A multi-channel treatment for the pion vector form factor

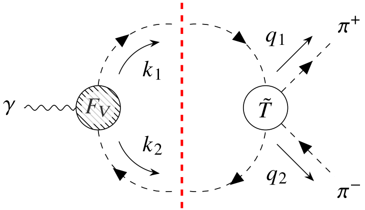
Definition: $\langle \pi^+(q_1)\pi^-(q_2)|\, j_\mu(0)\, |0\rangle = (q_1 - q_2)_\mu F_\pi^V(s)$, where $s = (q_1 + q_2)^2$.

Context: • The leading contribution to the HVP $\Longrightarrow$ relevant to $(g-2)_\mu$,
• Tension between theory and experiment $\Longrightarrow$ more precision needed,
• Tension between the experiments $\Longrightarrow$ inelastic channels are important.

Using dispersion relations,

$$F(s)_{ij} = \frac{1}{2\pi i}\int_{s_{\mathrm{thr}_1}}^{\infty} \mathrm{d}s' \, \frac{\mathrm{disc}\big(F(s')\big)_{ij}}{s' - s - i\epsilon}\,,$$

with $\mathrm{disc}\big(\mathcal{F}(s)\big) = 2iT^*(s)\rho(s)F(s)\,.$



We need a model that:
• preserves analyticity and unitarity, • maps to the Omnès–Muskhelishvili solution at low energies, • provides an accurate high-energy description, • includes contributions from coupled channels, • includes isospin-violating effects.

A two-potential model:

$$V_{\mathrm{B}}(s) = f_0/f(s) \quad \Longrightarrow \quad T_{\mathrm{B}} = \left(1 - V_{\mathrm{B}}\Pi\right)^{-1} V_{\mathrm{B}}\,;$$

$$V_{\mathrm{R}}(s) = -g^T G_{\mathrm{R}}(s) g\,, \quad G_{\mathrm{R}}^{kl}(s) = \frac{\delta_{kl}}{s - m_k^2} \quad \Longrightarrow \quad T_{\mathrm{R}} = \gamma_{\mathrm{out}}\left(1 - V_{\mathrm{R}}\Sigma\right)^{-1} V_{\mathrm{R}}\gamma_{\mathrm{in}}^\dagger\,.$$