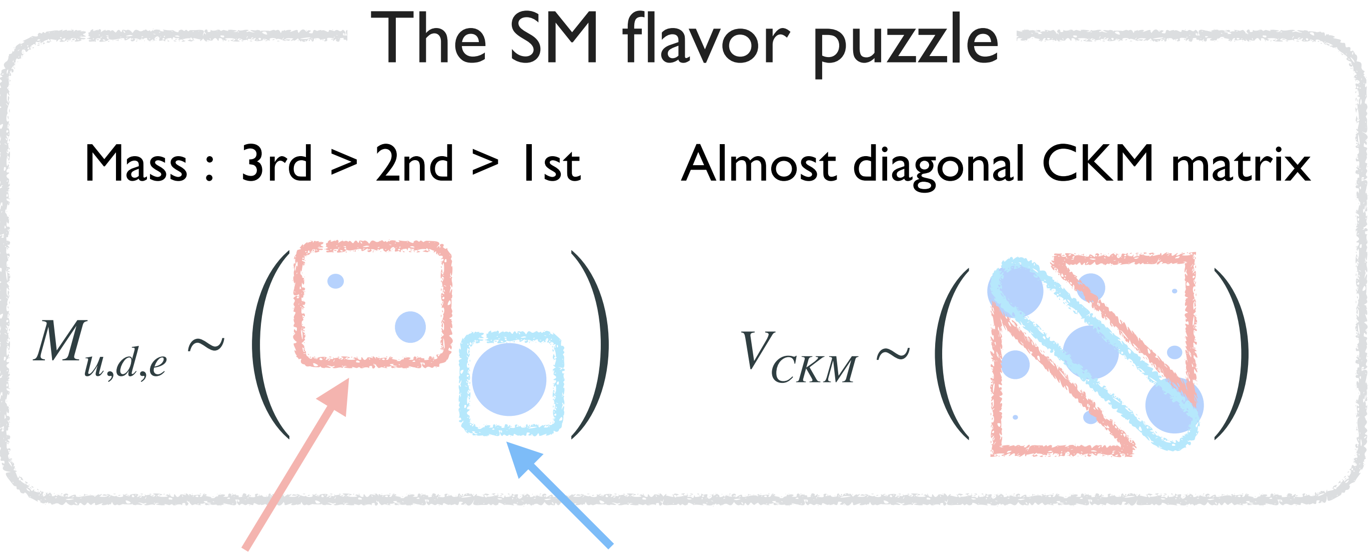
acting on 1st & 2nd generations only

3rd Yukawa coupling is allowed by the symmetry

- The symmetry is good approximation in the SM Yukawa

exact symmetry for $m_u, m_d, m_c, m_s = 0$ & $V_{CKM} = 1$

$\Rightarrow$ we only need small breaking terms



The SM flavor puzzle

Mass : 3rd > 2nd > 1st

Almost diagonal CKM matrix

$M_{u,d,e} \sim$

$V_{CKM} \sim$

U(2) breaking terms

allowed by U(2)

- Naturally small effects in FCNC observables assuming TeV-scale NP

$U(2)^5$ flavor symmetry

Barbieri, Isidori, Jones-Perez, Lodone, Straub [1105.2296]

Under $U(2)^3 = U(2)_Q \times U(2)_{u_R} \times U(2)_{d_R}$ symmetry

$$\psi = (\underbrace{\psi_1, \psi_2}_{\text{SU(2) doublet}}, \underbrace{\psi_3}_{\text{singlet}})$$

$$\mathcal{L}_{\text{Yuk}} = \begin{pmatrix} \bar{Q}^{(2)} & \bar{q}^3 \end{pmatrix} Y_u \begin{pmatrix} u^{(2)} \\ t \end{pmatrix} + \begin{pmatrix} \bar{Q}^{(2)} & \bar{q}^3 \end{pmatrix} Y_d \begin{pmatrix} d^{(2)} \\ b \end{pmatrix}$$

$Q^{(2)} = (Q^1, Q^2) \sim (2, 1, 1)$ $\quad Q^3 \sim (1, 1, 1)$

$u^{(2)} = (u^1, u^2) \sim (1, 2, 1)$ $\quad t \sim (1, 1, 1)$

$d^{(2)} = (d^1, d^2) \sim (1, 1, 2)$ $\quad b \sim (1, 1, 1)$

Unbroken symmetry

$$Y_d = y_b \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$U(2)_q$

$U(2)_d$

$U(2)^5$ flavor symmetry

Barbieri, Isidori, Jones-Perez,
Lodone, Straub [1105.2296]

Under $U(2)^3 = U(2)_Q \times U(2)_{u_R} \times U(2)_{d_R}$ symmetry

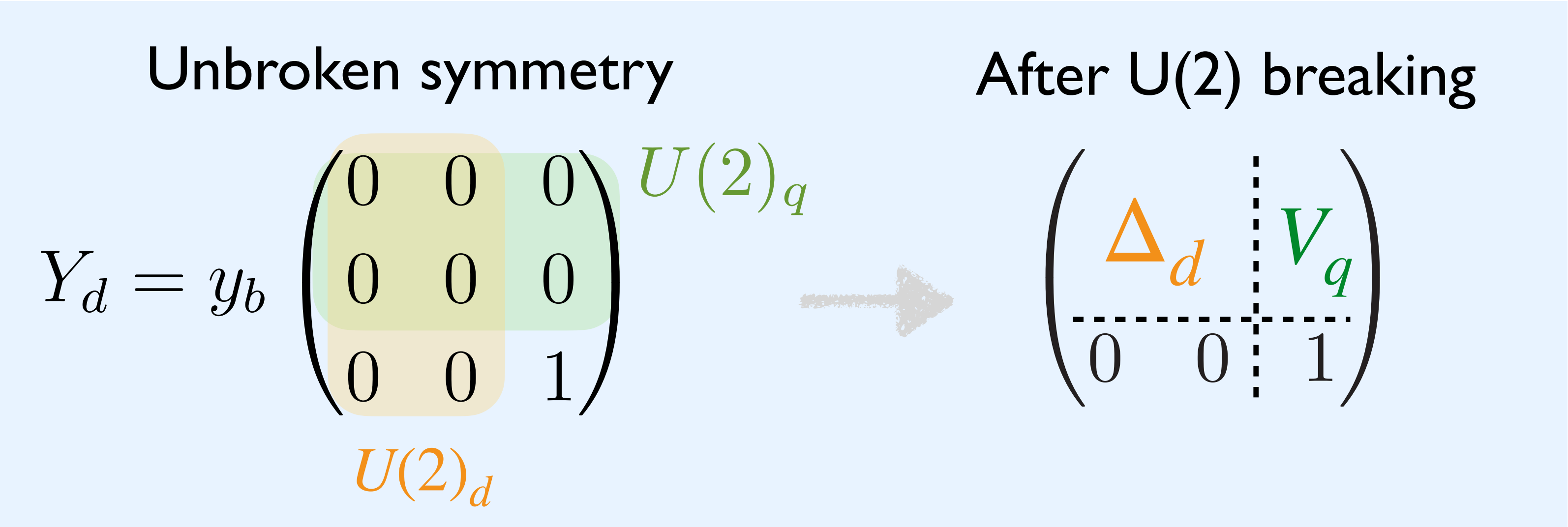
$$\psi = (\underbrace{\psi_1, \psi_2}_{\text{SU(2) doublet}}, \underbrace{\psi_3}_{\text{singlet}})$$

$$\mathcal{L}_{\text{Yuk}} = \begin{pmatrix} \bar{Q}^{(2)} & \bar{q}^3 \end{pmatrix} Y_u \begin{pmatrix} u^{(2)} \\ t \end{pmatrix} + \begin{pmatrix} \bar{Q}^{(2)} & \bar{q}^3 \end{pmatrix} Y_d \begin{pmatrix} d^{(2)} \\ b \end{pmatrix}$$

$Q^{(2)} = (Q^1, Q^2) \sim (2,1,1)$ $\quad Q^3 \sim (1,1,1)$

$u^{(2)} = (u^1, u^2) \sim (1,2,1)$ $\quad t \sim (1,1,1)$

$d^{(2)} = (d^1, d^2) \sim (1,1,2)$ $\quad b \sim (1,1,1)$



Unbroken symmetry

$$Y_d = y_b \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

After U(2) breaking

$$\left(\begin{array}{c|c} \Delta_d & V_q \\ \hline 0 \quad 0 & 1 \end{array}\right)$$

U(2) breaking (Spurion)

$V_q \sim (2,1,1),$

$\Delta_d \sim (2,1,\bar{2}),$

$\Delta_u \sim (2,\bar{2},1)$

$U(2)^5$ flavor symmetry

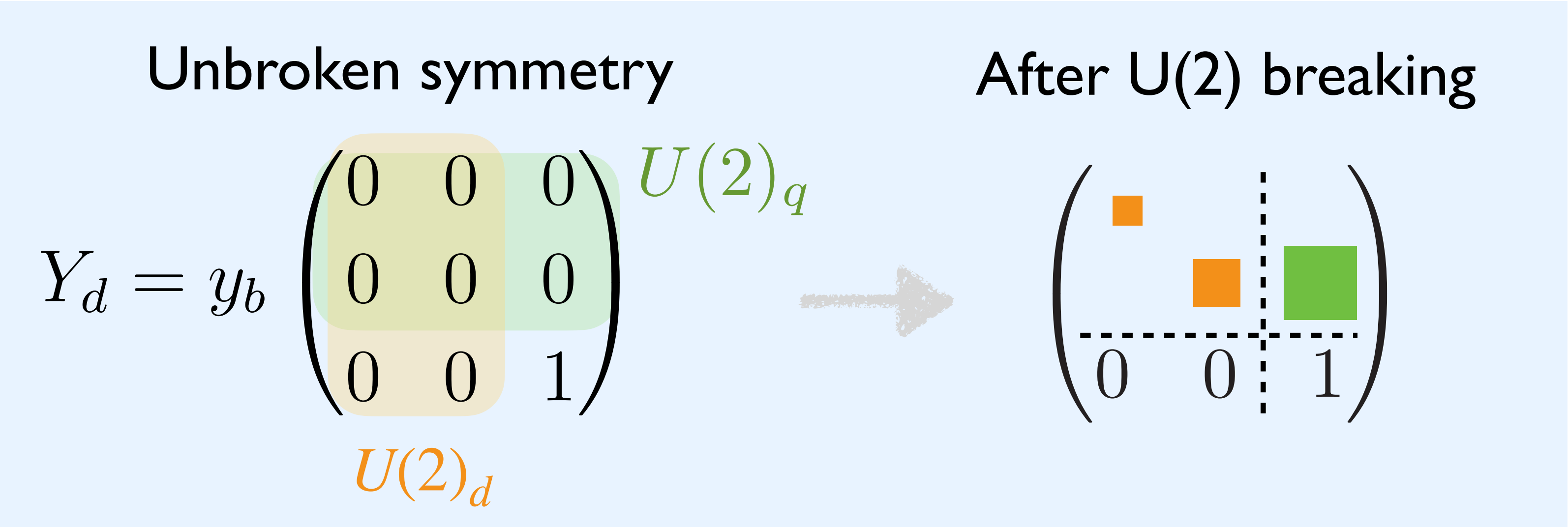
Barbieri, Isidori, Jones-Perez, Lodone, Straub [1105.2296]

Under $U(2)^3 = U(2)_Q \times U(2)_{u_R} \times U(2)_{d_R}$ symmetry

$$\psi = (\underbrace{\psi_1, \psi_2}_{\text{SU(2) doublet}}, \underbrace{\psi_3}_{\text{singlet}})$$

$$\mathcal{L}_{\text{Yuk}} = \begin{pmatrix} \bar{Q}^{(2)} & \bar{q}^3 \end{pmatrix} Y_u \begin{pmatrix} u^{(2)} \\ t \end{pmatrix} + \begin{pmatrix} \bar{Q}^{(2)} & \bar{q}^3 \end{pmatrix} Y_d \begin{pmatrix} d^{(2)} \\ b \end{pmatrix}$$

$$\begin{aligned} Q^{(2)} &= (Q^1, Q^2) \sim (2,1,1) & Q^3 &\sim (1,1,1) \\ u^{(2)} &= (u^1, u^2) \sim (1,2,1) & t &\sim (1,1,1) \\ d^{(2)} &= (d^1, d^2) \sim (1,1,2) & b &\sim (1,1,1) \end{aligned}$$



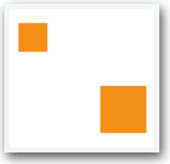
Unbroken symmetry

$$Y_d = y_b \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

After U(2) breaking

U(2) breaking (Spurion)

🟩 : $|V_q| \sim |V_{ts}| \sim \mathcal{O}(10^{-1})$



: $|\Delta_d| \sim \begin{pmatrix} y_d/y_b & \\ & y_s/y_b \end{pmatrix} \sim \begin{pmatrix} \mathcal{O}(10^{-3}) & \\ & \mathcal{O}(10^{-2}) \end{pmatrix}$

spurion order : $1 \gg$ 🟩 $\gg$ 🟧 $\gg$ 🔸 > 0

$\mathcal{O}(10^{-1})$ $\mathcal{O}(10^{-2})$ $\mathcal{O}(10^{-3})$

NP with $U(2)^5$

- The small breaking ensures small effects in rare processes
- Flavor symmetries are not necessarily fundamental symmetries of UV theory, but this effective approach is useful way for systematic NP analysis

e.g. Classification SMEFT operators under U(3) and U(2) A. Faroughy, Isidori, Wilsch, KY [2005.05366]

2499 in SMEFT $\xrightarrow[\text{flavor symmetry}]{}$ reduce number of independent parameters

$U(3)^5$ and MFV	drastic reduction : ~ 25 times smaller
$U(2)^5$	reduction : ~ one order smaller

- Interesting implication for phono.

e.g. B-anomalies are compatible with U(2) flavor symmetry Fuentes-Martin, Isidori, Pages, KY [1909.02519] etc.

what about effects on lepton sector, Muon $(g-2)_\mu$? *Isidori, Pages and Wilsch 2111.13724*

we also discuss EDM, LFV, electron $(g-2)_e$ *Tanimoto, KY 2310.16325*

Lepton flavor structure of LR operator in U(2)

$$X^n_{\alpha\beta}(\bar{\ell}_\alpha \Gamma e_\beta)\eta^n \qquad (n = Y, e\gamma)$$

(flavor structure) × ($\mathcal{O}(1)$ NP coefficients)

$U(2)_{L_L} \otimes U(2)_{E_R}$ breaking (Spurion)

$V_\ell \sim (2,1)$, $\Delta_e \sim (2,\bar{2})$

× $C^n, C^n_V, C^n_\Delta, C^n_{V\Delta}, C^n_{VV\Delta}$

$$(\bar{\ell}^{(2)}, \bar{\ell}^3)\left(\begin{array}{c:c} C^n_\Delta(\Delta_e)_{\alpha\beta}+C^n_{VV\Delta}(V_\ell)_\alpha(V_\ell^\dagger)_\gamma(\Delta_e)_{\gamma\beta} & C^n_V(V_\ell)_\alpha \\ \hdashline C^n_{V\Delta}(V_\ell^\dagger)_\alpha(\Delta_e)_{\alpha\beta} & C^n \end{array}\right)_{LR}\begin{pmatrix} e^{(2)} \\ e^3 \end{pmatrix}$$

parametrization of spurions

$$V_\ell = \begin{pmatrix} 0 \\ \epsilon_\ell \end{pmatrix}, \qquad \Delta_e = O_e^T\begin{pmatrix} \delta'_e & 0 \\ 0 & \delta_e \end{pmatrix}, \qquad O_e = \begin{pmatrix} c_e & s_e \\ -s_e & c_e \end{pmatrix}$$

$$\left(\begin{array}{cc:c} C^n_\Delta c_e \delta'_e & -C^n_\Delta s_e \delta_e & 0 \\ s_e\delta'_e(C^n_\Delta + C^n_{VV\Delta}\epsilon_\ell^2) & c_e\delta_e(C^n_\Delta + C^n_{VV\Delta}\epsilon_\ell^2) & C^n_V\epsilon_\ell \\ \hdashline C^n_{V\Delta}(s_e\epsilon_\ell\delta'_e) & C^n_{V\Delta}(c_e\epsilon_\ell\delta_e) & C^n \end{array}\right)_{LR}$$

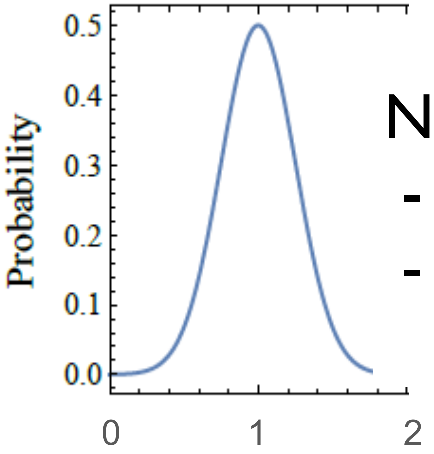
Numerical study

NP coefficients

magnitudes are $\mathcal{O}(1)$, phases are random

Yukawa $C^Y, C_V^Y, C_\Delta^Y, C_{V\Delta}^Y, C_{VV\Delta}^Y$

dipole $C^{e\gamma}, C_V^{e\gamma}, C_\Delta^{e\gamma}, C_{V\Delta}^{e\gamma}, C_{VV\Delta}^{e\gamma}$



Normal distribution
- Average 1
- Standard deviation 0.25

U(2) parameters

$$\frac{\delta_e'}{\delta_e} \simeq \frac{y_e}{y_\mu} \qquad \delta_e = (5.0 - 6.0) \times 10^{-2}\,, \qquad \delta_e' = (2.3 - 3.0) \times 10^{-4}$$

ϵ_ℓ and s_e are not constrained, but presume from quark sector

$$s_e = 0.01 - 0.1\,, \qquad \epsilon_\ell = 0.01 - 0.1$$

Calculation

take parameter regions which can realize $(g-2)_\mu$ anomaly

$(g-2)_\mu$ & $\mu \to e\gamma$ in U(2)

$$\underset{LR}{\mathcal{C}'_{e\gamma}} = \begin{pmatrix} \underset{ee}{\mathcal{C}'_{e\gamma}} & \underset{e\mu}{\mathcal{C}'_{e\gamma}} & \underset{e\tau}{\mathcal{C}'_{e\gamma}} \\ \underset{\mu e}{\mathcal{C}'_{e\gamma}} & \underset{\mu\mu}{\mathcal{C}'_{e\gamma}} & \underset{\mu\tau}{\mathcal{C}'_{e\gamma}} \\ \underset{\tau e}{\mathcal{C}'_{e\gamma}} & \underset{\tau\mu}{\mathcal{C}'_{e\gamma}} & \underset{\tau\tau}{\mathcal{C}'_{e\gamma}} \end{pmatrix}$$

$$\underset{\mu\mu}{\mathcal{C}'_{e\gamma}} \simeq |C_\Delta^{e\gamma}|\delta_e \left[\frac{1}{c_e} + c_e\epsilon_\ell^2 \left(\left|\frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}}\right| \cos\left(\arg C_{VV\Delta}^{e\gamma}\right) - \left|\frac{C_{VV\Delta}^{y}}{C_\Delta^{y}}\right| \frac{s_e^2}{c_e^2} \cos\left(\arg C_{VV\Delta}^{y}\right) \right)\right]$$
$$+ i|C_\Delta^{e\gamma}|c_e\delta_e\epsilon_\ell^2 \left(\left|\frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}}\right| \sin\left(\arg C_{VV\Delta}^{e\gamma}\right) - \left|\frac{C_{VV\Delta}^{y}}{C_\Delta^{y}}\right| \sin\left(\arg C_{VV\Delta}^{y}\right) \right)$$

$$\underset{e\mu}{\mathcal{C}'_{e\gamma}} \simeq |C_\Delta^{e\gamma}| s_e\delta_e\epsilon_\ell^2 \left(\frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} - \frac{C_{VV\Delta}^{y}}{C_\Delta^{y}} \right)$$

muon $(g-2)_\mu$ and $\mu \to e\gamma$

strong flavor alignment

$$\left|\frac{\underset{e\mu}{\mathcal{C}'_{e\gamma}}}{\underset{\mu\mu}{\mathcal{C}'_{e\gamma}}}\right| \approx \frac{s_e}{c_e}\epsilon_\ell^2 \left| \frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} - \frac{C_{VV\Delta}^{y}}{C_\Delta^{y}} \right| < 2.1 \times 10^{-5}$$

$s_e = \mathcal{O}(\sqrt{m_e/m_\mu}) \sim 10^{-1}$

$\epsilon_\ell \sim 10^{-1}$

→10^{-3} suppression by U(2) spurions

from $\mu \to e\gamma$

$$\frac{1}{\Lambda^2}|\underset{e\mu(\mu e)}{\mathcal{C}'_{e\gamma}}| < 2.1 \times 10^{-10}\,\mathrm{TeV}^{-2}$$

from muon $(g-2)_\mu$

$$\frac{1}{\Lambda^2}\mathrm{Re}\,[\underset{\mu\mu}{\mathcal{C}'_{e\gamma}}] \approx 1.0 \times 10^{-5}\,\mathrm{TeV}^{-2}$$

$$\longrightarrow \boxed{\left| \frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} - \frac{C_{VV\Delta}^{y}}{C_\Delta^{y}} \right| \lesssim 10^{-2}}$$

tight alignment condition

Isidori, Pages and Wilsch [2111.13724]

$(g-2)_\mu$ & $\mu \to e\gamma$ in U(2)

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$$\begin{aligned}\mathcal{C}'_{\underset{\mu\mu}{e\gamma}} &\simeq |C^{e\gamma}_\Delta|\delta_e\left[\frac{1}{c_e} + c_e\epsilon_\ell^2\left(\left|\frac{C^{e\gamma}_{VV\Delta}}{C^{e\gamma}_\Delta}\right|\cos\left(\arg C^{e\gamma}_{VV\Delta}\right) - \left|\frac{C^{y}_{VV\Delta}}{C^{y}_\Delta}\right|\frac{s_e^2}{c_e^2}\cos\left(\arg C^{y}_{VV\Delta}\right)\right)\right] \\ &+ i|C^{e\gamma}_\Delta|c_e\delta_e\epsilon_\ell^2\left(\left|\frac{C^{e\gamma}_{VV\Delta}}{C^{e\gamma}_\Delta}\right|\sin\left(\arg C^{e\gamma}_{VV\Delta}\right) - \left|\frac{C^{y}_{VV\Delta}}{C^{y}_\Delta}\right|\sin\left(\arg C^{y}_{VV\Delta}\right)\right) + c_e\delta_e\epsilon_\ell^2 C_{3rd}\end{aligned}$$

$$\mathcal{C}'_{\underset{e\mu}{e\gamma}} \simeq |C^{e\gamma}_\Delta| s_e\delta_e\epsilon_\ell^2\left(\frac{C^{e\gamma}_{VV\Delta}}{C^{e\gamma}_\Delta} - \frac{C^{y}_{VV\Delta}}{C^{y}_\Delta}\right) + s_e\delta_e\epsilon_\ell^2 C_{3rd}$$

3rd generation effects

$$C_{3rd} = \frac{C^{y*}_\Delta C^y_V + C^{y*}_{V\Delta}C^y}{|C^y|^2}\left(\frac{C^{e\gamma}}{C^y}C^y_V - C^{e\gamma}_V\right) + \frac{C^y_V}{C^y}\left(\frac{C^{e\gamma}_V}{C^y_V}C^y_{V\Delta} - C^{e\gamma}_{V\Delta}\right)$$

muon $(g-2)_\mu$ and $\mu \to e\gamma$

strong flavor alignment

$$\left|\frac{\mathcal{C}'_{\underset{e\mu}{e\gamma}}}{\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}}\right| \approx \frac{s_e}{c_e}\epsilon_\ell^2\left|\frac{C^{e\gamma}_{VV\Delta}}{C^{e\gamma}_\Delta} - \frac{C^{y}_{VV\Delta}}{C^{y}_\Delta} + \frac{C_{3rd}}{C^{e\gamma}_\Delta}\right| < 2.1\times10^{-5}$$

from $\mu \to e\gamma$

$$\frac{1}{\Lambda^2}|\mathcal{C}'_{\underset{e\mu(\mu e)}{e\gamma}}| < 2.1\times10^{-10}\,\mathrm{TeV}^{-2}$$

from muon $(g-2)_\mu$

$$\frac{1}{\Lambda^2}\mathrm{Re}\left[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}\right] \approx 1.0\times10^{-5}\,\mathrm{TeV}^{-2}$$

$s_e = \mathcal{O}(\sqrt{m_e/m_\mu}) \sim 10^{-1}$ $\epsilon_\ell \sim 10^{-1}$

→ 10^{-3} suppression by U(2) spurions

$$\longrightarrow \quad \left|\frac{C^{e\gamma}_{VV\Delta}}{C^{e\gamma}_\Delta} - \frac{C^{y}_{VV\Delta}}{C^{y}_\Delta} + \frac{C_{3rd}}{C^{e\gamma}_\Delta}\right| \lesssim 10^{-2}$$

Due to C_{3rd} effect, we still need alignment condition even if $\frac{C^{e\gamma}_{VV\Delta}}{C^{e\gamma}_\Delta} = \frac{C^{y}_{VV\Delta}}{C^{y}_\Delta}$

$(g-2)_\mu$ & $\mu \to e\gamma$ in U(2)

$$\underset{LR}{\mathcal{C}'_{e\gamma}} = \begin{pmatrix} \underset{ee}{\mathcal{C}'_{e\gamma}} & \underset{e\mu}{\mathcal{C}'_{e\gamma}} & \underset{e\tau}{\mathcal{C}'_{e\gamma}} \\ \underset{\mu e}{\mathcal{C}'_{e\gamma}} & \underset{\mu\mu}{\mathcal{C}'_{e\gamma}} & \underset{\mu\tau}{\mathcal{C}'_{e\gamma}} \\ \underset{\tau e}{\mathcal{C}'_{e\gamma}} & \underset{\tau\mu}{\mathcal{C}'_{e\gamma}} & \underset{\tau\tau}{\mathcal{C}'_{e\gamma}} \end{pmatrix}$$

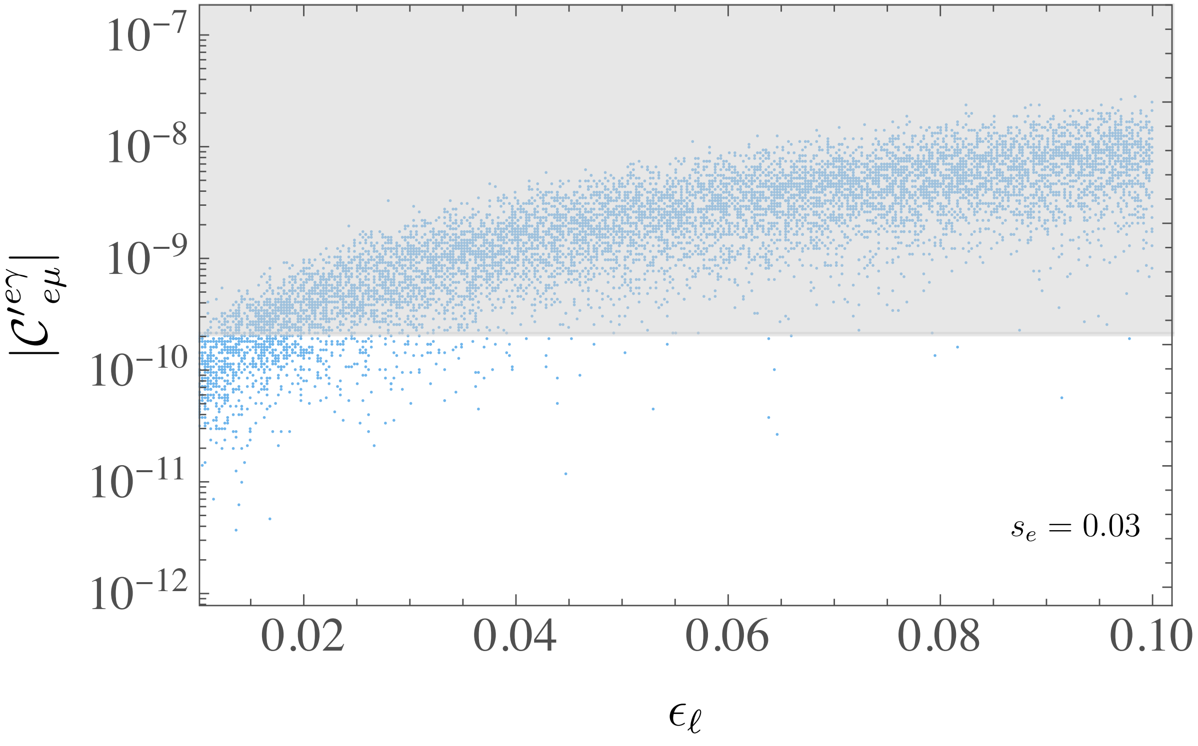
$$\begin{aligned}\underset{\mu\mu}{\mathcal{C}'_{e\gamma}} \simeq\ & |C^{e\gamma}_{\Delta}|\delta_e \left[\frac{1}{c_e} + c_e\epsilon_\ell^2 \left(\left|\frac{C^{e\gamma}_{VV\Delta}}{C^{e\gamma}_{\Delta}}\right| \cos\left(\arg C^{e\gamma}_{VV\Delta}\right) - \left|\frac{C^{y}_{VV\Delta}}{C^{y}_{\Delta}}\right| \frac{s_e^2}{c_e^2} \cos\left(\arg C^{y}_{VV\Delta}\right)\right)\right] \\ & + i|C^{e\gamma}_{\Delta}|c_e\delta_e\epsilon_\ell^2 \left(\left|\frac{C^{e\gamma}_{VV\Delta}}{C^{e\gamma}_{\Delta}}\right| \sin\left(\arg C^{e\gamma}_{VV\Delta}\right) - \left|\frac{C^{y}_{VV\Delta}}{C^{y}_{\Delta}}\right| \sin\left(\arg C^{y}_{VV\Delta}\right)\right) + c_e\delta_e\epsilon_\ell^2\, C_{3rd}\end{aligned}$$

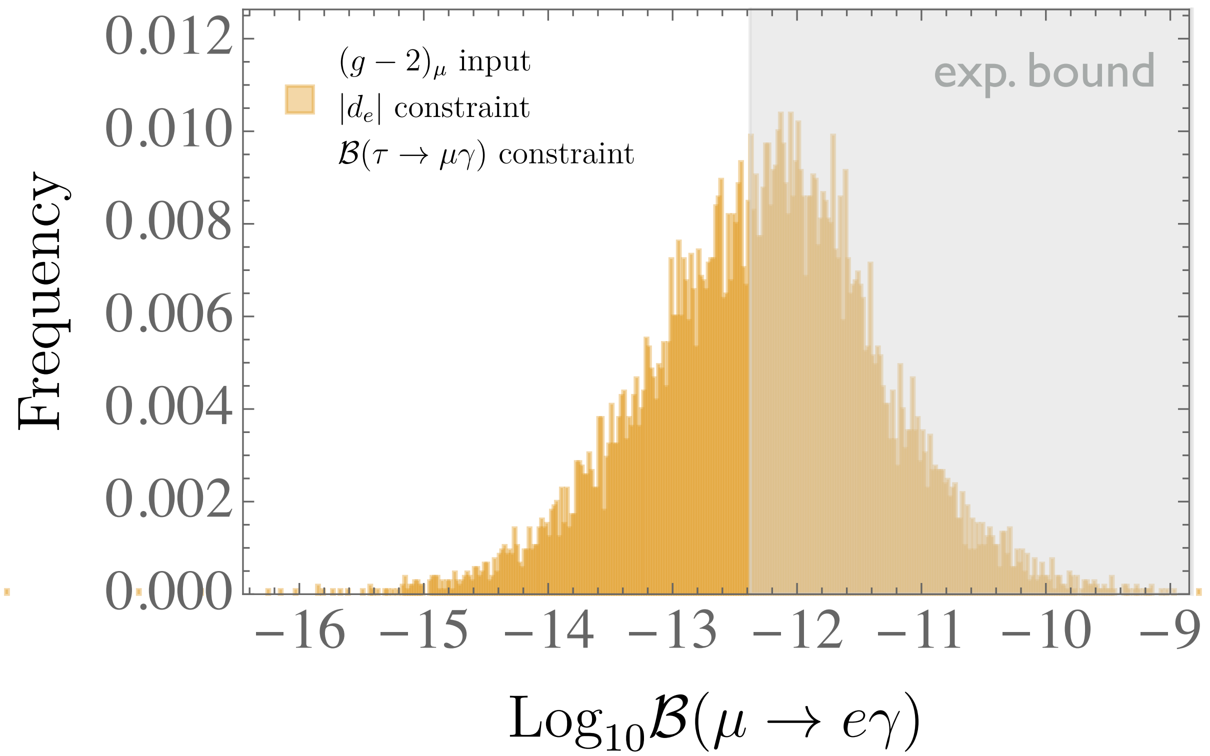
$$\underset{e\mu}{\mathcal{C}'_{e\gamma}} \simeq |C^{e\gamma}_{\Delta}|\, s_e\delta_e\epsilon_\ell^2 \left(\frac{C^{e\gamma}_{VV\Delta}}{C^{e\gamma}_{\Delta}} - \frac{C^{y}_{VV\Delta}}{C^{y}_{\Delta}}\right) + s_e\delta_e\epsilon_\ell^2\, C_{3rd}$$

3rd generation effects

$$C_{3rd} = \frac{C^{y*}_{\Delta}C^{y}_{V} + C^{y*}_{V\Delta}C^{y}}{|C^{y}|^2}\left(\frac{C^{e\gamma}}{C^{y}}C^{y}_{V} - C^{e\gamma}_{V}\right) + \frac{C^{y}_{V}}{C^{y}}\left(\frac{C^{e\gamma}_{V}}{C^{y}_{V}}C^{y}_{V\Delta} - C^{e\gamma}_{V\Delta}\right)$$

muon $(g-2)_\mu$ and $\mu \to e\gamma$





$(g-2)_\mu$ & EDM d_e in U(2)

muon $(g-2)_\mu$ and EDM d_e

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

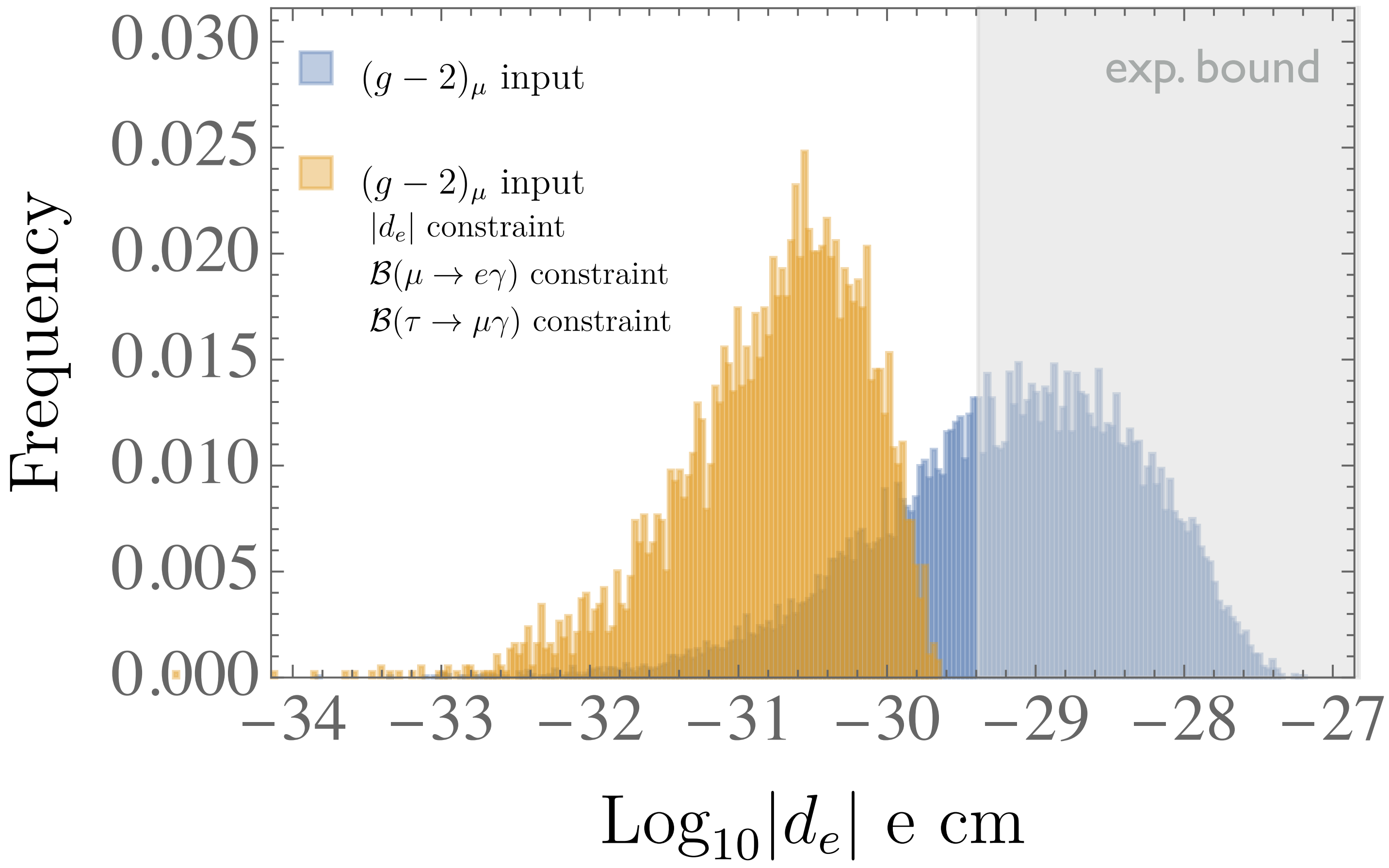
$$\left|\frac{\mathrm{Im}\ \mathcal{C}'_{\underset{ee}{e\gamma}}}{\mathrm{Re}\ \mathcal{C}'_{\underset{\mu\mu}{e\gamma}}}\right| \simeq s_e^2 \frac{\delta'^{\,2}}{\delta}\epsilon_\ell^2\, \mathrm{Im}\left(\frac{C^{e\gamma}_{VV\Delta}}{C^{e\gamma}_{\Delta}} - \frac{C^{y}_{VV\Delta}}{C^{y}_{\Delta}} + \frac{C_{3rd}}{|C^{e\gamma}_{\Delta}|}\right) < 1.8 \times 10^{-8}$$

$\sim 4 \times 10^{-9}$ close to the experimental upper bound

constraint from $\mu \to e\gamma$ more tight

$\vee$

constraint from EDM d_e in U(2)



$(g-2)_\mu$ & $(g-2)_e$ in U(2)

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

muon $(g-2)_\mu$ and electron $(g-2)_e$

U(2) relation $\frac{\mathrm{Re}\,\mathcal{C}'_{\underset{ee}{e\gamma}}}{\mathrm{Re}\,\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}} \approx \frac{\delta'}{\delta} = \frac{m_e}{m_\mu} \simeq 5\times10^{-3}$

$$\Delta a_e = \Delta a_\mu \frac{m_e}{m_\mu} \frac{\mathrm{Re}\,\mathcal{C}'_{\underset{ee}{e\gamma}}}{\mathrm{Re}\,\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}} \approx \boxed{\Delta a_\mu \times \left(\frac{m_e}{m_\mu}\right)^2} \sim 6.2\times10^{-14}$$

naive scaling

vs.

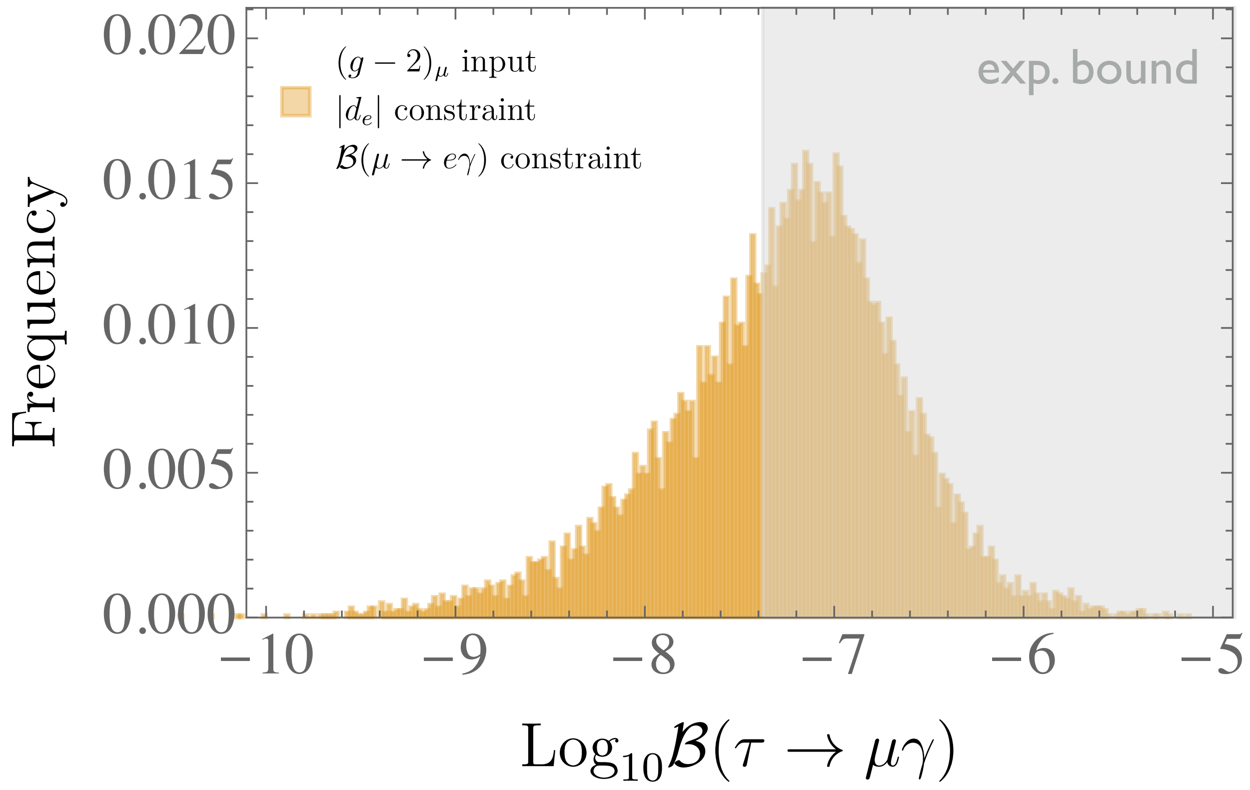
$$\Delta a_e^{Cs} = a_e^{\mathrm{Exp}} - a_e^{\mathrm{SM,CS}} = (-8.8\pm3.6)\times10^{-13}$$
$$\Delta a_e^{Rb} = a_e^{\mathrm{Exp}} - a_e^{\mathrm{SM,Rb}} = (4.8\pm3.0)\times10^{-13}\quad .$$

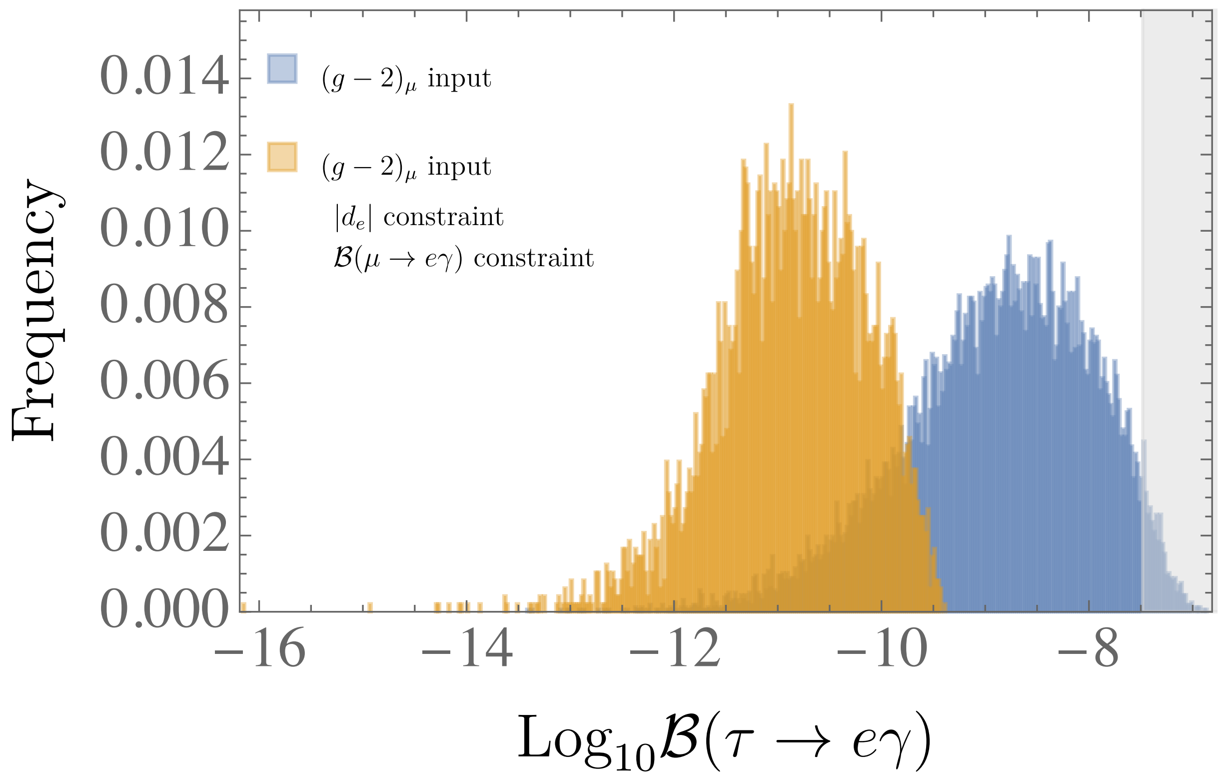
Predicted value is small of one order compared with the present observed one at present
Wait for the precise observation of the fine structure constant to test the framework

$(g-2)_\mu$ & $\tau \to \mu\gamma$, $\tau \to e\gamma$ in U(2)

muon $(g-2)_\mu$ and $\tau \to \mu\gamma$, $\tau \to e\gamma$

$$\mathcal{C}'_{\substack{e\gamma\\ LR}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma\\ ee}} & \mathcal{C}'_{\substack{e\gamma\\ e\mu}} & \mathcal{C}'_{\substack{e\gamma\\ e\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\ \mu e}} & \mathcal{C}'_{\substack{e\gamma\\ \mu\mu}} & \mathcal{C}'_{\substack{e\gamma\\ \mu\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\ \tau e}} & \mathcal{C}'_{\substack{e\gamma\\ \tau\mu}} & \mathcal{C}'_{\substack{e\gamma\\ \tau\tau}} \end{pmatrix}$$





→ Belle II $(BR \sim \mathcal{O}(10^{-9}))$

$(g-2)_\mu$ in U(2)

Tanimoto, KY 2310.16325

U(2) provides partial alignment in flavor space, but not enough for $C_{e\gamma,e\mu} \ll C_{e\gamma,\mu\mu}$

Third family contribution is significant because of non-negligible left-handed 2-3 mixing ϵ_ℓ

Predictions of LFV and EDM

input $(g-2)_\mu$ anomaly

$\mu \to e\gamma$	$\tau \to \mu\gamma$	$\tau \to e\gamma$	EDM d_e
$BR \sim 10^{-13}$	$BR \sim 10^{-8}$	$BR \sim 10^{-11}$	$\lvert d_e/e \rvert \lesssim 10^{-31}\text{cm}$

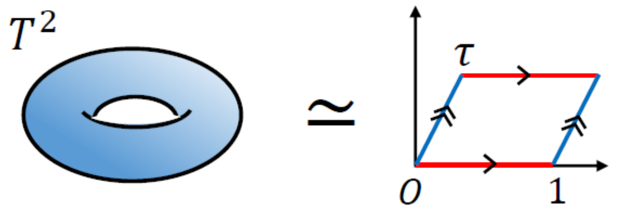
Possibly observations in the near future

$(g-2)_\mu$ in Modular symmetry

Kobayashi, Otsuka, Tanimoto, KY
2204.12325

Other possibility : Modular flavor symmetry

Compactification of the superstring theory



complex
modulus
parameter τ

modular transformation

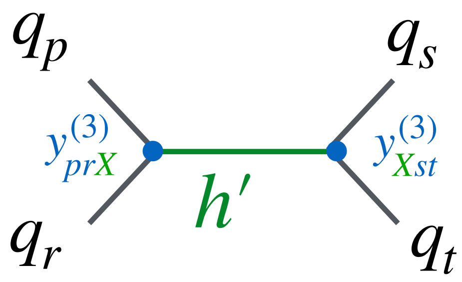
$$\tau \longrightarrow \gamma\tau = \frac{a\tau+b}{c\tau+d}$$

isomorphic

Modular symmetry $\simeq$ Discrete symmetry

← neutrino large mixing angle

Due to string ansatz, strong flavor alignment can be realized without tuning



$\cancel{\propto}$ Yukawa

muon $(g-2)_\mu$ and $\mu \to e\gamma$

strong flavor alignment

$$\left|\frac{C'_{\substack{e\gamma\\ e\mu}}}{C'_{\substack{e\gamma\\ \mu\mu}}}\right| = \frac{\tilde{\beta}_e}{\tilde{\alpha}_e}\left|1-\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right| < 2.1\times10^{-5} \longrightarrow |\delta_\beta-\delta_\alpha| < 1.4\times10^{-3}$$

without tuning between $\delta_{\alpha,\beta}$, $|\delta_\alpha| < \mathcal{O}(10^{-3})\,,$ $|\delta_\beta| < \mathcal{O}(10^{-3})$

Outline

- Introduction of muon g-2
- New physics interpretation of muon g-2
- Flavor symmetry and muon g-2
- **Summary**

Summary

Muon g-2 anomaly provides the most longstanding hint of New Physics

Possible NP :

enhancement mechanism or Light NP particle

strongly constrained by low energy physics and LHC search

NP for muon g-2 anomaly can also lead to potentially effect in leptonic EDMs and LFV

→ need strong alignment in flavor space

Flavor symmetry ?

Intensity frontier is crucial to probe NP