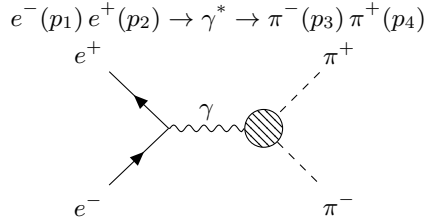


Calculation in sQED and $F \times$ sQED

Pion Vector Form Factor

$$\langle \pi^\pm(p') | j_{em}^\mu(0) | \pi^\pm(p) \rangle = \pm (p' + p)^\mu F_\pi((p' - p)^2)$$

Born approximation



$$\frac{d\sigma_{LO}}{d\cos\vartheta} = \frac{\alpha^2\pi}{4s\beta_e} \beta_\pi^3 (1 - \beta_e^2 \cos^2\vartheta) |F_\pi(s)|^2$$

NLO photonic corrections

$$\frac{d\sigma_{NLO}}{d\cos\theta} = \frac{d\sigma_{LO}}{d\cos\theta} \left(1 + \delta_{SV}^{ISR} + \delta_{SV}^{FSR} + \delta_{SV}^{IFI} \right) + \frac{d\sigma_H}{d\cos\theta}$$

Parton shower algorithm

$$\Pi(\varepsilon, Q^2) = \exp \left\{ -\frac{\alpha}{2\pi} \int_0^{1-\varepsilon} dz P(z) \int d\Omega_k \mathcal{I}(k) \right\}.$$

$$I_+^{QED}(\varepsilon) = \int_0^{1-\varepsilon} dz P_f(z) = -2 \ln \varepsilon - \frac{3}{2} + 2\varepsilon - \frac{1}{2}\varepsilon^2,$$

$$I_+^{sQED}(\varepsilon) = \int_0^{1-\varepsilon} dz P_s(z) = -2 \ln \varepsilon - 2 + 2\varepsilon$$

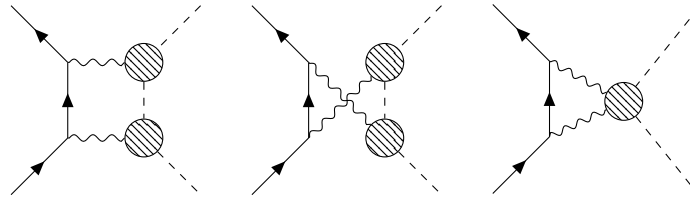
QED+sQED Parton Shower

$$d\sigma_{NLOPS} = F_{SV} \Pi(\varepsilon, Q^2) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=1}^n F_{H,i} \right) |\mathcal{M}_n^{PS}|^2 d\Phi_n$$

Handling the pion composite structure in loops

$$\delta_{V,FF}^{IFI}(\lambda) = \frac{2 \operatorname{Re} \{ F_\pi(s)^* \mathcal{M}_{LO,0}^\dagger \mathcal{M}_V^{IFI}(\lambda) \}}{|F_\pi(s)|^2 |\mathcal{M}_{LO,0}|^2} = \frac{2 \operatorname{Re} \{ F_\pi(s)^* \bar{\delta}_V^i \}}{|F_\pi(s)|^2}$$

FF = GVMD, FsQED



NLO calculation in the GVMD model

GVMD

$$F_\pi^{BW}(q^2) = \sum_{v=1}^{n_r} F_{\pi,v}^{BW}(q^2) = \frac{1}{c_t} \sum_{v=1}^{n_r} c_v \frac{\Lambda_v^2}{\Lambda_v^2 - q^2}$$

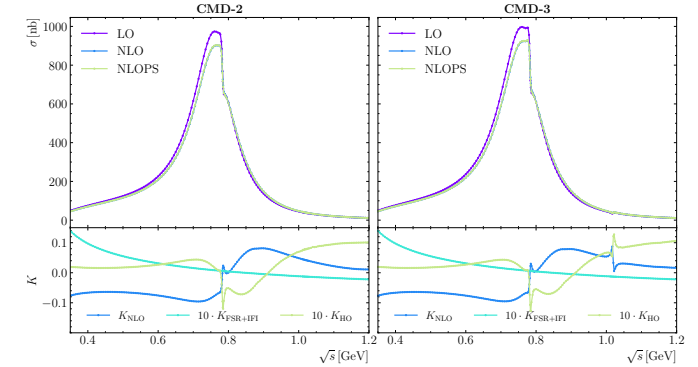
NLO calculation in the FsQED approach

Dispersion relation

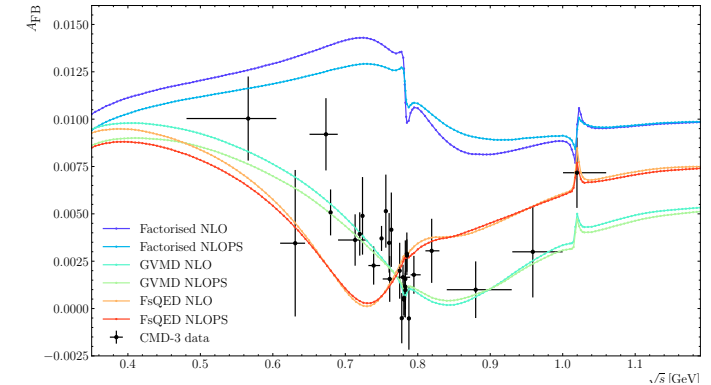
$$F_\pi(q^2) = 1 - \frac{q^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\operatorname{Im} F_\pi(s')}{q^2 - s'}$$

Numerical Results

Results for the integrated cross section



Results for the charge asymmetry



References

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- [2] Gilberto Colangelo, Martin Hoferichter, Joachim Monnard, and Jacobo Ruiz de Elvira. Radiative corrections to the forward-backward asymmetry in $e^+e^- \rightarrow \pi^+\pi^-$. *JHEP*, 08:295, 2022.
- [3] Fedor Ignatov and Roman N. Lee. Charge asymmetry in $e^+e^- \rightarrow \pi^+\pi^-$ process. *Phys. Lett. B*, 833:137283, 2022.