Lattice determination of the NLO HVP contributions to the $(g-2)_\mu$

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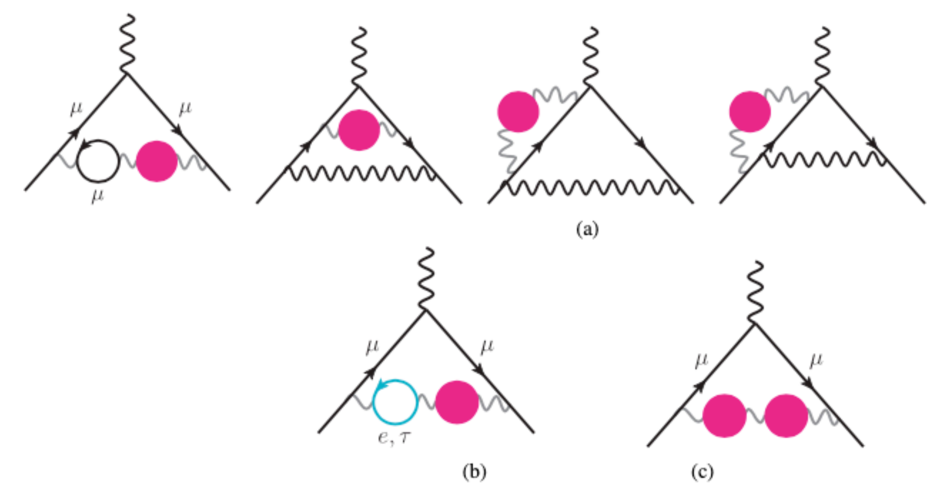
In this work, we present a full computation of the NLO HVP contributions to the muon $g-2$. Starting with a study of the Time-Momentum Representation (TMR) of the corresponding Kernels for the relevant contributions to the numerical implementation of QCD correlators obtained from lattice simulations.



[2]

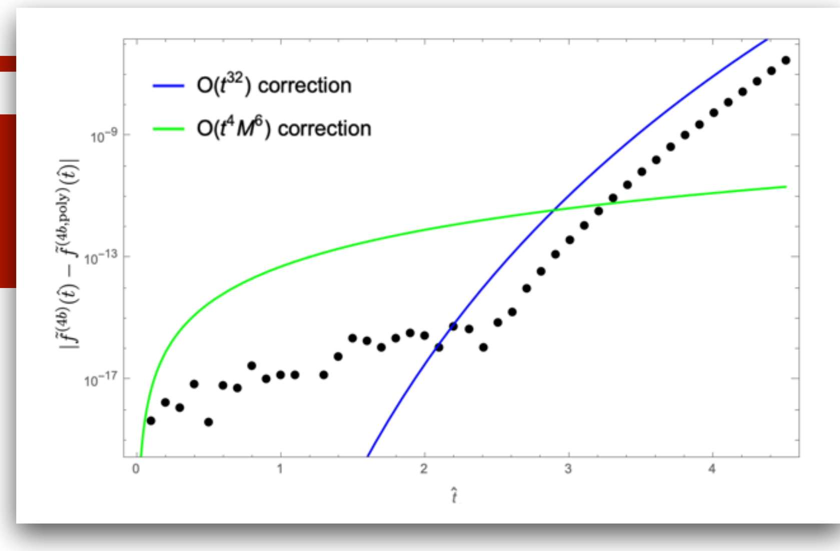
There are essentially three different types of α^{em} subleading HVP contributions to the $(g-2)_\mu$. When comparing with the LO contribution, these can be classified in diagrams **(a)** containing extra photon or muon lines, **(b)** containing a leptonic (*electron or tauon*) loop and **(c)** with an additional QCD insertion.
The TMR integral representation of these Kernels is

$$\tilde{f}^{(4a)}\left(t; m_\mu\right) = \int_0^\infty d\omega^2 \frac{4\pi^2 f_4(\omega^2; m_\mu)}{\omega^2}\left[\omega^2 t^2 - 4\sin^2\frac{\omega t}{2}\right], \qquad \tilde{f}^{(4b)}_l\left(t; m_\mu, m_l\right) = 2\int_0^\infty d\omega^2 \frac{4\pi^2 f_2(\omega^2; m_\mu)}{\omega^2}\left[\omega^2 t^2 - 4\sin^2\frac{\omega t}{2}\right] F^l\left(\omega^2; m_l^2\right),$$

$$\tilde{f}^{(4c)}\left(t, \tau; m_\mu\right) = \int_0^\infty d\omega^2 \frac{16\pi^4 f_2(\omega^2; m_\mu^2)}{\omega^4}\left[\omega^2 t^2 - 4\sin^2\frac{\omega t}{2}\right]\left[\omega^2 \tau^2 - 4\sin^2\frac{\omega \tau}{2}\right],$$

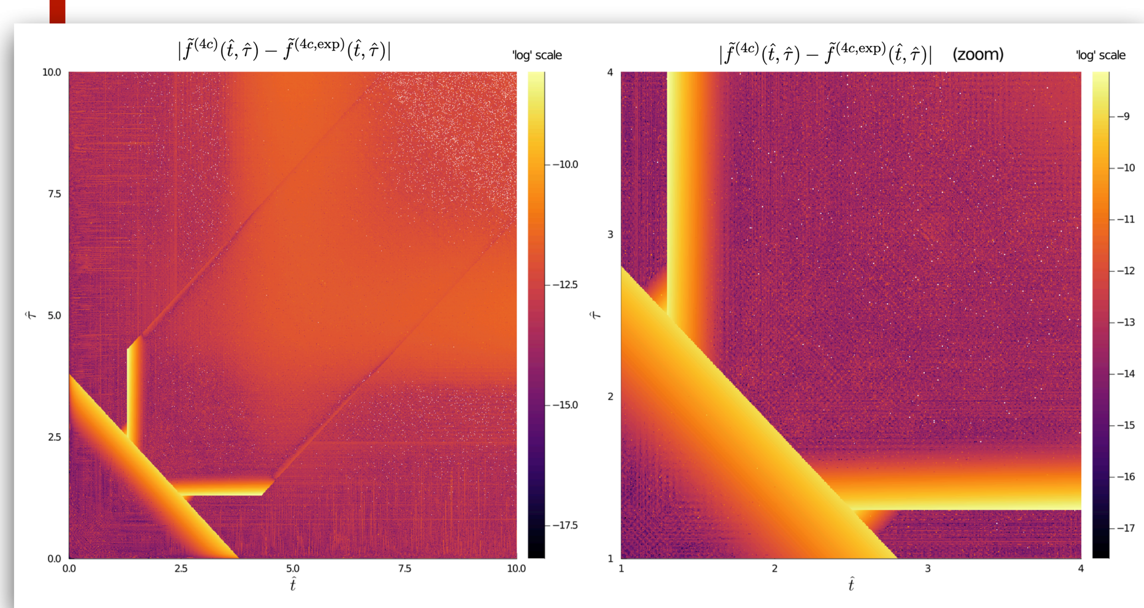
where f_2 and f_4 are the time-like LO and NLO Kernels [1] and F^l is the Lepton loop function [2].

These Kernels are then to be combined with a lattice calculation of the electromagnetic correlator $G(t) = -\frac{1}{3}\sum_{\mu=1}^{3}\sum_{\mathbf{x}\in\Lambda}\langle j_\mu(\mathbf{x}, t) j_\mu(0)\rangle$.



Suitable representation for the NLO Kernels

It is convenient to work with a simplified version of the TMR Kernels, a polynomial representation is best suited for the task. $\tilde{f}^{(4c)}$ can be analytically solved and then expanded around 5 different regions depending on the behaviour of the two Euclidean-time coordinates. For the (a) and (b) contributions, it is not possible to analytically solve the integral and another approach is required.



In [1] Balzani, Laporta and Passera developed an analytic method to obtain a polynomial expansion for contribution (a). First, the integral is conveniently split into small and large ω values which allow for different expansions around $t \sim 0$. For high t values, this approximation fails and another expansion is required.
We follow a similar approach for the electronic (b) contribution $l = e$, where one must first apply the method around a small leptonic mass ratio $M = m_e/m_\mu$ and then expand in the same way around $t \sim 0$.

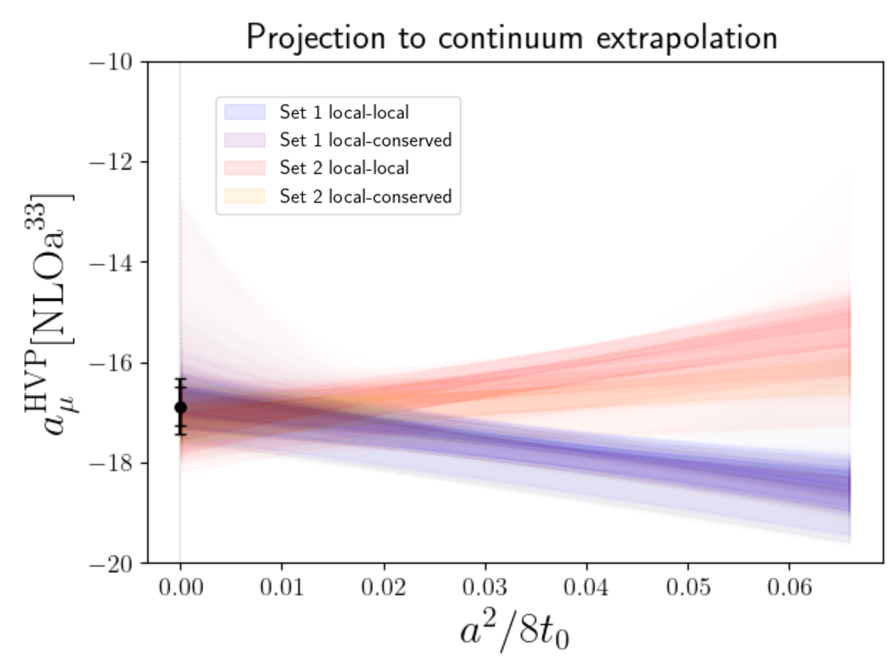
These polynomial representations are valid for all values of the Euclidean time up to a precision of 10^{-8} and can be tested against their numerical representation (shown for (b) in the upper-right corner and for (c) in the bottom-left).

First preliminary results

Eventually, the new Kernel representations are combined with lattice data from 12 CLS ensembles [3] with $N_f = 2+1$ flavours of $\mathcal{O}(a)$-improved Wilson quarks, to obtain a full preliminary determination of the sub-leading hadronic contribution to the $(g-2)_\mu$.
Following a decomposition in the iso-spin basis to simplify renormalisation, the total contribution can be symbolically expressed as

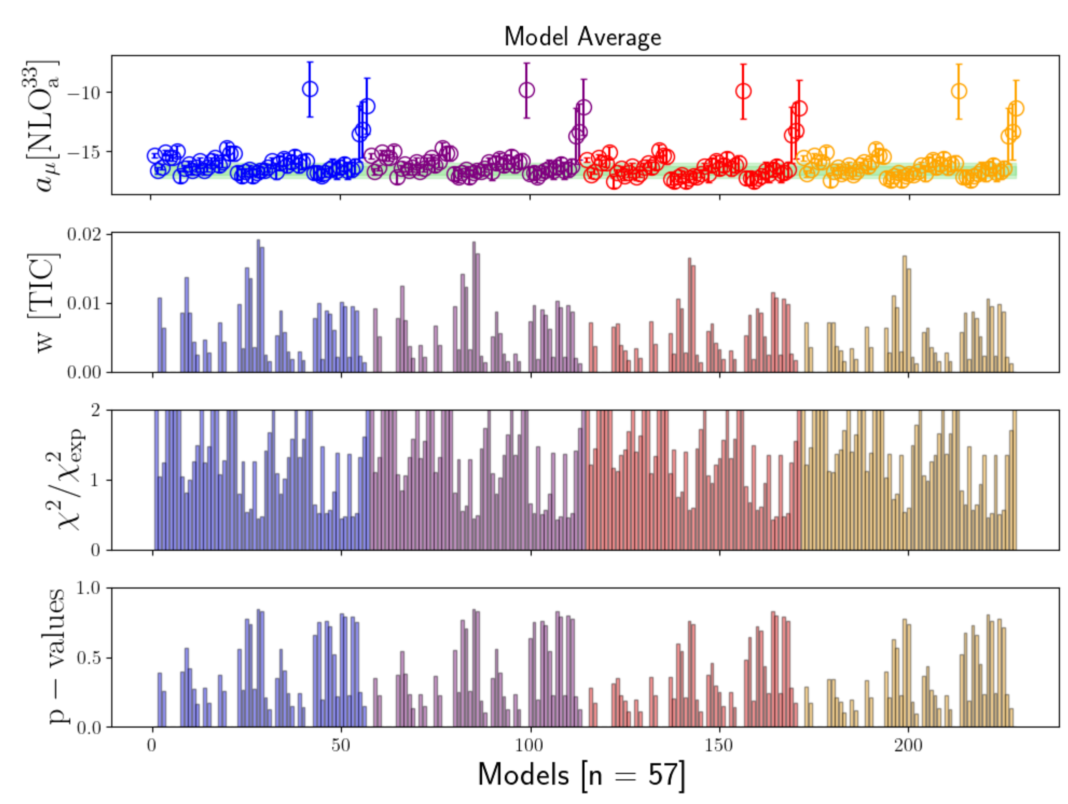
$$a_\mu^{\mathrm{hvp}}[\mathrm{NLO_{a\&b}}] = [\mathrm{Iso-vector}] + \frac{1}{3}[\mathrm{Iso-scalar}] + \frac{4}{9}[\text{charm conn.} + \text{charm disc.}] + \frac{2}{3\sqrt{3}}[\text{Iso-scalar} \leftrightarrow \text{charm}] + \ldots,$$

where the dots correspond to contributions from the bottom and top quarks, negligible at the current statistical precision.



We have applied 2 different sets of improvement coefficients with two discretizations each to constrain the continuum limit. Finite-volume effects are corrected for using the Hansen-Patella method [4]. To better control the signal-to-noise problem in the long distance regime of the correlators we have made use of the Bounding Method [3].

Lastly, we perform combined chiral and continuum extrapolations considering multiple different models. The results from different models are then combined into a weighted average using the Takeuchi Information Criterion (TIC) [5] in order to estimate the systematics arising from the spread of results from different models.



It is convenient to combine contributions (a) and (b) to substantially reduce cutoff effects in the chiral-continuum extrapolation, therefore achieving increased precision.
The preliminary results presented here still have a relatively large uncertainty due to the limited parameter space coverage. In the future, we will add more ensembles and increase statistics significantly.

$$a_\mu^{\mathrm{hvp}}[\mathrm{NLO_{a\&b}}] = -7.96(13)(19) + \frac{1}{3}[-5.50(10)(22)] + \frac{4}{9}[-1.430(16)(56) + 47(22)(12)\times 10^{-5}] + \frac{2}{3\sqrt{3}}[1.7(5.9)(7.2)\times 10^{-5}] = \mathbf{-10.43(15)(21)} \quad [2.47\%]$$

$$a_\mu^{\mathrm{hvp}}[\mathrm{NLO_c}] \overset{*}{=} 0.226(09)(12) + \frac{1}{9}[0.0810(52)(87)] + \frac{16}{81}[0.00213(50)(17)] + \frac{2}{3}[0.1253(60)(95)] + \frac{8}{9}[0.0180(05)(10)] + \frac{8}{27}[0.0087(19)(38)] = \mathbf{0.338(13)(14)} \quad [5.65\%]$$

$$a_\mu^{\mathrm{hvp}}[\mathrm{NLO}] = a_\mu^{\mathrm{hvp}}[\mathrm{NLO_{a\&b}}] + a_\mu^{\mathrm{hvp}}[\mathrm{NLO_c}] = \mathbf{-10.10(14)(21)} = \mathbf{-10.10(25)} \quad [2.50\%]$$

* Notice a different structure for the (c) contribution, this is caused by the $G(t) \times G(\tau)$ product in the integrand.

[1] Elisa Balzani, Stefano Laporta, and Massimo Passera. "Time-kernel for lattice determinations of NLO hadronic vacuum polarization contributions to the muon g-2". In: (June 2024). arXiv: 2406.17940 [hep-ph].
[2] B. Chakraborty et al. "Higher-order hadronic-vacuum-polarization contribution to the muon g-2 from lattice QCD". In: Physical Review D 98.9 (Nov. 2018). issn: 2470-0029.
[3] Antoine Gérardin et al. "Leading hadronic contribution to (g – 2)μ from lattice QCD with Nf = 2 + 1 flavours of O(a) improved Wilson quarks". In: Physical Review D 100.1 (July 2019). issn: 2470-0029.
[4] Maxwell T. Hansen and Agostino Patella. "Finite-Volume Effects in $(g-2)_\mu^{\mathrm{HVP,LO}}$". In: Physical Review Letters 123.17 (Oct. 2019). issn: 1079-7114.
[5] Ethan T. Neil and Jacob W. Sitison. "Improved information criteria for Bayesian model averaging in lattice field theory". In: Physical Review D 109.1 (Jan. 2024). issn: 2470-0029.